# Flow shop scheduling problem to reduce the makespan with transporting agent and loading - unloading time 

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#### Abstract

In flow shop scheduling problem n-number of jobs are processed through $m$ - number of machines to achieve the required objectives like minimizing make-span, reducing cost, meeting the due date etc. The main objectives of this flow shop problem are minimization of make-span and total flow time of jobs in process. In this paper we try to find the optimum solution for the problem of six machines arranged in series and jobs are assigned with weights. Transporting time, loading time and unloading time of the conveyor is considered separately. Johnson's algorithm is applied for finding the optimum solution. Total makespan and minimum weighted flow time is calculated.


KEY WORDS: - Flow-shop scheduling, Johnson's rule, loading time, unloading time, transporting time.

### 1.1 Introduction

Scheduling may contain single machine or more than one machine. In multiple machines model jobs are processed through multiple operations. There are two types of feasibility constraints- limits on number of machines and limitations on sequence of jobs processing on the machines. There are different heuristics used for solving these combinatorial optimization problems. For ex.- Genetic algorithm, Johnson's algorithm, particle warm method, branch and bound algorithm, tabu search method, ant colony algorithm etc. The objective function is defined on the basis of cost of the product and includes all constraints taken in consideration for scheduling.

In loading and unloading system conveyor system is also used at a large scale. In mechanical industries conveyors are used for transportation of heavy materials. They provides very fast service of transportation hence conveyors are the most famous in material handling and packing industries like mining, agricultural, computer, electronic, food industry, bottling and canning etc. Selection of conveyors is based on the actions of conveyors like transportation, categorization, size, weight and shape of material etc. In some industries robots are used to load or unload the material. In some industries robots are used to load the material and with the help of some other system material gets unloaded. For ex.- Use of robot is done to load the metal sheet in the press and after finishing the work it gets unloaded with the help of gravity. In plastic modelling and die casting the loading process is done without robot and unloading is done with robot. So loading and unloading procedure is very common in most of the industries for which different types of conveyers are used and the time required for this both activities are noted separately.

Mehrotra ${ }^{1}$ in her thesis considered flow shop scheduling for two and three machines in tandem and considered loading and unloading time along with transportation times and returning time. Bo Chen et al. ${ }^{2}$ considered a case of flow shop scheduling problem of three machines and $n$ jobs. He has used $\mathrm{O}(\mathrm{nlogn})$ time heuristic which depends on Johnson's algorithm to minimize the total completion time of the jobs. Ali Alahverdi ${ }^{3}$ summarized the data of scheduling literature on models with setup times for more than 300 papers. Qazi Shoeb Ahmad et al. ${ }^{4}$ have solved m machine n job problem with consideration of 3 machines 4 jobs arranged in series. The objective of solving problem is to reduce make-span. Mohammad Sadaqa et al. ${ }^{5}$ also used a concept of loading and unloading of jobs on machines. The flow shop problem is solved for two as well as three machines and objective of the solution is to reduce the make-span. The heuristic applied is meta-heuristic for Randomised priority search. Loading concept of outbound containers was studied by Quingcheng Zeng et al ${ }^{6}$. Kwei-Long Huang ${ }^{7}$ used loading and unloading concept in his research. He considered two types of scheduling synchronous and asynchronous.

Total six numbers of machines are arranged in tandem with five numbers of jobs. Jobs are also assigned with their importance that is weightage is given according to their importance. Six machines problem reduced into five four, then three and finally in two machines problem. Johnson's rule is applied for finding the sequence of jobs. Effect of breakdown interval is observed on machines and again the problem is redefined, solved and optimum solution is obtained. Total make-span and minimum weighted flow time is calculated [8, 9, 10].

Following result is used to solve the problem.
An optimal sequence is obtained by sequencing the item $i-1, i, i+1$ such that

 $\left.L T_{V_{i+1}}+d_{i+1}+U L_{S V_{i+1}}+V_{i+1}+L T V_{V Z_{i+1}}+e_{i+1}+U L_{V Z_{i+1}}+Z_{i+1}\right)$

$$
\begin{aligned}
& <\operatorname{Min}\left(\mathrm{P}_{\mathrm{i}+1}+\mathrm{LT}_{\mathrm{PQ}_{\mathrm{i}+1}}+\mathrm{a}_{\mathrm{i}+1}+\mathrm{UL}_{\mathrm{PQ}_{\mathrm{i}+1}}+\mathrm{Q}_{\mathrm{i}+1}+\mathrm{LT}_{\mathrm{QR}_{\mathrm{i}+1}}+\mathrm{b}_{\mathrm{i}+1}+\mathrm{UL}_{\mathrm{QR}_{\mathrm{i}+1}}+\mathrm{R}_{\mathrm{i}+1}+\mathrm{LT}_{\mathrm{RS}_{\mathrm{i}+1}}+\mathrm{c}_{\mathrm{i}+1}+\mathrm{UL}_{\mathrm{RS}_{\mathrm{i}+1}}+\mathrm{S}_{\mathrm{i}+1}+\right. \\
& \mathrm{LT}_{\mathrm{SV}_{\mathrm{i}+1}}+\mathrm{d}_{\mathrm{i}+1}+\mathrm{UL}_{\mathrm{SV}_{\mathrm{i}+1}}+\mathrm{V}_{\mathrm{i}+1}+\mathrm{LT}_{\mathrm{VZ}_{\mathrm{i}+1}}+\mathrm{e}_{\mathrm{i}+1}+\mathrm{UL}_{\mathrm{VZ}_{\mathrm{i}+1},} \mathrm{LT}_{\mathrm{PQ}_{\mathrm{i}}}+\mathrm{a}_{\mathrm{i}}+\mathrm{UL}_{\mathrm{PQ}_{\mathrm{i}}}+\mathrm{Q}_{\mathrm{i}}+\mathrm{LT}_{\mathrm{QR}_{\mathrm{i}}}+\mathrm{b}_{\mathrm{i}}+\mathrm{UL}_{\mathrm{QR}_{i}}+\mathrm{R}_{\mathrm{i}}+\mathrm{LT}_{\mathrm{RS}_{\mathrm{i}}}+ \\
& \left.c_{i}+U L_{R S_{i}}+S_{i}+{L T T V_{i}}+d_{i}+U L_{S_{i}}+V_{i}+L_{V Z_{i}}+e_{i}+U L_{V_{i}}+Z_{i}\right)
\end{aligned}
$$

