THE GENERALIZED HYERS –ULAM –RASSIAS STABILITY OF QUADRATIC FUNCTIONAL EQUATIONS WITH TWO VARIABLES

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ABSTRACT: In this paper,we consider functional equations involving a two variables examine some of these equations in greater detail and we study applications of cauchy's equation.using the generalized hyers-ulam-rassias stability of quaradic functional equations finding the solution of two variables(quaradic functional equations)

1.INTRODUCTION

We achieve the general solution and the generalized Hyers-Ulam-Rassias and Ulam-Gavruta-Rassias stabilities for quadratic functional equations

$$f(ax+by)+f(ax-by) = \left(\frac{b(a+b)}{2}\right)f(x+y) + \left(\frac{b(a+b)}{2}\right)f(x-y) + (2a^2-ab-b^2)f(x) + (b^2-ab)f(y) \quad (1)$$

where a, b are nonzero fixed integers with $b \neq \pm a, -3a$, and

$$f(ax+by)+f(ax-by) = 2a^2 f(x) + 2b^2 f(y) \quad (2)$$

for fixed integers a, b with $a, b \neq 0$ and $a \pm b \neq 0$.

In 1940, Ulam[19] proposed the stability problem for functional equations in the following question regarding to the stability of group homomorphism.

Let $(G_1, .)$ be a group and let $(G_2, *)$ be a metric group with the metric $d(.,.)$. Given $\in > 0$, there exist a $\delta > 0$, such that if a mapping $h: G_1 \to G_2$ satisfies the inequality

$$d(h(x.y), h(x) * h(y)) < \delta,$$

for all $x, y \in G_1$, then there exists a homomorphism $H: G_1 \to G_2$ with

$d(h(x), H(x)) < \in$, for all $x \in G_1$

In other words, under the conditions does a homomorphism exist near an approximately homomorphism generally, the concept of stability for a functional equation comes up when we the functional equation is replaced by an inequality which acts as a perturbation of that equation. Hyers [7] answered to the question affirmatively in 1941 so if $f: E \to E$ such that

$$\|f(x+y) - f(x) - f(y)\| \leq \delta, \quad \text{for all } x, y \in E, \quad (3)$$

and for some $\delta > 0$ where E,E are Banach spaces; then there exists a unique additive mapping $T: E \to E$ such that

$$\|f(x) - T(x)\| \leq \delta, \text{ for all } x \in E. \quad (4)$$

However, if $f(tx)$ is a continuous mapping at $t \in \mathbb{R}$ for each fixed $x \in E$then T is linear. In 1950, Hyers's theorem was generalized by Aoki for additive mappings and independently, in 1978, by Rassias [15] for linear mappings considering the Cauchy difference controlled by sum of powers of norms. This stability phenomenon is called the Hyers-Ulam-Rassias stability.

On the other hand, Rassias [15,16] considered the Cauchy difference controlled by a product of different powers of norm. However, there was a singular case; for this singularity a counterexample was given by Gavruta. This stability phenomenon is called the Ulam-Gavruta-Rassias stability. In addition, J.M. Rassias considered the mixed product–sum of powers of norms control function. This stability is called JM Rassias mixed product-sum stability.

The functional equation

$$f(x+y) + f(x-y) = 2f(x) + 2f(y), \quad (5)$$

is related to symmetric biadditive function and is called a quadratic functional equation naturally, and every solution of the quadratic equation (3.1.3) is said to be a quadratic function. It is well known that a function f between two real vector spaces is quadratic if and only if there exists a unique symmetric biadditive function B such that $f(x) = B(x, x)$ for all x where

$$B(x,y) = \frac{1}{4}\left(f(x+y) - f(x-y)\right)$$

(see [17]). Skof proved Hyers-Ulam-Rassias stability problem for quadratic functional equation for a class of functions $f: A - B$, where A is normed space and B is a Banach space, (see [17]. Cholewa [3] noticed that Skof's theorem is still true if relevant domain A alters to an abelian group. In 1992, Czerqik proved the Hyers-Ulam-Rassias stability of (1.3), Grabiec [6] generalized the result mentioned above.

