Effect of Hydrodynamic Film Lubrication on Vehicles on Wet Tarred Road Pavement

[1]EJEHSON PHILIP SULE, [2]ASHA SATURDAY, [3]OKWUEGO EMEKA CHARLES, [4]JOSEPH MICHEAL IRABODEMEH

[1]Scientific Equipment Development Institute SEDI,P. O .BOX 3205,Enugu ,Enugu State,Nigeria

[2,3,4] National Engineering Design And Development Institute (NEDDI),Nnewi,Anambra State,Nigeria

Abstract-Hydrodynamic lubrication is a medium found advantageous in thrust carriage in Journal bearings. As it saves the life of the Bearing and the rotating shaft by suspending the shaft in space at high speed to disallowed metal rubbing. Hydrodynamic film lubrication has adverse effect on vehicles on wet tarred road. Toppling of vehicles is imminent at sudden application of break(tyre lock) or on high speed on a wet tarred road because of the hydrodynamic film. This offers a high pressure and hydrodynamic lift force that lifts the vehicle besides frictionless condition caused by the film lubrication. Toppling becomes possible as the vehicle is slides off its course, and can cause grievous damage to life, property and vehicles on accident .

Index Terms: Hydrodynamic film lubrication, hydrodynamic pressure and lift force, high speed , sudden break application, wet road pavement, vehicle toppling.

INTRODUCTION

Transportation becomes vital in the quest for knowledge, communication, production, etc. Time space may not be far from the numerous reasons why people drives on high speed. Inspite of these many reasons – recklessness, drunkenness, dizziness, etc. Hydrodynamic film lubrication is also termed 'perfect' or 'thick' film lubrication is not ignored in on wet road pavement. Thin film lubrication is obtained when some of the requirements for thick film lubrications are not wholly met. In the thick film lubrication the sliding surfaces are completely separated by a lubricant, there is no metal to metal contact and the wear is minimized. The process involved in thick film lubrication is primarily one of viscous flow; the frictional resistance being attributed to the shearing of the lubricant.Tribology as the study of friction, wear, and lubrication. Tribological processes are involved whenever one solid slides or rolls against another, as in bearings, cams, gears, piston rings and cylinders, machining and metalworking, grinding, rock drilling, sliding electrical contacts, frictional welding, brakes, the striking of a match, music from a cello, articulation of human synovial joints (e.g., hip joints), machinery, and in numerous less obvious processes (e.g., walking, holding, stopping, writing, and the use of fasteners such as nails, screws, and bolts). friction is the tangential resistance that is offered to the sliding of one solid body over another.Friction is the result of many factors and cannot be treated as something as singular as density or even viscosity. Postulated sources of friction have included (1) the lifting of one asperity over another (increase in potential energy), (2) the interlocking of asperities followed by shear, (3) interlocking followed by plastic deformation or plowing, (4) adhesion followed by shear, (5) elastic hysteresis and waves of deformation, (6) adhesion or interlocking followed by tensile failure, (7) intermolecular attraction, (8) electrostatic effects, and (9) viscous drag.

The coefficient of friction, indicated in the literature by μ or f, is defined as the ratio F/W
where
F = friction force and
W = the normal load.

It is shows that friction is a force and not a property of a solid material or lubricant

Lubrication is a process of reducing friction *and/or* wear (or other forms of surface damage) between relatively moving surfaces by the application of a solid, liquid, or gaseous substance (i.e., a lubricant).The primary function of a lubricant is to reduce friction or wear or both between moving surfaces in contact with each other. Examples of lubricants are wide and varied. They include automotive engine oils, wheel bearing greases, transmission fluids, electrical contact lubricants, rolling oils, cutting fluids, preservative oils, gear oils, jet fuels, instrument oils, turbine oils, textile lubricants, machine oils, jet engine lubricants, air, water, molten glass, liquid metals, oxide films, talcum powder, graphite, molybdenum disulfide, waxes, soaps, polymers, and the synovial fluid in human joints

