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NUMERICAL SOLUTIONS OF FUZZY DIFFERENTIAL EQUATIONS BY MILNE'S METHOD

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ABSTRACT: In this dissertation, we interpret a fuzzy differential equation by using seikkala derivative .We investigate the problem of finding a numerical approximation of solutions. Adams-fifth order Predictor-Corrector method and Milne's fifth order Predictor –Corrector method are implemented and their error analysis which guarantees pointwise convergence is discussed. The methods applicability is illustrated by solving a first order fuzzy differential equation. Finally we compare the solutions obtained by Adams-fifth order Predictor-Corrector method and milne's fifth order Predictor-Corrector method.

INTRODUCTION

The concept of fuzzy derivative was first introduced by S.L Chang. L.A Zadeh in [6] it was followed up by D.Dubois, H.Prade in [7].who defined and used the extension principle. The fuzzy differential equation and the initial value problem were regularly treated by O.Kaleva in [8] and by S.Seikkala in [5]. The numerical method for solving fuzzy differential equations is introduced by O.Kaleva in [8] by the standard Eular method and by authors in [1,2] by Taylor method. In this work we replace the fuzzy differential equation by its parametric form and then solve numerically the new system. Which consider the two classic ordinary differential equations with initial condition.

The structure of this chapter organizes as follows: In section 1.2 some basic definitions and results are brought. Milne's fourth order predictor-corrector methods for solving fuzzy differential equations are proposed. Milne's fourth order predictor-corrector method Algorithm is discussed in section 3.3Two examples are presented in section 2And conclusion in section 3

1.1.Fuzzy sets:

The idea of fuzzy set was introduced by Lotfi Zadeh in 1960s as a means of handling uncertainity that is due to imprecision or vagueness rather than to randomness. Fuzzy sets were taken up with interests by engineers, computer scientists and operations researchers. While mathematicians have been involved with the development of fuzzy sets from the very beginning, it has really been in recent years only that fuzzy sets have started receiving serious consideration from a wider mathematical community. Many interesting mathematical problems are coming to the for and the mathematical foundations of the subject are firmly established and now it has emerged as an independent branch of applied Fuzzy sets are considered with respect to a nonempty base set X of elements of interest. The essential idea is that each element $x \in X$ is assigned a membership grade $u(x)$ taking values in $[0,1]$, with $u(x) = 0$ corresponding to non-membership, $0 < u(x) < 1$ to partial membership, and $u(x) = 1$ to full membership. According to Zadeh a fuzzy subset of X is a nonempty subset $\{(x, u(x)) : x \in X\}$ of $X \times [0,1]$ for some function $u: X \to [0,1]$. The function u itself used for the fuzzy set.

1.2. Definitions and Basic properties

Let $P_K(R^n)$ denote the family of nonempty compact convex substes of R^n. Addition and scalar multiplication in $P_K(R^n)$ as usual. Let A and B be two nonempty bounded subsets of (R^n). The distance between A and B is defined by the Housdroff metric,

$$d(A,B) = max\{\sup_{a\in A} \inf_{b\in B} \|a-b\|, \sup_{b\in B} \inf_{a\in A}$$

where$\|.\|$ denote the usual Euclidean norm in R^n. Then it is clear that $(P_K(R^n), d)$ becomes a complete metric space.

We denote the Housdroff semimetric by $\rho(A,B) = \sup_{a\epsilon A} \inf_{b\epsilon B} \|a-b\|$.It is clear that $\rho(A,B) = 0 \Leftrightarrow A \subset B$ and $\rho(A,C) \le \rho(A,B) + \rho(B,C)$, where A, B, C are nonempty bounded subsets of R^n and $\bar{B}$ denote the closure of B.

Also $d(A,B) = max[\rho(A,B), \rho(B,A)]$ and $\rho(A,B) = 0 \Leftrightarrow \bar{A} = \bar{B}$.

A fuzzy subset of R^n is defined in terms of a membership function which assigns. to each point $x\epsilon R^n$,a grade of membership in fuzzy set. Such a membership function $u: R^n \to I = [0,1]$ is used to denote the corresponding fuzzy set.

For each $\alpha\epsilon[0,1]$, the α-level set $[u]^{\alpha}$ of a fuzzy set u is subset of points $x\epsilon R^{n}$ with membership grade $u(x)$ of atleast α,that is

$$[u]^{\alpha}=\{x\epsilon R^{n}: u(x)\geq\alpha\}$$

The support $[u]^{0}$of a fuzzy set is then defined as the union of all its level sets, that is,

$$[u]^{0}=\overline{\bigcup_{\alpha\epsilon[0,1]}[u]^{\alpha}}.$$

An inclusion property follows immediately from the above definitions.