Where $P, Q, R, S, V, Z$ are six machines considered. $a_{i}, b_{i}, c_{i}, d_{i}, e_{i}-$ transporting time
$\mathbf{P}_{\mathbf{i}}, \mathbf{Q}_{\mathbf{i}}, \mathbf{R}_{\mathbf{i}}, \mathbf{S}_{\mathbf{i}}, \mathbf{V}_{\mathbf{i}}, \mathbf{Z}_{\mathbf{i}}$ :- Processing time of resp. machine
$\mathrm{LT}_{\mathrm{PQi}}$ :- loading time at machine P and similarly loading time considered at all machines
$\mathrm{UL}_{\mathrm{PQ}}$ :- unloading time at machine Q and similarly unloading time considered at all machines.

### 1.2 Problem Designing

We can prove the utility of the theorem mentioned for solving the numerical problem with loading and unloading time.
In the given problem six machines namely $\mathrm{P}, \mathrm{Q}, \mathrm{R}, \mathrm{S}, \mathrm{V}, \mathrm{Z}$ are arranged in series loading time, unloading time and transport time of articles $A_{1}, A_{2}, A_{3},-----A_{n}$ are considered. The transporting agent transport the items from machine $P$ to machine Q , machine $Q$ to machine $R$, machine $R$ to machine $S$ and machine $S$ to machine $V$ and $V$ to $Z$ in such a way that after delivering the articles to machine $Z$ without delay come back to machine $P$ for transferring the next item.

Let $\mathbf{a}_{\mathbf{i}}$ denotes transporting time that is the time required for transportation of the articles from P to Q ,
$\mathbf{b}_{\mathbf{i}}$ denotes transporting time required for transportation of the articles from Q to R ,
$\mathbf{c}_{\mathbf{i}}$ denotes transporting time required for transportation of the articles from $R$ to $S$
$\mathbf{d}_{\mathbf{i}}$ denotes transporting time required for transportation of the articles from S to V .
$\mathbf{e}_{\mathbf{i}}$ denotes transporting time required for transportation of the articles from V to Z .
For producing the articles machines required the set up time which is denoted by
$\mathbf{P}_{\mathbf{i}}$ :- Processing time of machine $\mathrm{P}, \mathbf{Q}_{\mathbf{i}}$ :- Processing time of machine Q ,
$\mathbf{R}_{\mathrm{i}}$ :- Processing time of machine $\mathrm{R}, \mathbf{S}_{\mathrm{i}}$ :- Processing time of machine S ,
$\mathbf{V}_{\mathbf{i}}$ :- Processing time of machine $\mathrm{V}, \quad \mathbf{Z}_{\mathrm{i}}$ :- Processing time of machine Z ,
All machines required time for loading items transported by transport agent so that machines will start processing, this time will be considered as loading time. After finishing the working of machine the articles will be transferred to the next machine, the time required for unloading of these articles on the particular machine is considered as unloading time.
$\mathrm{LT}_{\mathrm{PQ} i}$ :- loading time at machine $\mathrm{P}, \mathrm{LT}_{\mathrm{QRi}}$ :- loading time at machine Q
$\mathrm{LT}_{\mathrm{RSi}}$ :- loading time at machine $\mathrm{R}, \mathrm{LT}_{\mathrm{SV}_{\mathrm{i}}}$ :- loading time at machine S
$\mathrm{LT}_{\mathrm{VZi}}$ :- loading time at machine $\mathrm{V}, \quad \mathrm{UL}_{\mathrm{PQi}}$ :- unloading time at machine Q
$\mathrm{UL}_{\mathrm{QRi}}$ :- unloading time at machine $\mathrm{R}, \mathrm{UL}_{\mathrm{RSi}}$ :- unloading time at machine S
$\mathrm{UL}_{\mathrm{SV} i}$ :- unloading time at machine $\mathrm{V}, \mathrm{UL}_{\mathrm{VZi}}:$ - unloading time at machine Z
Six machines problem with loading and unloading time and transportation time is considered satisfying following conditions:

1) $\operatorname{Min}\left(P i+L T_{P Q i}+a_{i}+U L_{P Q i}\right) \geq \operatorname{Max}\left(L T_{P Q i}+a_{i}+U L_{P Q i}+Q_{i}\right)$
2) $\operatorname{Min}\left(L T T_{Q R i}+b_{i}+U L_{Q R i}+R_{i}\right) \geq \operatorname{Max}\left(Q_{i}+L_{Q R i}+b_{i}+U L_{Q R i}\right)$
3) $\operatorname{Min}\left(L T R_{R S i}+c_{i}+U L_{R S i}+S_{i}\right) \geq \operatorname{Max}\left(R_{i}+L T_{R S i}+c_{i}+U L_{R S i}\right)$
4) $\operatorname{Min}\left(L_{S V_{i}}+d_{i}+U L_{S V_{i}+} V_{i}\right) \geq \operatorname{Max}\left(S_{i}+L T_{S V_{i}}+d_{i}+U L_{S V_{i}}\right)$
5) $\operatorname{Min}\left(L T_{V Z i}+e_{i}+U L_{V Z i}+Z_{i}\right) \geq \operatorname{Max}\left(V_{i}+L_{V Z i}+e_{i}+U L_{V Z i}\right)$