Hydrodynamic lubrication adverse effect on high speed vehicles at sudden application of break on a wet tarred road is diversitating as a spectrum of lost of lives and properties have been recorded over the world. In hydrodynamic lubrication, the load is supported by the pressure developed due to relative motion and the geometry of the system. In the regime of hydrodynamic or fluid film lubrication, there is no contact between the solids. The film thickness is governed by the bulk physical properties of the lubricants, the most important being viscosity; friction arises purely from shearing of viscous lubricant. The discovery and engineering worlds had not relented on achieving high speed vehicles both on land and air. Effective Break systems had also been worked on quick release responds and dissipation of heat. At high speed there is a percentage lost in weight of moving bodies in motion.A Vehicle in motion has its tyres in relative sliding motion between the tyres surface and the tarred road surface. For stability of the Vehicle, the basic laws of friction must be upheld. Hence, the nature of surfaces in contact becomes important to avoid slippage. In era of 1970s and 80s most Nigeria roads were coarse or rough hence they possessed good grip even on wet condition. But the recent tarred road formation are of black smooth tar and are characterized to be smoother than those of 70s and 80s. Coarse surface will have high tractive force with vehicle tyres that the smooth surface.Wetness of road whether by oil spills or water due to spills or rainfall brought a reduction of friction, the degree of reduction friction indicates the degree of slippage of the surface.

Among other road surface conditions, slippery pavement during precipitation is of great concern to road safety authorities. Some statistics indicate that the number of accidents increases by up to two folds during rainy conditions [1],[2]. Loss of skid resistance affects driver's ability to control vehicle. In addition to increasing the stopping distance while braking, lower skid resistance reduces steering controllability since both braking and steering depend on tire-pavement friction. This means that drivers need to change their driving habits when facing wet driving conditions.

Skid resistance is known to be a function of pavement construction materials [5],[6], pavement roughness [7], and surface conditions [8]. As far as surface conditions are concerned, most of the work on wet pavement skid resistance appears to be dealing with the effect of the presence of precipitation water as a lubricant regardless of the other contaminants [9],[11] . Road pavement contaminants are not unexpected to constitute of wear debris from pavement materials, pollutants from the surrounding environment, hydrocarbons leaking from motor vehicles, carbon particles, other vapors from the exhaust products, tire rubber wear particles, brake pads wear products, and metal wear debris produced by moving vehicle parts.

FACTORS AFFECTING SLIPPAGE OF VEHICLE TIRE

Slippage of motion vehicle types on the road is dangerous for the fact that it can lead to accident due to diversion of the Vehicle from its course, toppling of the vehicle and collision with another vehicle or object on the road. The factors that can lead to slippage or lost of tractive forces between the vehicle tyre and the road are.

(i) Wear of the tyre
(ii) Coarseness or smoothness of the road
(iii) Wetness of the road whether oil spill or water due spills or rainfall.
(iv) Width of tyre
(v) Properties of the lubricant-water or other spills.

Also this could be consider as factors of importance in relation to skid resistance can be assigned to four categories:

a) those related to the pavement — pavement type, micro- and macro-texture;

b) those related to tire — rubber compound, tread design, inflation pressure;
c) those related to substance present on the interface of tire pavement — water, rubber deposits, oil contaminants;
d) those related to operation conditions — mode of operation, vehicle velocity, tire load

The first three groups are the major components of the tire-road pavement interface system: the tire, the road pavement, and the interface contaminant, mainly, water.

The wheel slip (W_s) is defined by the following formula:

$$W_s = \frac{V - R\omega}{V} \qquad (1)$$

Where

W_s: slip;
V: speed;
R: tire radius;
ω: angular velocity of the tire.

STATEMENT OF PROBLEM

Road pavement surfaces of the present days in Nigeria are characterized with smooth tar black surface. This could be thought to offer low wear rate to vehicle tires but the skid resistance is low. Hence ,there is inherent danger to vehicles ,lives and property when the surfaces of these are wet with water. Wet road surface is dangerous to vehicles base on the effect of hydrodynamic lubrication that can spin off vehicles off road thereby leading to accident.

SCOPE

This paper is done to x-rays the adverse effect of hydrodynamic lubrication film on high speed vehicles or vehicles at sudden application of breaks (tyre lock) on wet tarred road surface condition owning to the development of hydrodynamic pressure and hydrodynamic lift force. Focus is also given to analyze the factors that could lead to toppling of vehicles on high speed at sudden application of break or no break application on a wet road pavement.