2.Adams method

2.1 Convergence and stability

To integrate the system given in equation (12) from t_0 a prefixed T> t_0 the interval $[t_0,T]$ will be replaced by a set of discrete equally spaced gird point $t_0<t_1<t_2<\cdots<t_N=T$ which the exact solution $(\underline{y}(t,\alpha),\overline{y}(t,\alpha))$.The exact and approximate solutions at $t_n, 0\leq n\leq N$ are denoted by $y_n(t,\alpha)=\underline{y}_n(t,\alpha),\overline{y}_n(t,\alpha))$, and $y_n(t,\alpha)=(\underline{y}_n(t,\alpha),\overline{y}_n(t,\alpha))$,respectively.The grid points which the solution is calculated are $t_n=t_0+nh, h=\frac{(T-t_0)}{N}, 1\leq n\leq N$.

From (11),the polygon curves

$$\underline{y}(t,h,\alpha)=\{[t_0,y_0(\alpha)],[t_1,\underline{y}_1(\alpha)],\ldots,[t_N,\underline{y}_N(\alpha)]\}$$

wAre the Adams-Moulton approximates to $\underline{y}(t,\alpha)$ and $\overline{y}(t,\alpha)$,respectively,over the interval $t_0\leq t\leq t_N$. The following lemmas will be applied to show convergence of these approximates,i.e.

$$\mathrm{Lim}_{h\to 0}\,\underline{y}\,(t,h,\alpha)=\underline{y}\,(t,\alpha),$$

$$\lim_{h\to 0}\overline{y}\,(t,h,\alpha)=\overline{y}(t,\alpha)$$

Theroem 2.3

For arbitrary fixed α :0$\leq\alpha\leq 1$, the Adams Moultan four step approximates of converges to the exact solution $\underline{y}(t,\alpha),\overline{y}(t,\alpha)$ for $\underline{y},\overline{y}\in C^{5}[t_0,T]$.

Table 2..4

α	Adams-5		Exact solution	Error in Adams-5	
0.1	2.106668505	3.024088534	3.024088534	8.7741×10^{-8}	1.2595×10^{-7}
0.2	2.174625553	2.990110011	2.990110011	9.0571×10^{-8}	1.2453×10^{-7}
0.3	2.242582602	2.956131488	2.956131488	9.3401×10^{-8}	1.2312×10^{-7}
0.4	2.310539650	2.922152966	2.922152966	9.6231×10^{-8}	1.2171×10^{-7}
0.5	2.378496699	2.888174443	2.888174443	9.9062×10^{-8}	1.2028×10^{-7}
0.6	2.446453748	2.854195919	2.854195920	1.0189×10^{-7}	1.1887×10^{-7}
0.7	2.514410796	2.820217397	2.820217398	1.0472×10^{-7}	1.1745×10^{-7}
0.8	2.582367845	2.786238990	2.786238874	1.0755×10^{-7}	1.1604×10^{-7}
0.9	2.650324893	2.752260466	2.752260351	1.1038×10^{-7}	1.1462×10^{-7}
0.10	2.718281942	2.718281942	2.718281528	1.1321×10^{-7}	1.1321×10^{-7}

3.Milne's method.

Definition 3.1.

An m-step method for solving the initial-value peoblem is one whose difference equation for finding the approximation $y(t_{i+1})$ at the mesh point t_{i+1} can be represented by the following equation:

$$y(t_{i+1})=a_{m-1}\mathrm{y}(t_i)+a_{m-2}\mathrm{y}(t_{i-1})+\cdots+a_0\mathrm{y}(t_{i+1-m})\qquad(3.1)$$

$+h\{b_m f(t_{i+1}, y_{i+1}) + b_{m-1} f(t_i, y_i) + \cdots + b_0 f(t_{i+1-m}, y_{i+1-m})\}$,

For $i = m-1, m, \ldots, N-1$, such that $a = t_0 \le t_1 \le \cdots \le t_N = b$.

$h = \frac{(b-a)}{N} = t_{i+1} - t_i$, and $a_0, a_1, \ldots, a_{m-1}, b_0, b_1, \ldots, b_m$ are constants with the starting values

$y_0 = \alpha_0, y_1 = \alpha_1, y_2 = \alpha_2, \ldots, y_{m-1} = \alpha_{m-1.}$
When $b_m = 0$, the method is known as explicit,since (3.1) gives y_{i+1} explicit in terms of previously determined values.When $b_m \neq 0$, the method is know as implicit,since y_{i+1} occurs on both sides of (3.1) and is specified only implicitly.