Throughout this chapter, assume that a, b are fixed integers with $a, b \neq 0$, we introduce the following functional equations, which are different from

$$f(ax+by) + f(ax-by) = \frac{b(a+b)}{2} f(x+y) + \frac{b(a+b)}{2} f(x-y) + (2a^2 - ab - b^2) f(x) + (b^2 - ab) f(y), \quad (6)$$

where $b \neq \pm a, -3a$, and (1.3)

$$f(ax+by) + f(ax-by) = 2a^2 f(x) + 2b^2 f(y), \quad (7)$$

where $b \neq \pm a$.

1.1 Banach Space

A Banach space isa vector space X over the field R of real numbers, which is equipped with a norm and which is complete with every Cauchy sequence, $\{x_n\}$ in X, there exists an element x in X such that

$$\lim_{n \to \infty} X_n = x,$$

(or equivalently)

$$\lim_{n\to\infty}\|x_n - x\|_x = 0.$$

The vector space structure allows one to relate the behaviour of Cauchy sequences to that of converging series of vectors. A normed space X is a banach space. If and only if each absolutely convergent series X converges.

$$\sum_{n=1}^{\infty}\|V_n\|x < \infty \;\; => \sum_{n=1}^{\infty} V_n \; converges\, in\;\; X.$$

Completeness of a normed space preserved if the given norm is replaced by an equivalent one.

All norms on a finite – dimensional vector space are equivalent. Every finite – dimensional normed space over R or C is a banach space.

1.2. Solution of (1.5), (1.6)

Let X and Y be real vector spaces. We here present the general solution of (1.5), (1.6).

Theorem 1.2.1

A function $f: X \to Y$ satisfies the functional equation (1.3) if and only if $f: X \to Y$ satisfies the functional equation (1.5). Therefore, every solution of functional equation (1.5) is also a quadratic function.

Proof.

Let f satisfy the functional equation (1.3). Putting $x = y = 0$ in (1.3), we get $f(0) = 0$. Set $x = 0$ in (1.3) to get $f(-y) = f(y)$. Letting $y = x \; and \; y = 2x$ in (1.3), respectively, we obtain that $f(2x) = 4f(x)$ and $f(3x) = 9f(x)$ for all $x \in X$. By induction, we lead to $f(kx) = k^2 f(x)$ for all positive integers k. Replacing x and y by $2x + y$ and $2x - y$ in (1.3), respectively, gives

$f(2x + y) + f(2x - y) = 8f(x) + 2f(y)$ for all $x, y \in X$.

Using (1.3) and (1.2), we lead to

$f(2x + y) + f(2x - y) = 2f(x + y) + 2f(x - y) + 4f(x) - 2f(y)$ (1.2.2)

for all $x, y \in X$. Suppose that $k \neq 0$ is a fixed integer by using (1.2.2), we get $kf(+y) + kf(-y) - 2kf(x) - 2kf(y) = 0$for all $x, y \; \epsilon \; X$. (1.2.3)

get Using (1.2.2) and (1.2.3) we obtain

$f(2x + y) + f(2x - y) = (2 + k)f(x + y) + (2 + k)f(x - y) + 2(2 - k)f(x) - 2(1 + k)f(y)$ (1.2.4)

for all $x, y \in X$. Replacing x and y by $3x + y$ and $3x - y$ in (1.3), respectively, then using (1.3) and (2.3), we have

$f(3x + y) + f(3x - y) = (3 + k)f(x + y) + (3 + k)f(x - y) + 2(6 - k)f(x) - 2(2 + k)f(y)$ for all $x, y \in X$. (1.2.5)

By using the above method, by

$$f(ax + y) + f(ax - y) = (a + k)f(x + y) + (a + k)f(x - y) + 2(a^2 - a - k)f(x) - 2(a1)f(y)$$

for all $x, y \in X$ (1.2.6)

induction, we infer that and each positive integer $a \geq 1$.

for a negative integer $a \leq -1$, replacing a by $-a$ one can easily prove the validity of (2.6). Therefore (1.2.3) implies (1.2.6) for any integer $a \neq 0$. First,

$f(bx + y) + f(bx - y) = (b + k)f(x + y) + (b + k)f(x - y) + 2(b^2 - b - k)f(x) - 2(b + k - 1)f(y)$ (1.2.7)

it is noted that (1.2.6) also implies the following equation for all integers $b \neq 0$. Setting $y = 0$ in (1.2.7) gives $f(bx) = b^2 f(x)$. Substituting y with by into (1.2.7), one gets

$$(b + k)f(x + by) + (b + k)f(x - by) = b^2 f(x + y) + b^2 f(x - y) - 2(b^2 - b - k)f(x) + 2b^2(b + k - 1)f(y)$$

for all $x, y \in X$ (1.2.8)