## Step I:

Suppose FIVE machines E, $F, G, O$, $W$ are assumed with the service time $E_{i}, F_{i}, G_{i}, O_{i}$, Wi resp.Where,
$\mathrm{E}_{\mathrm{i}}=\mathrm{P}_{\mathrm{i}}+\mathrm{LT}_{\mathrm{PQi}_{i}}+\mathrm{a}_{\mathrm{i}}+\mathrm{UL}_{\mathrm{PQi}}+\mathrm{Q}_{\mathrm{i}}+\mathrm{LT}_{\mathrm{QRi}}+\mathrm{b}_{\mathrm{i}}+\mathrm{UL}_{\mathrm{QRi}}$
$\mathrm{F}_{\mathrm{i}}=\mathrm{LT}_{\mathrm{PQi}}+\mathrm{a}_{\mathrm{i}}+\mathrm{UL}_{\mathrm{PQi}}+\mathrm{Q}_{\mathrm{i}}+\mathrm{LT}_{\mathrm{QRi}}+\mathrm{b}_{\mathrm{i}}+\mathrm{UL}_{\mathrm{QRi}}+\mathrm{R}_{\mathrm{i}}$
$\mathrm{G}_{\mathrm{i}}=\mathrm{LT}_{\mathrm{QRi}}+\mathrm{b}_{\mathrm{i}}+\mathrm{UL}_{\mathrm{QRi}}+\mathrm{R}_{\mathrm{i}}+\mathrm{LT}_{\mathrm{RSi}}+\mathrm{c}_{\mathrm{i}}+\mathrm{UL}_{\mathrm{RSi}}+\mathrm{S}_{\mathrm{i}}$
$\mathrm{O}_{\mathrm{i}}=\mathrm{LT}_{\mathrm{RSi}}+\mathrm{c}_{\mathrm{i}}+\mathrm{UL}_{\mathrm{RSi}}+\mathrm{S}_{\mathrm{i}}+\mathrm{LT}_{\mathrm{SVi}}+\mathrm{d}_{\mathrm{i}}+\mathrm{UL}_{\mathrm{SVi}}+\mathrm{V}_{\mathrm{i}}$
$\mathrm{W}_{\mathrm{i}}=\mathrm{LT}_{\mathrm{SVi}}+\mathrm{d}_{\mathrm{i}}+\mathrm{UL}_{\mathrm{SVi}}+\mathrm{V}_{\mathrm{i}}+\mathrm{LT}_{\mathrm{VZ}}+\mathrm{e}_{\mathrm{i}}+\mathrm{UL}_{\mathrm{VZi}}+\mathrm{Z}_{\mathrm{i}}$

| Article | $\mathrm{E}_{\mathrm{i}}$ | $\mathrm{F}_{\mathrm{i}}$ | $\mathrm{G}_{\mathrm{i}}$ | $\mathrm{O}_{\mathrm{i}}$ | $\mathrm{W}_{\mathrm{i}}$ | $\mathrm{wt}_{\mathrm{i}}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathrm{A}_{1}$ | $\mathrm{E}_{1}$ | $\mathrm{~F}_{1}$ | $\mathrm{G}_{1}$ | $\mathrm{O}_{1}$ | $\mathrm{~W}_{1}$ | $\mathrm{wt}_{1}$ |
| $\mathrm{~A}_{2}$ | $\mathrm{E}_{2}$ | $\mathrm{~F}_{2}$ | $\mathrm{G}_{2}$ | $\mathrm{O}_{2}$ | $\mathrm{~W}_{2}$ | $\mathrm{wt}_{2}$ |
| $\mathrm{~A}_{3}$ | $\mathrm{E}_{3}$ | $\mathrm{~F}_{3}$ | $\mathrm{G}_{3}$ | $\mathrm{O}_{3}$ | $\mathrm{~W}_{3}$ | $\mathrm{wt}_{3}$ |
| $\mathrm{~A}_{4}$ | $\mathrm{E}_{4}$ | $\mathrm{~F}_{4}$ | $\mathrm{G}_{4}$ | $\mathrm{O}_{4}$ | $\mathrm{~W}_{4}$ | $\mathrm{wt}_{4}$ |
| $\mathrm{~A}_{5}$ | $\mathrm{E}_{5}$ | $\mathrm{~F}_{5}$ | $\mathrm{G}_{5}$ | $\mathrm{O}_{5}$ | $\mathrm{~W}_{5}$ | $\mathrm{wt}_{5}$ |

Step II: Now this problem will be reduced into two machine problem.
But first we have to convert it into three machines $\mathrm{H}, \mathrm{K}, \mathrm{L}$ and then by considering two fictitious machines M and N the problem will be converted into two machines problem.
$\mathrm{Hi}=\mathrm{E}_{\mathrm{i}}+\mathrm{F}_{\mathrm{i}}, \mathrm{Ki}=\mathrm{F}_{\mathrm{i}}+\mathrm{G}_{\mathrm{i}}, \mathrm{Li}=\mathrm{G}_{\mathrm{i}}+\mathrm{O}_{\mathrm{i}, \mathrm{Ji}}=\mathrm{O}_{\mathrm{i}}+\mathrm{W}_{\mathrm{i}}$
Therefore $D_{1}, D_{2}, D_{3}$ are represented as follows:
$\mathrm{D}_{1}=\mathrm{H}_{\mathrm{i}}+\mathrm{K}_{\mathrm{i}}, \mathrm{D}_{2}=\mathrm{K}_{\mathrm{i}}+\mathrm{L}_{\mathrm{i}}, \mathrm{D}_{3}=\mathrm{L}_{\mathrm{i}}+\mathrm{J}_{\mathrm{i}}$
Let M and N are two fictitious machines given by
$\mathrm{M}=\mathrm{D}_{1}+\mathrm{D}_{2}, \quad \mathrm{~N}=\mathrm{D}_{2}+\mathrm{D}_{3}$
Also 1) If $\min (\mathrm{M}, \mathrm{N})=\mathrm{M}_{\mathrm{i}}$ then $\mathrm{M}_{\mathrm{i}}{ }^{\prime}=\mathrm{M}_{\mathrm{i}}-\mathrm{wt}_{\mathrm{i}}$ and $\mathrm{N}^{\prime}{ }_{\mathrm{i}}=\mathrm{N}_{\mathrm{i}}$
2) If $\min (M, N)=N_{i}$ then $M_{i}^{\prime}=M_{i}$ and $N_{I}^{\prime}=N_{i}+w t_{i}$