AIM AND OBJECTIVE

The aims and objective of this work are outlined as follows

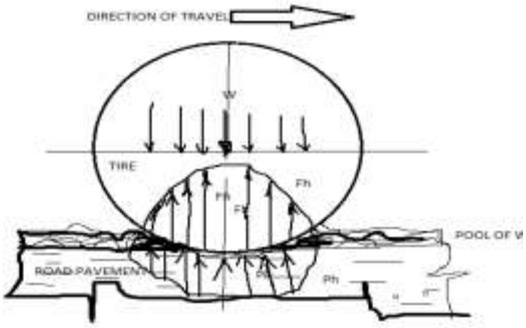
(i) To analyze the effect of hydrodynamic lubrication film on wet tarred road on high speed vehicle or sudden application of break (tyre lock) on a wet tarred road pavement.
(ii) To x-ray mathematically the effect of hydrodynamic pressure and lift force on the toppling of vehicles
(iii) To show other factors that can lead to toppling of vehicles.
(iv) To suggest solutions to this problem.

SIGNIFICANT

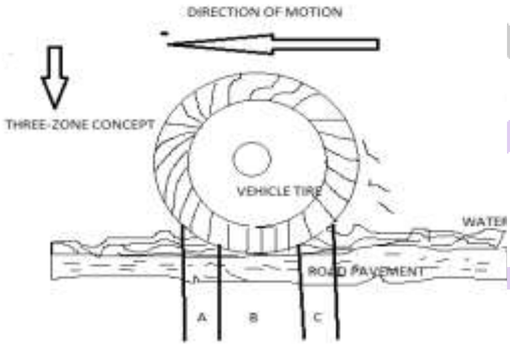
To create awareness and suggestions on the effect of hydrodynamic lubrication on vehicles under wet tarred road pavement condition.

MECHANICS OF WATER-TIRE INTERACTION

This lubrication mode concept clarifies the three-zone concept proposed by Gough (1959). Gough (1959) originally suggested it for sliding locked-wheel traction, and later Moore (1966) carried the concept further to cover the case of a rolling tire. The two figures (a) and (b) demonstrates the phenomenon three zones are illustrated as in (b) of Figure 1.



a. hydrodynamic pressure, P_h, Hydrodynamic lift force, F_h, distribution, weight of the vehicle distributed on the tyre W_z Speed of the vehicle, u_b



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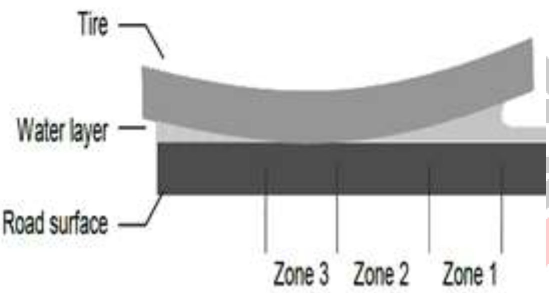


II

b.three –zone concept

A: Sinkage Zone (the tyre step on & splash water)(Hydrodynamic Lubrication)
B: Transition Zone (film generation)(Partial Hydrodynamic Lubrication)
C: Actual Contact Zone (little or nothin film generation)(Boundary Layer Lubrication)

Fig 1.Tire Sliding on Wetted Road Pavement Surface

THREE-ZONE CONCEPT (MOORE 1966)

The three zone concept is a division of the contact length of the tyre and wet road and are explained to show where the maximum hydro dynamic pressure and hydrodynamic lift force that cause toppling of vehicles

ZONE A: SINKAGE, OR SQUEEZE-FILM ZONE

In the pool of water on a road pavement, the forward part of water would normally be considered the contact area under dry conditions floats on a thin film of water, the thickness of which decreases progressively as individual tread elements traverse the contact area. The zone is formed due to the displacement inertia of the intercepted water film. This corresponds to the extreme situation elasto-hydrodynamic lubrication. The frictional force developed is strongly dependent on the bulk properties of the lubricant, mainly the viscosity and velocity gradient in the lubricant film. Hydroplaning occurs when the total hydrodynamic lift force acting on the tire equals the sum of the weight of the tire plus the downward vertical loading upon it (Browne 1975). As either speed or water film thickness increases, the fully developed Zone A would replace both Zone B and Zone C and the tire would eventually appear as skidding on the film of water.