Replacing y by in (1.2.6). We observe that

$$f(ax + by) + f(ax - by) = (a + k)f(x + by) + (a + k)f(x - by) + 2(a^2 - a - k)f(x) - 2(a + k - 1)f(by)$$

for all $x, y \in X$ (1.2.9)

Hence, according to (1.2.8) and (1.2.9), we get

$(b + k)f(ax + by) + (b + k)f(ax - by) = b^2(a + k)f(x + y) + b^2(a + k)f(x - y) + 2\big(a^2(b + k) - b^2(a + k)\big)f(x) - 2b^2(a - b)f(y)$ for all $x, y \in X$. (2.10)

In particular, if we substitute $k = b$ in (1.2.10) and dividing it by $2b$, we conclude that f satisfies (1.2.5)

Let f satisfy the functional equation (1.2.5), for nonzero fixed integers a, b with $b \neq \pm a, -3a$. Putting $x = y = 0$ in (1.2.5), we get

$$(2a^2 - ba + b^2 - 2)f(0) = 0,$$

So

$$\left(2a - \tfrac{b+\sqrt{16-7b^2}}{2}\right)\left(a - \tfrac{b-\sqrt{16-7b^2}}{4}\right)f(0) = 0,$$

but since $a, b \neq 0$ and $b \neq \pm a, -3a$,

therefore $f(0) = 0$. Setting $y = 0$ in (1.5) gives $f(ax) = a^2 f(x)$ for all $x \in X$. Letting $y = -y$ in (1.5), we get

$$f(ax - by) + f(ax + by) = \frac{b(a + b)}{2}f(x - y) + \frac{b(a + b)}{2}f(x + y) + (2a^2 - ab - b^2)f(x) + (b^2 - ab)f(-y)$$

(2.1) for all $x, y \in X$. (2.13)

If we compare (1.5) with (2.13), then since $a, b \neq 0$ and $b \neq \pm a, -3a$, we conclude that $f(-y) = f(y)$ for all $y \in X$. Letting $x = 0$ in (1.5) and using the evenness of f give $f(by) = b^2 f(y)$ for all $y \in X$. Therefore for all $x \in X$, we get $f(abx) = a^2 b^2 f(x)$. Replacing x and y by bx and ay in (1.5), respectively, we have

$$a^2 b^2 f(x + y) + a^2 b^2 f(x - y) = \frac{b(a + b)}{2}f(bx + ay) + \frac{b(a + b)}{2}f(bx - ay) + b^2(2a^2 - ab - b^2)f(x) + a^2(b^2 - ab)f(y)$$

(1.2.14)

for all $x, y \in X$. On the other hand, if we interchange x with y in

$$f(ay + bx) + f(ay - bx) = \frac{b(a + b)}{2}f(y + x) + \frac{b(a + b)}{2}f(-x) + (2a^2 - ab - b^2)f(y) + (b^2 - ab)f(x)$$

for all $x, y \in X$. (1.2.15)

(1.2.5), we obtain but since f is even, it follows

$$f(bx + ay) + f(bx - ay) = \frac{b(a + b)}{2}f(x + y) + \frac{b(a + b)}{2}f(-y) + (b^2 - ab)f(x) + (2a^2 - ab - b^2)f(y)$$

for all $x, y \in X$. (1.2.16)

Hence, according to (1.2.14) and from (1.2.15) that

$$a^2b^2f(x+y)+a^2b^2f(x-y) = \frac{b(a+b)}{2}\left[\frac{b(a+b)}{2}(f(x+y) + f(x-y)) +(b^2-ab)f(x)+(2a^2-ab-)f(y)\right] +b^2(2a^2-ab-b^2)f(x)+ a^2(b^2-ab)f(y) \quad (1.2.17)$$

for all $x,y \in X$. So from (1.2.17)

$$\frac{b^2}{4}(4a^2-(a+b)^2)\big(f(x+y)+f(x-y)\big) = \frac{b^2}{2}(3a^2-2ab-b^2)f(x) +\frac{b^2}{2}(3a^2-2ab-b^2)f(y)$$

we have (2.16), we obtain that for all $x,y \in X$. But since $a,b \neq 0$ and $b \neq \pm a, -3a$, we conclude that

$$f(x+y)+f(x-y)=2f(x)+2f(y) \text{for all } x,y \in X. \quad (1.2.19)$$

Therefore, f satisfies (1.2.3)

Theorem 1.2.2

A function $f: X \to Y$ satisfies the functional equation (1.1.3) if and only if $f: X \to Y$ satisfies the functional equation (1..1.6). Therefore, every solution of functional equation(1.1.6) is also a quadratic function.