## Step III:

For getting the proper sequence the new problem is defined as follows and represented in the table form: $M^{\prime} i=M^{\prime} i / w t_{i}$ and $N^{\prime} i$ $=\mathrm{N}^{\prime} \mathrm{i} / \mathrm{wt}_{\mathrm{i}}$

| Article | $\mathrm{M}_{\mathrm{i}} / \mathrm{wt}_{\mathrm{i}}$ | $\mathrm{N}_{\mathrm{i}} / \mathrm{wt}_{\mathrm{i}}$ | $\mathrm{wt}_{\mathrm{i}}$ |
| :---: | :--- | :--- | :--- |
| $\mathrm{A}_{1}$ | $\mathrm{M}^{\prime}{ }_{1} / \mathrm{wt}_{1}$ | $\mathrm{~N}^{\prime} /{ }_{1} \mathrm{wt}_{1}$ | $\mathrm{wt}_{1}$ |
| $\mathrm{~A}_{2}$ | $\mathrm{M}^{\prime}{ }_{2} / \mathrm{wt}_{2}$ | $\mathrm{~N}^{\prime}{ }_{2} / \mathrm{wt}_{2}$ | $\mathrm{wt}_{2}$ |
| $\mathrm{~A}_{3}$ | $\mathrm{M}_{3}{ }_{3} / \mathrm{wt}_{3}$ | $\mathrm{~N}^{\prime}{ }_{3} / \mathrm{wt}_{3}$ | $\mathrm{wt}_{3}$ |
| $\mathrm{~A}_{4}$ | $\mathrm{M}_{4}{ }_{4} / \mathrm{wt}_{4}$ | $\mathrm{~N}^{\prime}{ }_{4} / \mathrm{wt}_{4}$ | $\mathrm{wt}_{4}$ |
| $\mathrm{~A}_{5}$ | $\mathrm{M}_{5}{ }_{5} / \mathrm{wt}_{5}$ | $\mathrm{~N}^{\prime}{ }_{5} / \mathrm{wt}_{5}$ | $\mathrm{Wt}_{5}$ |

## Step IV:

Considering the breakdown interval and the effect of this time interval should be observed on all jobs. If the jobs are affected by this time interval then the difference of the interval will be added. With the effect of this breakdown interval, the problem will be redefined as follows:

1) If the job is affected by the breakdown interval $\left(u_{1}-u_{2}\right)$ then
$P_{i}^{\prime}=P_{i}+\left(u_{1}-u_{2}\right), \quad Q_{i}^{\prime}=Q_{i}+\left(u_{1}-u_{2}\right), R_{i}^{\prime}=R_{i}+\left(u_{1}-u_{2}\right)$,
$S_{i}^{\prime}=S_{i}+\left(u_{1}-u_{2}\right), \quad V_{i}^{\prime}=V_{i}+\left(u_{1}-u_{2}\right), \quad Z_{i}^{\prime}=Z_{i}+\left(u_{1}-u_{2}\right)$
2) If the job is not affected by the breakdown interval $\left(u_{1}-u_{2}\right)$ then
$\mathrm{P}_{\mathrm{i}}^{\prime}=\mathrm{P}_{\mathrm{i}}, \quad \mathrm{Q}_{\mathrm{i}}^{\prime}=\mathrm{Q}_{\mathrm{i}}, \quad \mathrm{Ri}^{\prime}=\mathrm{R}_{\mathrm{i}}, \quad \mathrm{Si}^{\prime}=\mathrm{S}_{\mathrm{i}}, \quad \mathrm{Vi}^{\prime}=\mathrm{V}_{\mathrm{i}}, \quad \mathrm{Zi}^{\prime}=\mathrm{Z}_{\mathrm{i}}$,
Step V: Applying steps I, II, III, IV the problem has been solved for getting the optimal sequence. The scheduling of all the course of action is in such a way that the minimum time should be required for getting the optimum solution or whole production For finding the algorithm the above information can be symbolized and represented in table for finding the sequence by using Johnson's rule of sequencing.

### 1.3 Numerical Example:

Table 8.3.1

| 皆 | - |  | त | \% | 0 | $\stackrel{(\underset{z}{0}}{\leftrightarrows}$ |  | $)_{5}^{\text {g }}$ | $\sim$ | 先 | 0 | ) | $\dot{\sim}$ | $\stackrel{5}{5}$ | - | $5^{5}$ | 7 | $\stackrel{\text { N }}{\stackrel{\text { ® }}{\square}}$ | \% | ${ }_{5}^{\text {N }}$ | N | \% |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{A}_{1}$ | 6 | 2 | 4 | 3 | 5 | 2 |  | 4 | 6 | 2 | 2 | 5 | 6 | 1 | 3 | 6 | 6 | 2 | 3 | 5 | 8 | 2 |
| $\mathrm{A}_{2}$ | 5 | 3 | 6 | 3 | 2 | 2 | 5 | 2 | 5 | 1 | 3 | 6 | 5 | 3 | 2 | 5 | 7 | 3 | 2 | 6 | 7 | 4 |
| $\mathrm{A}_{3}$ | 4 | 3 | 5 | 2 | 4 | 2 | 2 | 4 | 6 | 2 | 3 | 4 | 6 | 2 | 3 |  | 6 | 3 | 2 | 6 | 7 | 3 |
| $\mathrm{A}_{4}$ | 5 | 4 | 3 | 2 | 4 | 3 | 4 | 2 | 7 | 2 | 2 | 3 | 8 | 1 | 3 | 4 | 8 | 2 | 2 | 6 | 8 | 5 |
| $\mathrm{A}_{5}$ | 8 | 2 | 3 | 1 | 3 | 2 | 6 | 1 | 8 | 1 | 2 | 4 | 8 | 1 | 2 | 5 | 8 | 1 | 2 | 6 | 9 | 2 |

The given data satisfies the following conditions
$\operatorname{Min}\left(\mathrm{Pi}+\mathrm{LT}_{\mathrm{PQi}_{i}}+\mathrm{a}_{\mathrm{i}}+\mathrm{UL}_{\mathrm{PQi}}\right)=15$
$\operatorname{Max}\left(\mathrm{LT}_{\mathrm{PQi}}+\mathrm{a}_{\mathrm{i}}+\mathrm{UL}_{\mathrm{PQi}}+\mathrm{Q}_{\mathrm{i}}\right)=15$
Similarly other inequalities are also satisfied.