ZONE B: DRAPING, OR TRANSITION ZONE

The draping zone begins when the tire elements, having penetrated the squeeze film, start to drape over the major asperities of the surface and to make contact with the lesser asperities. A mixed lubrication regime exists, which is called partial boundary. Partial dynamic hydroplaning may occur at ordinary speeds, but the uplift forces are not great enough to develop a full dynamic hydroplaning (Balmer and Gallaway 1983).

ZONE C: ACTUAL CONTACT, OR TRACTIVE ZONE

This is the zone where the tire, after draping, having attained a vertical equilibrium position on the surface. In this position, boundary-layer lubrication becomes dominant which results in an intimated contact between the tire and road pavement. The length of this region depends on vehicle velocity. The frictional force is a function of the properties of the contacting solids and of the lubricant at their common interface. The general traction versus speed relationship of tires can be explained on the basis of the three-zone concept. The zone will vary in length according to the water film thickness and traction performance that is mainly related to vehicle speed. On this basis, the wet traction modes can also explained in terms of hydrodynamic, boundary layer and partial boundary lubrication respectively (Veith 1976), which comes from the lubrication theory. The classification of friction mode is very useful for the analysis of the effect of water and travel speed on skid resistance.

SPEED

The influence of vehicle speed on skid resistance is very much dependent upon the properties of the tire and road surface. For any road surface, friction force is basically independent of speed., wet skid resistance decreases dramatically with increasing speed, but that decrease is also related to other factors such as water film thickness, tire tread, pattern and depth, and road pavement properties.

ANALYSIS

ANALOGY OF HYDRODYNAMIC FILM LUBRICATION AS APPLIED TO JOURNAL BEARING.

Journal bearings do not have a constant central axis, hence its eccentric nature provides the shock or thrust absorbing ability during high speed of the shaft to prevent metal robbing that can lead to wear of the bearing materials and the shaft, the bearing is packed with oil grease which is behind the phenomenon – hydrodynamic film lubrication.

the film provides a hydrodynamic pressure on the rotating shaft and suspends it on space to prevent robbing during high speed rotation of the shaft. The pressure built by the film is high enough to carry this thrust or load bearing on shaft. Hence it is greater than the maximum load at high speed to suspend the loaded rotating shaft. This could offer analogy to the tire-wet tarred road pavement interaction ,the pressure built cause a high lift force the lift the vehicle off its course.

APPLICATION ON WET TARRED ROAD

Tire- tarred road pavement skid resistance has long been recognized as the primary factor of controlling vehicle direction and speed, as well as ensuring short braking distances. Numerous studies show that a large number of accidents were directly attributed to the lack of skid resistance between tire and pavement, and accident risk actually increased when the pavement was wet. Wet weather skidding figures predominate among causes of traffic accidents (OECD 1984 Wambold et al. 1986). From the law of friction, the nature of surface in contact, the surface area affects the level of frictional force between the vehicle tyres and the tarred road surface. Hence, good tractive force, or grip between the tyres and the road is attained between the new tyre with grooves and coarse and dried road surfaces. The surface energy possessed by the road surface as well as its frictional ability is reduced by the addition of water [water spill or rainfall] on the surface. The wetness of tarred road becomes significant in discussing hydrodynamic film lubrication for the reduction of surface energy and frictional force on the road. At high speed, vehicles or moving objects loose weights and becomes dynamically inbalance due to the aerodynamic environment they are exposed to; drag and lift forces become prevailing.