Proof. If f satisfies the functional equation (1.2.3), then f satisfies the functional equation (1.2.5). Now combining (1.1.3) with (1.1.5), we have

$$f(ax+by)+f(ax-by) = \frac{b(a+b)}{2}\big(2f(x)+2f(y)\big) + (2a^2-ab-b^2)f(x) +(b^2-ab)f(y) \quad \text{for all } x,y \in X.$$

So from (1.2.20), we conclude that f satisfies (1.1.6). Let f satisfy the functional equation (1.1.6) for fixed integers a,b with $a \neq 0, b \neq 0$ and $a \pm b \neq 0$. Putting $x=y=0$ in (1.6), we get $(2(a^2+b^2)-2)f(0)=0$, and since$a \neq 0, b \neq 0$, therefore $f(0)=0$. Setting $y=0$ in (1.1.6) gives $f(ax)=a^2f(x)$ for all $x \in X$. Letting $y := -y$ in (1.1.6), we have

$$f(ax-by)+f(ax+by)=2a^2f(x)+2b^2f(-y) \text{ for all } x,y \in X. \quad (1.2.21)$$

If we compare (1.2.6) with (1.2.21), then since $a,b \neq 0$ and $a \pm b \neq 0$, we obtain that $f(-y)=f(y)$ for all $y \in X$. Letting $x=0$ in (1.2.6) and using the evenness of f gives $f(by) = b^2f(y)$ for all $y \in X$. Therefore for $x \in X$, we get $f(abx)=a^2b^2f(x)$. Replacing x and y by bx and ay in (1.6), respectively, we have

$$f(abx-aby)+f(abx+aby)=2a^2f(bx)+ 2b^2f(ay) \text{for all } x,y \in X. \quad (2.22)$$

Now, by using $f(ax)=a^2f(x), f(bx)=b^2f(x)$and $f(abx)=a^2b^2f(x)$, it follows from (2.22) that

$$f(x+y)+f(x-y)=2f(x)+2f(y) \quad \text{for all } x,y \in X. \quad (1.2.23)$$

Which completes the proof of the theorem.

Corollary 2.3 *(Proposition 2.1). A function $f: X \to Y$ satisfies the following functional equation:*

$$f(ax+y)+f(ax-y)=2a^2f(x)+2f(y) \textit{for all } x,y \in X. \quad (1.2.24)$$

If and only if $f: X \to Y$ satisfies the functional equation (1.3) for all $x,y \in X$.

Proof. Assume that b=1 in functional equation (1.1.6) and apply Theorem (.2.2).

3.3. Stability

We now investigate the generalized Hyers-Ulam-Rassias and Ulam-Gavruta-Rassias stabilities problem for functional equations (1.5), (1.6). From this point n, let X be a real vector and let Y be a Banach space. Before taking up the main subject, we define he difference operator

$$\Delta_f(x,y)=f(ax+by)+f(ax-by) -\frac{b(a+b)}{2}f(x+y) -\frac{b(a+b)}{2}f(x-y) -(2a^2-ab-b^2)f(x)-(b^2-ab)f(y) > \Delta_f: X \times X \to Y \text{ by} \quad (2.18)\ (3.1)$$

for all $x,y \in X$ and a,b fixed integers such that $a,b \neq 0$ and $a \pm b \neq 0$ where $f: X \to Y$ is a given function.