## Step I:

Suppose FIVE machines E, F, G, O and W are assumed with the service time $\mathrm{E}_{\mathrm{i},} \mathrm{F}_{\mathrm{i}}, \mathrm{G}_{\mathrm{i}}, \mathrm{O}_{\mathrm{i}}$ and Wi resp. where,
$\mathrm{E}_{\mathrm{i}}=\mathrm{P}_{\mathrm{i}}+\mathrm{LT}_{\mathrm{PQi}}+\mathrm{a}_{\mathrm{i}}+\mathrm{UL}_{\mathrm{PQi}}+\mathrm{Q}_{\mathrm{i}}+\mathrm{LT}_{\mathrm{QRi}}+\mathrm{b}_{\mathrm{i}}+\mathrm{UL}_{\mathrm{QRi}}$
$\mathrm{F}_{\mathrm{i}}=\mathrm{LT}_{\mathrm{PQi}}+\mathrm{a}_{\mathrm{i}}+\mathrm{UL}_{\mathrm{PQi}}+\mathrm{Q}_{\mathrm{i}}+\mathrm{LT}_{\mathrm{QRi}}+\mathrm{b}_{\mathrm{i}}+\mathrm{UL}_{\mathrm{QRi}}+\mathrm{R}_{\mathrm{i}}$
$\mathrm{G}_{\mathrm{i}}=\mathrm{LT}_{\mathrm{QRi}}+\mathrm{b}_{\mathrm{i}}+\mathrm{UL}_{\mathrm{QRi}}+\mathrm{R}_{\mathrm{i}}+\mathrm{LT}_{\mathrm{RSi}}+\mathrm{c}_{\mathrm{i}}+\mathrm{UL}_{\mathrm{RSi}}+\mathrm{S}_{\mathrm{i}}$
$\mathrm{O}_{\mathrm{i}}=\mathrm{LT}_{\mathrm{RSi}}+\mathrm{c}_{\mathrm{i}}+\mathrm{UL}_{\mathrm{RSi}}+\mathrm{S}_{\mathrm{i}}+\mathrm{LT}_{\mathrm{SVi}}+\mathrm{d}_{\mathrm{i}}+\mathrm{UL}_{\mathrm{SVi}}+\mathrm{V}_{\mathrm{i}}$
$\mathrm{W}_{\mathrm{i}}=\mathrm{LT}_{\mathrm{SVi}}+\mathrm{d}_{\mathrm{i}}+\mathrm{UL}_{\mathrm{SVi}}+\mathrm{V}_{\mathrm{i}}+\mathrm{LT}_{\mathrm{VZ}}+\mathrm{e}_{\mathrm{i}}+\mathrm{UL}_{\mathrm{VZi}}+\mathrm{Z}_{\mathrm{i}}$

Table 8.3.2

| Article | $\mathrm{E}_{\mathrm{i}}$ | $\mathrm{F}_{\mathrm{i}}$ | $\mathrm{G}_{\mathrm{i}}$ | $\mathrm{O}_{\mathrm{i}}$ | $\mathrm{W}_{\mathrm{i}}$ | $\mathrm{wt}_{\mathrm{i}}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathrm{A}_{1}$ | 29 | 29 | 30 | 31 | 34 | 2 |
| $\mathrm{~A}_{2}$ | 28 | 28 | 29 | 32 | 35 | 4 |
| $\mathrm{~A}_{3}$ | 26 | 28 | 29 | 31 | 34 | 3 |
| $\mathrm{~A}_{4}$ | 27 | 29 | 31 | 31 | 34 | 5 |
| $\mathrm{~A}_{5}$ | 26 | 26 | 32 | 31 | 34 | 2 |

## Step II:

But first we have to convert it into four machines $\mathrm{H}, \mathrm{K}, \mathrm{L}, \mathrm{J}$ and then into three fictitious machines $\mathrm{D}_{1}, \mathrm{D}_{2}, \mathrm{D}_{3}$ and then into two fictitious machines M and N the problem will be converted into two machines problem.

## Table 8.3.3

| Article | $\mathrm{Hi}=\mathrm{E}_{\mathrm{i}}+\mathrm{F}_{\mathrm{i},}$ | $\mathrm{Ki}=\mathrm{F}_{\mathrm{i}}+\mathrm{G}_{\mathrm{i}}$ | $\mathrm{Li}=\mathrm{G}_{\mathrm{i}}+\mathrm{O}_{\mathrm{i},}$ | $\mathrm{Ji}=\mathrm{O}_{\mathrm{i}}+\mathrm{W}_{\mathrm{i}}$ | $\mathrm{wt}_{\mathrm{i}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{A}_{1}$ | 58 | 59 | 61 | 65 | 2 |
| $\mathrm{~A}_{2}$ | 56 | 57 | 61 | 67 | 4 |
| $\mathrm{~A}_{3}$ | 54 | 57 | 60 | 65 | 3 |
| $\mathrm{~A}_{4}$ | 56 | 60 | 62 | 65 | 5 |
| $\mathrm{~A}_{5}$ | 52 | 58 | 63 | 65 | 2 |

As mentioned earlier step now this four machine problem is reduced to three machine machines $D_{1}, D_{2}, D_{3}$ where

Table 8.3.4

| Article | $\mathrm{D}_{1}=\mathrm{Hi}+\mathrm{K}_{\mathrm{i}}$ | $\mathrm{D}_{2}=\mathrm{Ki}+\mathrm{L}_{\mathrm{i}}$ | $\mathrm{D}_{3}=\mathrm{L}_{\mathrm{i}}+\mathrm{J}_{\mathrm{i}}$ | $\mathrm{wt}_{\mathrm{i}}$ |
| :---: | :---: | :---: | :---: | :---: |
| $\mathrm{A}_{1}$ | 117 | 120 | 126 | 2 |
| $\mathrm{~A}_{2}$ | 113 | 118 | 128 | 4 |
| $\mathrm{~A}_{3}$ | 111 | 117 | 125 | 3 |
| $\mathrm{~A}_{4}$ | 116 | 122 | 127 | 5 |
| $\mathrm{~A}_{5}$ | 110 | 121 | 128 | 2 |