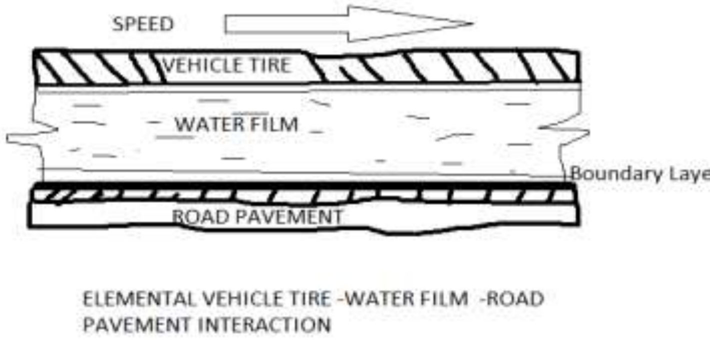
ASSUMPTIONS

In his theoretical analysis, some of Reynolds assumptions will be made:

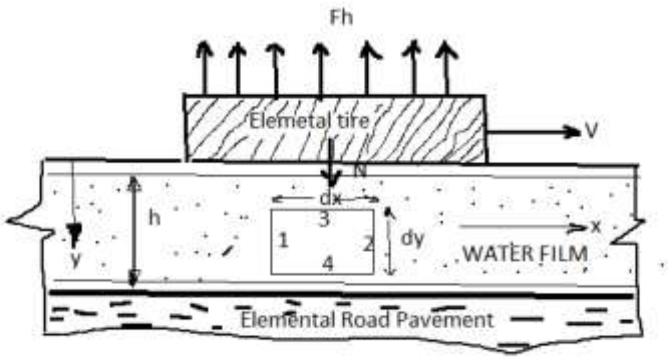
a) The fluid is Newtonian.
b) The fluid is incompressible.
c) The viscosity is constant throughout the film.
d) The pressure does not vary in the axial direction.
. e) The film pressure is constant in the y direction. Thus the pressure depends on the x coordinate only.
f) The velocity of particle of lubricant in the film depends only on the coordinates x and y.

g) The effect of inertial and gravitational force is neglected.
h) The fluid experience laminar flow

Velocity distributions due to relative motion and pressure buildup were developed and added together. The solution of the basic Reynolds equation for a particular bearing configuration results in a pressure distribution throughout the film as a function of viscosity, film shape, and velocity In hydrodynamic lubrication, large forces are developed in small clearances, when the surfaces are slightly convergent and one surface is in motion so that the fluid is "wedged" into the decreasing space
Consider a plane rectangular hard body which slides at some speed over a stationary plane
hard surface and allow the body to have the freedom to rotate around horizontal axis, the body pivots and a wedge shaped of liquid builds up between the plane surfaces. This effect is the
basis of sliding bearing lubrication. It has been shown that the minimum film thickness i.e. the
minimum clearance between the two surfaces is proportional to the square root of the relative
sliding speed. It has also been shown that this proportionality holds for a soft elastomer sliding over a hard surface



a.Hydrodynamic Lubrication (water Wedge Region) in tire-road pavement



b.analysis of hydrodynamic pressure and lift force
Fig2. Pressure and viscous forces acting on an element of lubricant.

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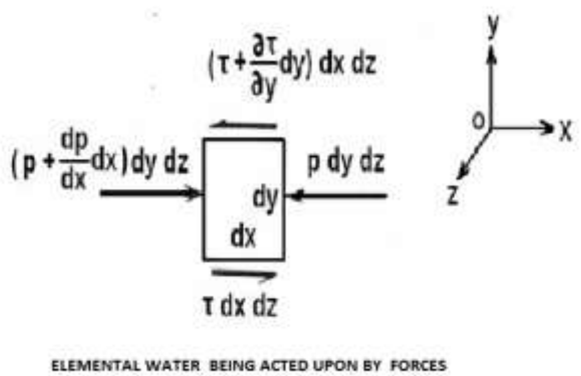

fig 3. Elemental water being acted upon by forces

Forces acting on a fluid element of height dy, width dx, velocity u, and top to bottom velocity gradient du is considered. For the equilibrium of forces in the x direction acting on the fluid element acting on the fluid element in fig.3

$$-Pdydz + \tau dxdz + \left(P + \frac{dP}{dx}\,dx\right) dydz - \left(\tau + \frac{\partial\tau}{\partial y} dy\right) dxdz = 0 \qquad (2)$$