With consideration definition 3.1, the multistep method is

Milne's explicit five-step method:

$$y_0 = \alpha_0, y_1 = \alpha_1, y_2 = \alpha_2, y_3 = \alpha_3, y_4 = \alpha_4$$

$$y_{i+1} = y_{i-4} + \frac{h}{144}[95f(t_{i-4}, y_{i-4}) - 50f(t_{i-3}, y_{i-3}) + 600f(t_{i-2}, y_{i-2}) - 350f(t_{i-1}, y_{i-1}) + 425f(t_i, y_i)],$$

Where $i = 4,5, \ldots, N-1$.

Milne's implicit four-step method:

$y_1 = \alpha_1, y_2 = \alpha_2, y_3 = \alpha_3, y_4 = \alpha_4,$

$$y_{i+4} = y_i + \frac{h}{90}[29f(t_{i+1}, y_{i+1}) + 124f(t_i, y_i) + 24f(t_{i-1}, y_{i-1}) + 4f(t_{i-2}, y_{i-2}) - 4f(t_{i-1}, y_{i-1})].$$

Where$i = 4,5, \ldots, N-1$.

Definition 3.2. Associated with the difference equation

$y_{i+1} = a_{m-1} y_i + a_{m-2} y_{i-1} + \cdots + a_0 y_{i+1-m} + hF(t_i, h, y_{i+1}, y_i, \ldots, y_{i+1-m})$. (3.2)

$$y_0 = \alpha, y_1 = \alpha_1, \ldots, y_{m-1} = \alpha_{m-1},$$

Is a polynomial,called the characteristic polynomial of the method given by

$p(\lambda) = \lambda^m - a_{m-1}\lambda^{m-1} - a_{m-2}\lambda^{m-2} - \cdots - a_1\lambda - a_0$.

If $|\lambda| \le 1$ for each $i = 1,2, \ldots, m$, and all roots with absolute value 1 are simple roots,then the difference method is said to satisfy the root condition.

ALGORITHM: 3.3

Fix $k \in z^+$.To approximate the solution of following fuzzy initial value problem.

$x_k' = f(t_{k,i}, x(t_{k,i}), \lambda_k(x_k))$

$$\underline{y}^\alpha(t_{k,i-1}) = \underline{\alpha}_0, \underline{y}^\alpha(t_{k,i}) = \underline{\alpha}_1, \underline{y}^\alpha(t_{k,i+1}) = \underline{\alpha}_2, \underline{y}^\alpha(t_{k,i+2}) = \underline{\alpha}_3, \underline{y}^\alpha(t_{k,i+3}) = \underline{\alpha}_4$$

$$\overline{y}^\alpha(t_{k,i-1}) = \overline{\alpha}_0, \overline{y}^\alpha(t_{k,i}) = \overline{\alpha}_1, \overline{y}^\alpha(t_{k,i+1}) = \overline{\alpha}_2, \overline{y}^\alpha(t_{k,i+2}) = \overline{\alpha}_3, \overline{y}^\alpha(t_{k,i+3}) = \overline{\alpha}_4.$$

Positive integer N_k is chosen.

Step 1. Let$h = \frac{t_{k+1} - t_k}{N_k}$.

$\underline{w}^\alpha(t_{k,0}) = \underline{\alpha}_0, \underline{w}^\alpha(t_{k,1}) = \underline{\alpha}_1, \underline{w}^\alpha(t_{k,2}) = \underline{\alpha}_2, \underline{w}^\alpha(t_{k,3}) = \underline{\alpha}_3, \underline{w}^\alpha(t_{k,4}) = \underline{\alpha}_4,$

$$\overline{w}^\alpha(t_{k,0}) = \overline{\alpha}_0, \overline{w}^\alpha(t_{k,1}) = \overline{\alpha}_1, \overline{w}^\alpha(t_{k,2}) = \overline{\alpha}_2, \overline{w}^\alpha(t_{k,3}) = \overline{\alpha}_3, \overline{w}^\alpha(t_{k,4}) = \overline{\alpha}_4,$$

Step 2. Let$i = 1$,

Step 3. Let

$$\underline{w}^{(0)\alpha}(t_{i+4}) = \underline{w}^{\alpha} + \frac{h}{144}\left[95\underline{f}^{\alpha}(t_{i-1}, w(t_{i-1})) - 50\overline{f}^{\alpha}(t_i, w(t_i)) + 600\underline{f}^{\alpha}(t_{i+1}, w(t_{i+1})) - 350\overline{f}^{\alpha}(t_{i+2} w(t_{i+2})) + 425\underline{f}^{\alpha}(t_{i+3}, w(t_{i+3}))\right].$$

$$\overline{w}^{(0)\alpha}(t_{i+4}) = \overline{w}^{\alpha}(t_{i-1}) + \frac{h}{144}[95\underline{f}^{\alpha}(t_{i-1}, w(t_{i-1}) - 50\overline{f}^{\alpha}(t_i, w(t_i)) + \quad 600\overline{f}^{\alpha}(t_{i+1}, w(t_{i+1})) - 350\underline{f}^{\alpha}(t_{i+2}, w(t_{i+2})) + 425\overline{f}^{\alpha}(t_{i+3}, w(t_{i+3}))].$$

Step 4. Let $t_{i+4} = t_0 + (i+4)h$.