Let M and N are two fictitious machines.
Table 8.3.5

| Article | $\mathrm{M}=\mathrm{D}_{1}+\mathrm{D}_{2}$ | $\mathrm{~N}=\mathrm{D}_{2}+\mathrm{D}_{3}$ | $\mathrm{wt}_{\mathrm{i}}$ |
| :---: | :---: | :---: | :---: |
| $\mathrm{A}_{1}$ | 237 | 246 | 2 |
| $\mathrm{~A}_{2}$ | 231 | 246 | 4 |
| $\mathrm{~A}_{3}$ | 228 | 242 | 3 |
| $\mathrm{~A}_{4}$ | 238 | 249 | 5 |
| $\mathrm{~A}_{5}$ |  |  | 2 |

Also1) If $\min (\mathrm{M}, \mathrm{N})=\mathrm{M}_{\mathrm{i}}$ then $\mathrm{M}_{\mathrm{i}}{ }^{\prime}=\mathrm{M}_{\mathrm{i}}-\mathrm{wt}_{\mathrm{i}}$ and $\mathrm{N}_{\mathrm{i}}=\mathrm{N}_{\mathrm{i}}$
2) If $\min (M, N)=N_{i}$ then $M_{i}^{\prime}=M_{i}$ and $N^{\prime}{ }_{I}=N_{i}+w t_{i}$

Table 8.3.6

| Article | $\mathrm{M}_{\mathrm{i}}^{\prime}$ | $\mathrm{N}_{\mathrm{i}}$ | $\mathrm{wt}_{\mathrm{i}}$ |
| :---: | :---: | :---: | :---: |
| $\mathrm{A}_{1}$ | 235 | 246 | 2 |
| $\mathrm{~A}_{2}$ | 227 | 246 | 4 |
| $\mathrm{~A}_{3}$ | 225 | 242 | 3 |
| $\mathrm{~A}_{4}$ | 233 | 249 | 5 |
| $\mathrm{~A}_{5}$ | 229 | 249 | 2 |

Step III; Now the ratio of weights with Mi and Ni is taken so that we can decide the sequence of jobs according to Johnson's rule.

Table 8.3.7

| Article | $\mathrm{M}_{\mathrm{i}}^{\prime} / \mathrm{w}_{\mathrm{i}}$ | $\mathrm{N}^{\prime}{ }_{\mathrm{i} /} \mathrm{w}_{\mathrm{i}}$ | $\mathrm{wt}_{\mathrm{i}}$ |
| :---: | :---: | :---: | :---: |
| $\mathrm{A}_{1}$ | 117.5 | 123 | 2 |


| $\mathrm{A}_{2}$ | 56.75 | 61.5 | 4 |
| :---: | :---: | :---: | :---: |
| $\mathrm{~A}_{3}$ | 75 | 80.66 | 3 |
| $\mathrm{~A}_{4}$ | 46.6 | 49.8 | 5 |
| $\mathrm{~A}_{5}$ | 114.5 | 124.5 | 2 |

By Johnson＇s rule the optimal sequence obtained for above reduced problem is $4,2,3,5,1$ ．Then the time required for total processing of articles by using above scheduling sequence i．e minimum time for entire production can be calculated by considering the time required by the transporting agent when it returns back to machine $\mathrm{M}_{1}$ to load the next article and also the time when it reaches to machine $\mathrm{M}_{2}$ for unloading of an article．

Table 8．3．8

| $\stackrel{0}{2}$ | － |  | $\begin{array}{r} \stackrel{\rightharpoonup}{2} \\ \stackrel{H}{a} \end{array}$ | － | 5 | 0 |  | 令 | － | $\stackrel{\substack{a \\ S \\ S}}{ }$ | $\underline{\sim}$ |  | 年 |  | $\stackrel{\text { ¢ }}{5}$ | ir |  | $\begin{gathered} \stackrel{5}{6} \\ \stackrel{y}{3} \end{gathered}$ | － | 方 | $>$ |  | $\stackrel{\stackrel{N}{\rightleftarrows}}{\underset{\sim}{2}}$ | － | $\stackrel{N}{3}$ | $\cdots$ |  | 3 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | I | O |  |  |  | I | O |  |  |  | I | O |  |  |  |  | O |  |  |  | I | O |  |  |  | I | O |  |
| $\begin{gathered} \mathrm{A} \\ 4 \end{gathered}$ | 0 | 5 | 4 | 3 | 2 | $\begin{aligned} & 1 \\ & 4 \end{aligned}$ | 18 | 3 | 4 | 2 | $\begin{aligned} & 2 \\ & 7 \end{aligned}$ | 34 | 2 | 2 | 3 | $\begin{aligned} & 4 \\ & 1 \end{aligned}$ | 49 | 1 | 3 | 4 | 57 | 65 | 2 | 2 | 6 | 75 | 83 | 5 |
| A | 5 | 10 | 3 | 6 | 3 | $\begin{aligned} & 2 \\ & 2 \end{aligned}$ | 24 |  | 5 | 2 | $\begin{aligned} & 3 \\ & 6 \end{aligned}$ | 41 | 1 | 3 | 6 | $\begin{aligned} & 5 \\ & 5 \end{aligned}$ | 60 | 3 | 2 | 5 | 70 | 77 | 3 | 2 | 6 | 89 | 96 | 4 |
| A <br> 3 | 10 | 14 | 3 | 5 | 2 | $\begin{aligned} & 2 \\ & 6 \end{aligned}$ | 30 | 2 | 2 | 4 | 4 5 | 51 | 2 | 3 | 4 | $\begin{aligned} & 6 \\ & 4 \end{aligned}$ | 70 | 2 | 3 | 5 | 80 | 86 | 3 | 2 | 6 | 10 2 | 10 9 | 3 |
| A | 14 | 22 | 2 | 3 | 1 | $\begin{aligned} & 3 \\ & 1 \end{aligned}$ | 34 | 2 | 6 | 1 | $\begin{aligned} & 5 \\ & 2 \end{aligned}$ | 60 | 1 |  | 4 | $\begin{aligned} & 7 \\ & 4 \end{aligned}$ | 82 | 1 | 2 | 5 | 91 | 99 | 1 | 2 | 6 | $\begin{gathered} 11 \\ 5 \end{gathered}$ | $\begin{gathered} 12 \\ 4 \end{gathered}$ | 2 |
| A | 22 | 28 | 2 | 4 | 3 | $\begin{aligned} & 3 \\ & 7 \end{aligned}$ | 42 | 2 | 3 | 4 | $\begin{aligned} & 6 \\ & 4 \end{aligned}$ | 70 | 2 | 2 | 5 | 8 7 | 93 | 1 | 3 | 6 | 10 5 | $\begin{gathered} 11 \\ 1 \end{gathered}$ | 2 | 3 | 5 | 12 9 | 13 7 | 2 |