which reduces to

$$\frac{dP}{dx} = \frac{\partial\tau}{\partial y} \quad (3)$$

The viscosity equation given as

$$\mu = \frac{Fh}{AU} \qquad (4)$$

Shear stress due to the fluid flow

$$\tau = \mu \frac{\partial U}{\partial y} (5)$$

From solid mechanics ,the shear stress is

$$\tau = \frac{F}{A} \qquad (6)$$

where the partial derivatives is used since the velocity u depends upon both x and y. Substituting eqn (5) in (6), we get

$$\frac{dP}{dx} = \mu \frac{\partial^2 U}{\partial y^2} \qquad (7)$$

Rearranging the terms, we get

$$\frac{\partial^2 U}{\partial y^2} = \frac{1}{\mu}\frac{dP}{dx} \qquad (8)$$

Holding x constant and integrating twice with respect to y gives

$$\frac{\delta U}{\delta y} = \frac{1}{\mu}\left(\frac{dP}{dx} y + C_1\right) \qquad (9)$$

$$U = \frac{1}{\mu}\left(\frac{dP}{dx}\frac{y^2}{2} + C_1\, y + C_2\right) (10)$$

The assumption of no slip between the lubricants and the boundary surfaces gives boundary conditions enabling C1 and C2 to be evaluated:
u=0 at y=0, u=U at y=h

$$C_1 = \frac{U\mu}{h} - \frac{h}{2}\frac{dP}{dx}(y^2 - hy) + \frac{U}{h} y \qquad (11)$$

$$C_2 = 0 \qquad (12)$$

Substituting the values of C1 and C2 in Equation (9)

$$U = \frac{1}{2\mu}\frac{dP}{dx}(y^2 - hy) + \frac{U}{h} y \qquad (13)$$

Velocity Distribution of the Lubricant Film shown in Fig.4 consists of two terms on the right hand side

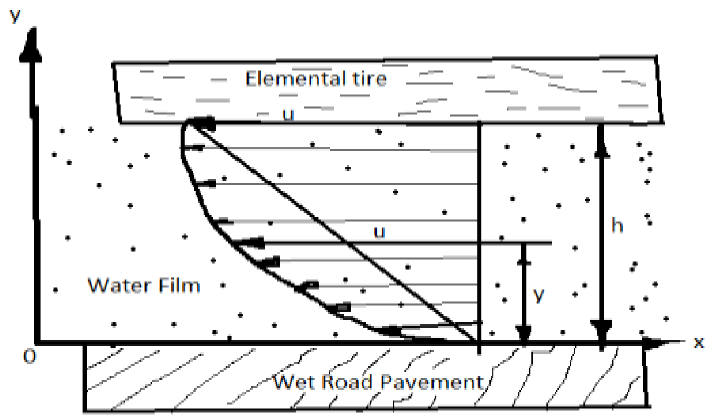

Fig. 4. A representation of Velocity distribution ,U,in the fluid film

$$U = \frac{1}{2\mu}\frac{dP}{dx}(y^2 - hy) + \frac{U}{h} y \qquad (14)$$

This equation (12) describes Velocity Distribution of the Lubricant Film has a parabolic and a linear –dashed part

At the section when pressure is a maximum and the velocity gradient is linear.

$$\frac{dP}{dy} = 0 \qquad (15)$$

Hence

$$U = \frac{u}{h} y \ (16)$$

At this maximum hydrodynamic pressure ,there is a maximum hydrodynamic lifting force that is responsible to lifting the vehicle out of its course and topples or slides to result to accident. A hydrodynamic upward thrust ahead the contact of the tyre is determined by water layer depth(thickness) and the velocity of the travelling vehicle.This could also be analysed by considering the interaction of the moving surface with the lubricant in contact, the equations governing the flow of the lubricant in the roughness interaction zone [16] by considering rough surface road pavement can be written as:

$$\eta \frac{\partial^2 u_1}{\partial z^2} - k(u_1 - U) = \frac{\partial P}{\partial x} \qquad (17)$$

In $\frac{h}{2} \leq z \leq \frac{h}{2} + a$

Where

Hydrodynamic pressure= P

$$k = \frac{\mu}{\emptyset} \quad (18)$$

$u_1$be the velocity, parameter$\emptyset$ corresponds to permeability of porous matrix at the surfaces and η be the effective viscosity. The pressure P is assumed to be constant across the film. The second term $k(u_1 - U)$ on the left hand side of equation (17) corresponds to be resistance offered by the roughness asperities to the flowing fluid. It is assumed to be proportional to the relative velocity.