$$\tilde{\varphi}(x) = \sum_{i=\frac{1-j}{2}}^{\infty} \frac{1}{a^{2ij}}\varphi(a^{ij}x,0) < \infty$$

$$\lim_{n\to\infty} \frac{1}{a^{2nj}}\varphi(a^{nj}x, a^{nj}y) = 0 \quad (3.2)$$

Theorem 3.3.1

Let $j \in \{-1,1\}$ be fixed, and let $\varphi: X \times X \to [0,\infty)$ be a function such that for all $x,y \in X$. Suppose that $f: X \to Y$ be a function satisfies

$$\|\Delta_f(x,y)\| \le \varphi(x,y) \text{for all } x,y \in X.$$

Furthermore, assume that $f(0)=0$ in (3.3.4) for the case j=1. Then there exists a unique quadratic function $Q: X \to Y$ such that

$$(2.20)\ \|f(x)-Q(x)\| \le \frac{1}{2a^{1+j}}\tilde{\varphi}\left(\frac{x}{a^{(1-j)/2}}\right), \quad \text{for all } x \in X.$$

Proof. For $j=1$, putting $y=0$ in (3.4), we have $\|2f(ax)-2a^2f(x)\| \le \varphi(x,0)$ for all $x \in X$.

$$\text{So } \left\|f(x)-\frac{1}{a^2}f(ax)\right\| \le \frac{1}{2a^2}\varphi(x,0) \text{ for all } x \in X. \quad (3.7)$$

Replacing x by ax in (3.3.7) and dividing by a^2 and summing the resulting inequality with (3.7), we get

$$\left\|f(x)-\frac{1}{a^4}f(a^2x)\right\| \le \frac{1}{2a^2}\left(\varphi(x,0)+\frac{\varphi(ax,0)}{a^2}\right) \text{ for all } x \in X. \quad (3.8)$$

Hence

$$\left\|\frac{1}{a^{2k}}f(a^kx)-\frac{1}{a^{2m}}f(a^mx)\right\| \le \frac{1}{2a^2}\sum_{i=k}^{m-1}\frac{1}{a^{2i}}\varphi(a^ix,0)$$

nonnegative integers m and k with $m > k$ and for all $x \in X$. It follows from (3.2) and (3.9) that the sequence $\{(1/a^{2n})f(a^nx)\}$ is a Cauchy sequence for all $x \in X$. Since Y is complete, the sequence $\{(1/a^2)f(a^nx)\}$ converges. So one can define the function $Q: X \to Y$ by

$$Q(x) = \lim_{n\to\infty}\frac{1}{a^{2n}}f(a^nx) \text{for afor all } x \in X \quad (3.10)$$

for all $x \in X$. By (3.3.3) for j=1 and (3.3.4)

$$\|\Delta_Q(x,y)\| = \lim_{n\to\infty}\frac{1}{a^{2n}}\|\Delta_f(a^nx,a^ny)\| \le \lim_{n\to\infty}\frac{1}{a^{2n}}\varphi(a^nx,a^ny)=0 \quad (3.11)$$

for all $x,y \in X$. So $\Delta_Q(x,y)=0$. By Theorem (2.1), the function $Q: X \to Y$ is quadratic. Moreover, letting $k=0$ and passing the limit $m \to \infty$ in (3.9), we get the inequality (3.5) for $j=1$.

Now, let $Q: X \to Y$ be another quadratic function satisfying (.1.5) and (3.5).
Then we have

$$\|Q(x) - Q(x)\| = \frac{1}{a^{2n}}\|Q(a^n x) - Q(a^n x)\|$$
$$\leq \frac{1}{a^{2n}}(\|Q(a^n x) - f(a^n x)\| + \|Q(a^n x) - f(a^n x)\|) \quad (3.12)$$
$$\leq \frac{1}{a^2 a^{2n}}\tilde{\varphi}(a^n x, 0),$$

which tends to zero as $n \to \infty$ for all $x \in X$. So we can conclude that $Q(x) = Q(x)$ for all $x \in X$. This proves the uniqueness of Q.
Also, for $j = -1$, it follows from (3.3.6) that

$$\left\|f(x) - a^2 f(\tfrac{x}{a})\right\| \leq \tfrac{1}{2}\varphi(\tfrac{x}{a}, 0) \text{ for all } x \in X.$$