Effect of breakdown interval：［35，41］
The effect of breakdown interval is on jobs $\mathrm{Q}_{\mathrm{i}} 1$ and Ri 2 ；hence the original problem is converted into new problem．If the job is affected by the breakdown interval then
$P_{i}^{\prime}=P_{i}+\left(u_{2}-u_{1}\right), \quad Q_{i}^{\prime}=Q_{i}+\left(u_{2}-u_{1}\right)$,
$\mathrm{R}_{\mathrm{i}}^{\prime}=\mathrm{R}_{\mathrm{i}}+\left(\mathrm{u}_{2}-\mathrm{u}_{1}\right)$ and $\mathrm{S}_{\mathrm{i}}^{\prime}=\mathrm{S}_{\mathrm{i}}+\left(\mathrm{u}_{2}-\mathrm{u}_{1}\right)$
Therefore repeating the same procedure of step $1,2,3$ for finding the sequence and getting the optimal solution．
Table 8．3．9

|  | $\sim$ | $\stackrel{\ddot{\partial}}{\underset{1}{6}}$ | ส | $\stackrel{\ddot{2}}{\stackrel{\circ}{1}}$ | 0 | $\stackrel{\text { \％}}{\substack{\text { ¢ }}}$ | － | $\stackrel{\text { \％}}{\substack{0}}$ | a | $\stackrel{\text { 気 }}{\stackrel{\sim}{4}}$ | ט＇ | $\frac{\sqrt[n]{5}}{\underset{5}{2}}$ | $\dot{\square}$ | $\underset{y}{\stackrel{5}{6}}$ | － | 5 | $>$ | $\stackrel{\text { N }}{\stackrel{\text { a }}{\square}}$ | ט | $\stackrel{y}{*}_{3}^{5}$ | N | 3 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{A}_{1}$ | 6 | 2 | 4 | 3 | 11 | 2 | 3 | 4 | 6 | 2 | 2 | 5 | 6 | 1 | 3 | 6 | 6 | 2 | 3 | 5 | 8 | 2 |


| $\mathrm{A}_{2}$ | 5 | 3 | 6 | 3 | 2 | 2 | 5 | 2 | 11 | 1 | 3 | 6 | 5 | 3 | 2 | 5 | 7 | 3 | 2 | 6 | 7 | 4 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathrm{~A}_{3}$ | 4 | 3 | 5 | 2 | 4 | 2 | 2 | 4 | 6 | 2 | 3 | 4 | 6 | 2 | 3 | 5 | 6 | 3 | 2 | 6 | 7 | 3 |
| $\mathrm{~A}_{4}$ | 5 | 4 | 3 | 2 | 4 | 3 | 4 | 2 | 7 | 2 | 2 | 3 | 8 | 1 | 3 | 4 | 8 | 2 | 2 | 6 | 8 | 5 |
| $\mathrm{~A}_{5}$ | 8 | 2 | 3 | 1 | 3 | 2 | 6 | 1 | 8 | 1 | 2 | 4 | 8 | 1 | 2 | 5 | 8 | 1 | 2 | 6 | 9 | 2 |

As in step I six machine problems is initially converted into five machines by considering five fictitious machines as follows:
Table 8.3.10

| Article | $\mathrm{E}_{\mathrm{i}}$ | $\mathrm{F}_{\mathrm{i}}$ | $\mathrm{G}_{\mathrm{i}}$ | $\mathrm{O}_{\mathrm{i}}$ | $\mathrm{W}_{\mathrm{i}}$ | $\mathrm{wt}_{\mathrm{i}}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathrm{A}_{1}$ | 35 | 35 | 30 | 31 | 34 | 2 |
| $\mathrm{~A}_{2}$ | 28 | 34 | 35 | 32 | 35 | 4 |
| $\mathrm{~A}_{3}$ | 26 | 28 | 29 | 31 | 34 | 3 |
| $\mathrm{~A}_{4}$ | 27 | 29 | 31 | 31 | 34 | 5 |
| $\mathrm{~A}_{5}$ | 26 | 26 | 32 | 31 | 34 | 2 |

Following step II we have
Table 8.3.11

| Article | H | K | L | J | $\mathrm{wt}_{\mathrm{i}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{A}_{1}$ | 70 | 65 | 61 | 65 | 2 |
| $\mathrm{~A}_{2}$ | 62 | 69 | 67 | 67 | 4 |
| $\mathrm{~A}_{3}$ | 54 | 57 | 60 | 65 | 3 |
| $\mathrm{~A}_{4}$ | 56 | 60 | 62 | 65 | 5 |
| $\mathrm{~A}_{5}$ | 52 | 58 | 63 | 65 | 2 |

Now this four machine problem is reduced to three machine by considering three fictitious machines $D_{1}, D_{2}, D_{3}$ where
Table 8.3.12

| Article | $\mathrm{D}_{1}$ | $\mathrm{D}_{2}$ | $\mathrm{D}_{3}$ | $\mathrm{wt}_{\mathrm{i}}$ |
| :---: | :---: | :---: | :---: | :---: |
| $\mathrm{A}_{1}$ | 135 | 126 | 126 | 2 |
| $\mathrm{~A}_{2}$ | 131 | 136 | 134 | 4 |
| $\mathrm{~A}_{3}$ | 111 | 117 | 125 | 3 |


| $\mathrm{A}_{4}$ | 116 | 122 | 127 | 5 |
| :---: | :---: | :---: | :---: | :---: |
| $\mathrm{~A}_{5}$ | 110 | 121 | 128 | 2 |

