IDEMPOTENT & COMPLEMENT LAW IN INTUITIONISTIC FUZZY SETS

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Abstract: In this paper, various operations in Intuitionistic Fuzzy Sets are discussed. Some theorems are proved for establishing the properties of intuitionistic fuzzy operators with respect to different intuitionistic fuzzy sets.

Keywords: Intuitionistic Fuzzy Set (IFS), Intuitionistic Fuzzy set Operators.

1. INTRODUCTION
L.A. Zadeh [5] introduced the notion of a Fuzzy sub set μ of a Set X as a function from X to [0,1]. After the introduction of Fuzzy sets by L.A.Zadeh [5], the Fuzzy concept has been introduced in almost all branches of Mathematics. Then the concept of Intuitionistic Fuzzy Set (IFS) was introduced by Atanassov [1] as a generalization of the notation of a Fuzzy set.

2. PRELIMINARIES

Definition 2.1 - Crisp Sets:
Either the element belongs to the set or it does not is known as Crisp set.

Definition 2.2 - Fuzzy set:
A fuzzy set is built from a reference set called universe of discourse. Assume that U = {x1, x2,....., xn} is the universe of discourse, then a fuzzy set A in U (A ⊂ U) is defined as a set of ordered pairs

{(x_i, μ_A(x_i))}  

Where x_i ∈ U, μ : U → [0,1] is the membership function of A and, μ_A(x) ∈ [0,1] is the degree of membership of x in A.

Definition 2.3 - Intuitionistic Fuzzy Set:
An Intuitionistic Fuzzy Set A in a non empty set X is an object having the form

A= { (x, μ_A(x), ν_A(x)) | x∈X } where the functions μ_A : X→ [0,1] and ν_A : X→ [0,1] denote the degrees of membership and non membership of the element x∈X to A respectively and satisfy 0≤μ_A(x) + ν_A(x) ≤ 1 for all x∈X. The family of all intuitionistic fuzzy sets in X denoted by IFS (X).

Definition 2.4 - Intuitionistic Fuzzy Set Operations:
For every two IFSs A and B we define the following relations and operations:

A ⊂ B  iff  (∀x ∈ E)(μ_A(x) ≤ μ_B(x) & ν_A(x) ≥ ν_B(x))
A ⊃ B  iff  (B ⊃ A)
A=B  iff  (∀x ∈ E)(μ_A(x) = μ_B(x) & ν_A(x) = ν_B(x))

\[ A = \{ (x, ν_A(x), μ_A(x)) / x ∈ E \} \]
\[ \bar{A} = \{ (x, min(μ_A(x), μ_B(x)), max(ν_A(x), ν_B(x))) / x ∈ E \} \]
\[ A ∩ B = \{ (x, min(μ_A(x), μ_B(x)), max(ν_A(x), ν_B(x))) / x ∈ E \} \]
\[ A ∪ B = \{ (x, max(μ_A(x), μ_B(x)), min(ν_A(x), ν_B(x))) / x ∈ E \} \]
\[ A + B = \left\{ (x, \mu_A(x) + \mu_B(x) - \mu_A(x)\mu_A(x), \nu_A(x)\nu_A(x)) \middle| x \in E \right\} \]

\[ A \cdot B = \left\{ (x, \mu_A(x)\mu_B(x), \nu_A(x) + \nu_B(x) - \nu_A(x)\nu_B(x)) \middle| x \in E \right\} \]

\[ A \odot B = \left\{ \left( x, \frac{\mu_A(x) + \mu_B(x)}{2}, \frac{\nu_A(x) + \nu_B(x)}{2} \right) \middle| x \in E \right\} \]

\[ A \Delta B = \left\{ \left( x, \sqrt{\mu_A(x)\mu_B(x)}, \sqrt{\nu_A(x)\nu_B(x)} \right) \middle| x \in E \right\} \]

\[ A \odot \bar{B} = \left\{ \left( x, 2, \frac{\mu_A(x)\mu_B(x)}{\mu_A(x) + \mu_B(x)}, 2, \frac{\nu_A(x)\nu_B(x)}{\nu_A(x) + \nu_B(x)} \right) \middle| x \in E \right\} \]

3. IDEMPOTENT & COMPLEMENT LAW IN INTUITIONISTIC FUZZY SETS

**THEOREM 3.1:**

Prove that \( A \cap A = A \)

**PROOF:**

\[ A \cap A = \left\{ (x, \min(\mu_A(x), \mu_A(x)), \max(\nu_A(x), \nu_A(x))) \middle| x \in E \right\} \]

\[ = \left\{ (x, \mu_A(x), \nu_A(x)) \middle| x \in E \right\} \]

\[ = A \]

Hence it completes the proof.