Hence

$$\left\|a^{2k}(f\left(\frac{x}{a^k}\right) - a^{2m} f(\frac{x}{a^m})\right\| \leq \frac{1}{2}\sum_{i=k}^{m-1} a^{2i}\varphi\left(\frac{x}{a^{i+1}}, 0\right) \quad (3.3.14)$$

for all nonnegative integers m and k with $m > k$ and for all $x \in X$.It follows from (3.3.14) that the sequence $\{a^{2n} f(x/a^n)\}$ is a Cauchy sequence for all $x \in X$. Since Y is complete, the sequence $\{a^{2n} f(x/a^n)\}$ converges. So one can define the function $Q: X \to Y$ by

$$Q(x) = \lim_{n\to\infty} a^{2n} f(\frac{x}{a^n}) \text{ for all } x \in X. \quad (3.15)$$

By (3.3) for j=-1 and (3.4),

$$\|\Delta_Q(x, y)\| = \lim_{n\to\infty} a^{2n}\left\|\Delta_f(\frac{x}{a^n}, \frac{y}{a^n})\right\| \leq \lim_{n\to\infty} a^{2n}\varphi(\frac{x}{a^n}\frac{y}{a^n}) = 0, \quad (3.16)$$

for all $x, y \in X$. So $\Delta_Q(x, y) = 0$. By Theorem (2.1), the function $Q: X \to Y$ is quadratic. Moreover, letting $k = 0$ and passing the limit $m \to \infty$ in (3.14), we get the inequality (3.5) for $j = -1$. The rest of the proof is Similar to the proof of previous section.

From Theorem (3.1), we obtain the following corollaries concerning the JM Rassias mixed product –sum stability of the Functional equation (1.5).

Corollary 3.3.2

Let $\varepsilon, p, q \geq 0$ and $r, s > 0$ be real numbers such that $p, q < 2$ and $r + s \neq 2$. Suppose that a function $f: X \to Y$ satisfies

$$\|\Delta_f(x, y)\| \leq \varepsilon(\|x\|^p + \|y\|^q + \|x\|^r\|y\|^s) \text{ for all } x, y \in X. \quad (3.17)$$

Then there exists a unique quadratic function $Q: X \to Y$ such that

$$\|f(x) - Q(x)\| \leq \frac{\varepsilon}{2(a^2 - a^p)}\|x\|^p \text{ for all } x \in X. \quad (3.18)$$

Proof. In Theorem (3.1), put $j = 1$ and $\varphi(x, y) = \varepsilon(\|x\|^p + \|y\|^q + \|x\|^r\|y\|^s)$.

Corollary 3.3

Let $\varepsilon, p, q \geq 0$ and $r, s > 0$ be real numbers such that $p, q > 2$ and $r + s \neq 2$. Suppose that a function $f: X \to Y$ with $f(0) = 0$ satisfies (3.17) for all $x, y \in X$. Then there exists a unique quadratic function $Q: X \to Y$ such that

$$\|f(x) - Q(x)\| \leq \frac{\varepsilon}{2(a^p - a^2)}\|x\|^p \text{ for all } x \in X. \quad (3.19)$$

Proof. In Theorem (3.1), put $j = -1$ and $\varphi(x, y) = \varepsilon(\|x\|^p + \|y\|^q + \|x\|^r\|y\|^s)$.

$$\tilde{\varphi}(x) = \sum_{i=1-j)/2}^{\infty}\frac{1}{a^{2ij}}\varphi(a^{ij}x, 0) < \infty \quad (3.20)$$

Theorem 3.4

Let $j \epsilon \{-1,1\}$ be fixed, and let $\varphi: X \times X \to [0, \infty)$ be a function such that

$$\lim_{n\to\infty}\frac{1}{a^{2nj}}\varphi(a^{nj}x, a^{nj}y) = 0 \quad \text{for all } x, y \in X.$$

Suppose that $f: X \to Y$ be a function satisfies

$$\|f(ax + by) + f(ax - by) - 2a^2 f(x) - 2b^2(f(y)\| \leq \varphi(x, y) \quad (3.21)$$

for all $x, y \in X$. (3.13)

Furthermore, assume that $f(0) = 0$ in (3.21) *for the case* $j = 1$. *Then there exists a unique quadratic function* $Q: X \to Y$ *such that*

$$\|f(x) - Q(x)\| \leq \frac{1}{2a^{1+j}}\tilde{\varphi}(\frac{x}{a^{1-j/2}}), \text{for all } x \in X.$$

Proof. The proof is similar to the proof of Theorem (3.1).

CONCLUSION:

Finaly we conclude that we obtain the general solution and the generalized hyers-ulam rassias stability for a quaradic functional equations.

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