Let M and N are two fictitious machines given by
Table 8.3.13

| Article | Mi | Ni | $\mathrm{wt}_{\mathrm{i}}$ |
| :---: | :---: | :---: | :---: |
| $\mathrm{A}_{1}$ | 261 | 252 | 2 |
| $\mathrm{~A}_{2}$ | 267 | 270 | 4 |
| $\mathrm{~A}_{3}$ | 228 | 242 | 3 |
| $\mathrm{~A}_{4}$ | 238 | 249 | 5 |
| $\mathrm{~A}_{5}$ | 231 | 249 | 2 |

## Also,

1) If $\min (M, N)=M_{i}$ then $M_{i}^{\prime}=M_{i}-w t_{i}$ and $N_{i}^{\prime}=N_{i}$
2) If $\min (M, N)=N_{i}$ then $M_{i}^{\prime}=M_{i}$ and $N_{1}^{\prime}=N_{i}+w t_{i}$

Table 8.3.14

| Article | $\mathrm{M}_{\mathrm{i}}^{\prime}$ | $\mathrm{N}_{\mathrm{i}}$ | $\mathrm{wt}_{\mathrm{i}}$ |
| :---: | :---: | :---: | :---: |
| $\mathrm{A}_{1}$ | 259 | 252 | 2 |
| $\mathrm{~A}_{2}$ | 263 | 270 | 4 |
| $\mathrm{~A}_{3}$ | 225 | 242 | 3 |
| $\mathrm{~A}_{4}$ | 233 | 249 | 5 |
| $\mathrm{~A}_{5}$ | 229 | 249 | 2 |

Step III;Now the ratio of weights with Mi and Ni is taken so that we can decide the sequence of jobs according to Johnson's rule.
Table 8.3.15

| Article | $\mathrm{M}_{\mathrm{i}}^{\prime} / \mathrm{w}_{\mathrm{i}}$ | $\mathrm{N}_{\mathrm{i} /}^{\prime} \mathrm{w}_{\mathrm{i}}$ | $\mathrm{wt}_{\mathrm{i}}$ |
| :---: | :---: | :---: | :---: |
| $\mathrm{A}_{1}$ | 129.5 | 126 | 2 |
| $\mathrm{~A}_{2}$ | 65.75 | 67.5 | 4 |


| $\mathrm{A}_{3}$ | 75 | 80.66 | 3 |
| :---: | :---: | :---: | :---: |
| $\mathrm{~A}_{4}$ | 46.6 | 49.8 | 5 |
| $\mathrm{~A}_{5}$ | 114.5 | 124.5 | 2 |

By Johnson's rule the optimal sequence obtained for above reduced problem is $4,2,3,5,1$. Then the time required for total processing of articles by using above scheduling sequence i.e minimum time for entire production can be calculated by considering the time required by the transporting agent when it returns back to machine $\mathrm{M}_{1}$ to load the next article and also the time when it reaches to machine $\mathrm{M}_{2}$ for unloading of an article.

Table 8.3.16

|  | - |  | ¢ | テ | \% | 0 |  | $\xrightarrow{-3}$ | - | $\stackrel{z}{3}$ | $\sim$ |  | $\left\lvert\, \begin{gathered} \stackrel{2}{c} \\ \leftrightarrows \end{gathered}\right.$ |  |  | $\dot{\sim}$ |  | 気 |  | 5 | $>$ |  | $\stackrel{5}{4}$ | - | $\stackrel{\text { y }}{\substack{5 \\ 3}}$ | N |  | 3 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | I | O |  |  |  | I | O |  |  |  |  |  |  |  |  |  | 0 |  |  |  | I | O |  |  |  | I | O |  |
| $\mathrm{A}_{4}$ | 0 | 5 | 4 | 3 | 2 | 14 | 18 | 3 | 4 | 2 | 27 | 34 | 2 | 2 | 3 | 41 | 49 | 1 | 3 | 4 | 57 | 65 | 2 | 2 | 6 | 75 | 83 | 5 |
| $\mathrm{A}_{2}$ | 5 | 10 | 3 | 6 | 3 | 22 | 24 | 2 | 5 | 2 | 36 | 47 | 1 | 3 | 6 | 57 | 62 | 3 | 2 | 5 | 72 | 79 | 3 | 2 | 6 | 90 | 97 | 4 |
| $\mathrm{A}_{3}$ | 10 | 14 | 3 | 5 | 2 | 26 | 30 | 2 | 2 | 4 | 51 | 57 | 2 | 3 | 4 | 66 | 72 | 2 | 3 | 5 | 84 | 90 | 3 | 2 | 6 | 103 | 110 | 3 |
| $\mathrm{A}_{5}$ | 14 | 22 | 2 | 3 | 1 | 31 | 34 | 2 | 6 | 1 | 58 | 66 | 1 | 2 | 4 | 76 | 84 | 1 | 2 | 5 | 95 | 103 | 1 | 2 | 6 | 116 | 125 | 2 |
| $\mathrm{A}_{1}$ | 22 | 28 | 2 | 4 | 3 | 37 | 48 | 2 | 3 | 4 | 70 | 76 | 2 | 2 | 5 | 89 | 95 | 1 | 3 | 6 | 109 | 115 | 2 | 3 | 5 | 130 | 138 | 2 |

## Result:

Minimum weighted flow time (MWFT)
$=(83 * 5)+(97-5) * 4+(110-4) * 3+(125-8) * 2+(138-6) * 2$
$=415+368+318+234+264 / 5+4+3+2+2$
$=1599 / 16$
$=100$ hours

## Conclusion:

From above table it is shown that the time gets reduced for total production by using the sequence obtained with the help of Johnson's rule. The total elapsed time for the complete process is 138 hrs and minimum weighted flow time is 100 hrs. Use of Johnson's algorithm gives the proper sequence of jobs which is helpful in decreasing the total time of the process. If we increase the number of machines the makespan may get increased. The same problem can be solved by using the parallel arrangement of machines.

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