**THEOREM 3.2:**

Prove that \( A \odot A = A \)

**PROOF:**

\[ A \odot A = \left\{ \left( x, \frac{\mu_A(x) + \mu_A(x)}{2}, \frac{\nu_A(x) + \nu_A(x)}{2} \right) \middle| x \in E \right\} \]

\[ = \left\{ (x, \mu_A(x), \nu_A(x)) \middle| x \in E \right\} \]

\[ = A \]

Hence it completes the proof.

**THEOREM 3.3:**

Prove that \( A \Delta A = A \)

**PROOF:**

\[ A \Delta A = \left\{ (x, \sqrt{\mu_A(x)\mu_A(x)}, \sqrt{\nu_A(x)\nu_A(x)}) \middle| x \in E \right\} \]

\[ = \left\{ (x, \mu_A(x), \nu_A(x)) \middle| x \in E \right\} \]

\[ = A \]
Hence it completes the proof.

**THEOREM 3.4:**

Prove that $A \cap A = A$

**PROOF:**

$A \cap A = \{ \{ x, \mu_A(x), \mu_A(x) \} / x \in E \} = A$

Hence it completes the proof.

**THEOREM 3.5:**

Prove that $A \cap B = A \cup B$

**PROOF:**

$A = \{ \{ x, \nu_A(x), \mu_A(x) \} / x \in E \}

B = \{ \{ x, \nu_B(x), \mu_B(x) \} / x \in E \}$

$A \cap B = \{ \{ x, \min(\nu_A(x), \nu_B(x)), \max(\mu_A(x), \mu_B(x)) \} / x \in E \} = A \cup B$

Hence it completes the proof.

**THEOREM 3.6:**

Prove that $A \cup B = A \cap B$

**PROOF:**

$A = \{ \{ x, \nu_A(x), \mu_A(x) \} / x \in E \}

B = \{ \{ x, \nu_B(x), \mu_B(x) \} / x \in E \}$

$A \cup B = \{ \{ x, \max(\nu_A(x), \nu_B(x)), \min(\mu_A(x), \mu_B(x)) \} / x \in E \} = A \cap B$
Hence it completes the proof

**THEOREM 3.7:**

Prove that \( A + B = A \cdot B \)

**PROOF:**

L.H.S: 
\[
\overline{A} = \left\{ (x, \nu_A(x), \mu_A(x)) \mid x \in E \right\}
\]

\[
\overline{B} = \left\{ (x, \nu_B(x), \mu_B(x)) \mid x \in E \right\}
\]

\[
\overline{A} + \overline{B} = \left\{ (x, \nu_A(x) + \nu_B(x), \mu_A(x) \cdot \mu_B(x)) \mid x \in E \right\}
\]

\[
\overline{A} + \overline{B} = \left\{ (x, \nu_A(x) + \nu_B(x) - \nu_A(x) \cdot \nu_B(x), \mu_A(x) \cdot \mu_B(x)) \mid x \in E \right\}
\]

\[
\overline{A} + \overline{B} = \left\{ (x, \mu_A(x) \cdot \mu_B(x), \nu_A(x) + \nu_B(x) - \nu_A(x) \cdot \nu_B(x)) \mid x \in E \right\}
\]

Hence it completes the proof.

**THEOREM 3.8:**

Prove that \( A \circ B = A \circ B \)

**PROOF:**

L.H.S: 
\[
\overline{A} = \left\{ (x, \nu_A(x), \mu_A(x)) \mid x \in E \right\}
\]

\[
\overline{B} = \left\{ (x, \nu_B(x), \mu_B(x)) \mid x \in E \right\}
\]

\[
\overline{A} \circ \overline{B} = \left\{ (x, \nu_A(x) + \nu_B(x), \mu_A(x) \cdot \mu_B(x)) \mid x \in E \right\}
\]

\[
\overline{A} \circ \overline{B} = \left\{ (x, \frac{\nu_A(x) + \nu_B(x)}{2}, \frac{\mu_A(x) + \mu_B(x)}{2}) \mid x \in E \right\}
\]

Hence it completes the proof.

**Conclusion**

We have defined different operations of Intuitionistic Fuzzy Sets. Using these, we have proved different relations between these operators in the intuitionistic fuzzy sets.
REFERENCES