The continuity equation in the lubricant zone is given by:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (19)$$

Integrating (13) across the film thickness from to we get,

$$\frac{\partial Q}{\partial x} + [V]_0^h = 0 \quad (20)$$

Using (12) in the above equation, the generalized Reynolds equation for the lubrication between two rough surfaces with squeezing velocity is obtained as:

$$\frac{\partial}{\partial x}\left[F_1 \frac{\partial P}{\partial x}\right] + \frac{\partial}{\partial x}\left[U\left(a + \frac{h}{2}\right)\right] = -V \quad (21)$$

The relationship between the effective retardation forces and the ratio of the wet contact zone and the total tire foot print is derived below:

$$F_V = PA_C \quad (22)$$

where

Fv=the total vertical force at the tire road interface;
P=tire pressure;
Ac=the total area of nominal tire foot print
However Ac can be also written as

$$A_c = A_{wet} + A_{dry} \quad (23)$$

As it is the area of dry contact within the total tire/road contact area which gives rise to the frictional forces at the tire/road interface ;

$$FH = \mu_0 F_V(dry) \quad (24)$$

where

FH=friction force at tire/road interface as horizontal component.
Fv(dry)=effective vertical force in the dry zone of contact area;
μ0=braking force coefficient between tire and road in dry contact conditions
However

$$F_V(dry) = PA_{dry} \quad (25)$$

Therefore

$$FH = \mu_0 PA_{dry} \quad (26)$$

and re-arranging equation(23) gives

$$A_{dry} = A_c\left(1 - \frac{A_{wet}}{A_c}\right) \quad (27)$$

Therefore substituting equation (27) into equation (25) and (26) gives

$$FH = \mu_0 PA_c\left(1 - \frac{A_{wet}}{A_c}\right) \quad (28)$$

$$F_V(dry) = PA_{dry}$$

$$F_V(dry) = PA_c\left(1 - \frac{A_{wet}}{A_c}\right) \quad (29)$$

Dividing across by Fv gives the retardation coefficient i.e.

$$\mu = \frac{FH}{F_V} \quad (30)$$

finally after substitution and re-arranging the final equation which represents the proposed model is obtained

$$\mu = \mu_0\left(1 - \frac{A_{wet}}{A_c}\right) \quad (31)$$

The time of contact tc is equal to the length of the contact patch divided by the speed V.
This means that when the tire is stationary the contact time is infinite. Consequently the ratio
of the wet zone of the contact area, Awet, to the total contact area, Ac, is equal to zero. Therefore
the full dry value of friction is available when the tire is locked or break is applied meaning when the tire is stationary.

RECOMMENDATION AND CONCLUSSION

This research work reveals the following recommendations:

Modern tarred road pavements are constructed with very smooth surface with tar especially in Nigeria we suggest the tarred road surface should be made coarse or rough to reduce hydrodynamic effect under wet condition and this standard should be maintained by Road constructions companies to ensure high skid resistance.

Vehicles owner should use good tyres with grooves for good grip and reduction of hydrodynamic effect under wet condition.

Traffic signals- Speed limit and signs should be indicated in areas like slopes that are liable to effect of hydrodynamic lubrication under wet condition.

Although pavement contamination with water alone reduces tire-pavement skid resistance, other pavement contaminants play an important rule in further deterioration of this resistance . hence all tarred road pavement should be cleared of these contaminants.

Campaign should be organized for road users on the adverse effect of hydrodynamic lubrication on tarred road pavements under wet condition via national dailies and TV stations.

In a nutshell, in spite of the numerous advantages of hydrodynamic lubrications in engineering and tribology, it is dangerous to life ,property and vehicles since it can lead to accident. In fact ,many accidents on rainy season has a greater percent caused as a result of road wetness as it is dangerous for sudden application of break or tyre lock or being on high speed. Moderate speed, new tyre or good tyre conditions, rough or coarse road surface initiations, and making road surfaces free of contaminants are factors that can curb effect of hydrodynamic lubrication.

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