Step 5. Let

$$\begin{cases} \underline{w}^{\alpha}(t_{i+3}) = \underline{y}^{\alpha}(t_{i+2}) + \dfrac{h}{90}[9\underline{f}^{\alpha}(t_{i+3}, w(t_{i+3})) + 124\underline{f}^{\alpha}(t_{i+2}, w(t_{i+2}) \\ \qquad +24\overline{f}^{\alpha}(t_{i+1}, w(t_{i+1})) + 4\underline{f}^{\alpha}(t_i, w(t_i)) - \overline{f}^{\alpha}(t_{i-1}, w(t_{i-1}))], \\ \overline{w}^{\alpha}(t_{i+3}) = \overline{w}^{\alpha}(t_{i+2}) + \dfrac{h}{90}[29\overline{f}^{\alpha}(t_{i+3}, w(t_{i+3})) + 124\overline{f}^{\alpha}(t_{i+2}, w(t_{i+2})) \\ \qquad +24\underline{f}^{\alpha}(t_{i+1}, w(t_{i+1})) + 4\overline{f}^{\alpha}(t_i, w(t_i)) - \underline{f}^{\alpha}(t_{i-1}, w(t_{i-1}))]. \end{cases}$$

Step 6.$i = i + 1$

Step 7. If $i \le N - 4$ go to step 3.

Step 8. Algorithm will be completed and $(\underline{w}^{\alpha}(t_{k+1}), \overline{w}^{\alpha}(t_{k+1})$ approximates real value of $(\underline{x}^{\alpha}(t_{k+1}), \overline{x}^{\alpha}(t_{k+1})$.

Theorem 3.3 For arbitrary fixed $\alpha : 0 \le \alpha \le 1$, the Milne's expilicit four step approximates of (4.9) converges to the exact solution $\underline{y}(\text{t}, \alpha), \overline{y}(t, \alpha) for\ \underline{y}, \overline{y} \in c^5[t_0, T]$.

Example: 3.4 Consider the fuzzy initial value problem,

$$y'(t) = y(t), \qquad t \in I = [0,1],$$

$$y(0) = [0.75 + 0.25\alpha, 1.125 - 0.125\alpha], \quad 0 < \alpha < 1$$

$$y(0,1) = [(0.75 + 0.25\alpha)e^{0.1}, (1.125 - 0.125\alpha)e^{0.1}],$$

$$y(0,2) = [(0.75 + 0.25\alpha)e^{0.2}, (1.125 - 0.125\alpha)e^{0.2}],$$

$$y(0,3) = [(0.75 + 0.25\alpha)e^{0.3}, (1.125 - 0.125\alpha)e^{0.3}],$$

$$y(0,4) = [(0.75 + 0.25\alpha)e^{0.4}, (1.125 - 0.125\alpha)e^{0.4}],$$

The exact solution at $t = 1$ is given by

$$Y(1; \alpha) = [(0.75 + 0.25\alpha)e, \&\&\& \qquad (1.125 - 0.125\alpha)e], \quad 0 < \alpha < 1.$$

by using the Milne's-fifth order predictor-corrector method the following results are obtained:

Table 3.5

α	Milne's-5		Exact solution	Error in Milne's-5	
0.1	0.856507462	1.229502646	1.229502646	-2.5646e-14	-2.5646e-14
0.2	0.881136735	1.215688010	1.215688010	-2.6423e-14	-2.6423e-14
0.3	0.911766007	1.201873373	1.201873373	-2.7311e-14	-2.7311e-14
0.4	0.939395280	1.188058737	1.188058737	-2.8089e-14	-2.8089e-14
0.5	0.967024553	1.174244100	1.174244100	-2.8977e-14	-2.8977e-14
0.6	0.994653826	1.160429464	1.160429464	-2.9754e-14	-2.9754e-14
0.7	1.022283099	1.146614828	1.146614828	-3.0420e-14	-3.0420e-14
0.8	1.049912372	1.132800191	1.132800191	-3.1530e-14	-3.1530e-14
0.9	1.077541645	1.118985555	1.118985555	-3.2196e-14	-3.2196e-14
0.10	1.105170918	1.103170918	1.103170918	-3.3085e-14	-3.3085e-14

CONCLUSION :

In this chapter the iterative solutions of for finding the numerical solution of fuzzy differential equations are provided. Comparison of solutions of table 2.4 and 3.5 shows that the method proposed here gives better solution of Milne's method.

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