On (i,j)*- Soft Preopen Sets In Soft Bitopological Spaces

1Mrs. R.Vijaya Chandra, 2Ms. M.R. Dhivyaa
1Head and Assistant Professor, 2M.Phil Scholar
Department of Mathematics, Navarasam Arts and Science College for Women, Erode, Tamilnadu, In

Abstract: The purpose of this paper is to introduce the concept of (i,j)*-soft preopen sets and (i,j)*-soft preclosed sets, (i,j)*-soft preinterior, (i,j)*-soft preclosure in soft bitopological space and some of their properties are also obtained. Also we state and prove the condition for collection of (i,j)*-soft preopen sets to be a soft bitopology.

Keywords: Soft Set, Soft Topology, (i,j)*-Soft Preopen sets, (i,j)*-Soft Preclosed sets, (i,j)*-Soft Preinterior, (i,j)*-Soft Preclosure sets.

1. INTRODUCTION

The concept of soft set theory was initiated by Russian researcher D. Molodtsov [1] in the year 1999 as a recent mathematical tool and has been applied in several directions, such as smoothness of functions, game theory, operation research, engineering physics, economics, social science, Riemann integration, etc. Maji, Biswas, Roy [2], studied the theory of soft sets. They discussed the basic soft set definition with examples. The Muhammad shabir and mumazza naz [3], introduced notion of soft topological spaces which are defined to be over an initial universe with a fixed set of parameter. Bitopological spaces was introduced by J. C. Kelly [8] in the year 1963 and specify a bitopological space, (A, δ₁, δ₂) to be a set A for topologies δ₁ and δ₂ on A and introduced the systematic study of bitopological space. The concept of soft bitopological space was introduced by Basavaraj M. Ittanagi [5] in the year 2014. Andrijevic and Mashour [4] gave many results on preopen sets in general topology. Some properties of soft bitopological spaces was investigated by G. Senel et al [6]. Shabir Hussain et al [7] obtained some results on soft topological spaces. This paper aims at developing a soft bitopological via , (i,j)*-soft preopen sets.

PREMILINARIES

Definition 1.1 [1]: Let V = (v₁, v₂, ..., vₙ) be an initial universal set, T be the set of all parameters and ρ(A) expressed the power set of V, A be a non-empty subset of T. A pair (P, A) expressed as P̃A is called a soft set over A. Where P is a map from A to ρ(A).

Definition 1.2 [3]: A soft set (P, T) in soft topological space is known as null soft set if it is expressed as P̃∅ if for every e ∈ T, P(e) = ∅.

A soft set (P, T) in soft topological space is known as absolute soft set if it is expressed as P̃A or Ã if for every e ∈ T, P(e) = Ã.

Definition 1.3 [3]: A topology on a set A is a collection σ of subset of A having the following properties:

(i) Null soft sets and absolute soft sets belongs to σ.

(ii) Union of some member of soft set in σ is in σ, i.e., If \( \{ (P̃_j, T) \}_j \in \sigma \), then \( \bigcup_{j \in \sigma} (P̃_j, T) \in \sigma \).

(iii) Intersection of some dual of soft set in σ is in σ, i.e., If \( (P̃_j, T) \in \sigma \), then \( (P̃_j, T) \cap (Q̃_j, T) \in \sigma \).

A set A for which a topology σ has been specified is known as soft topological space. An any member of σ are known as (i,j)*-soft open set in A and complement of them are called (i,j)*-soft closed set in A.

Definition 1.4 [5]: Let A be a non empty soft set over the universe V, δ₁ & δ₂ are dual diverse soft topologies over A. Then (A, δ₁, δ₂) is called a soft bitopological space.

Definition 1.5 [8]: Let (Q, T) be a soft set in soft bitopological space (A, δ₁, T). Then

(i) The (i,j)*-soft interior of (Q, T) is the soft set,

\( (i,j)*-\text{int} (Q, T) = \bigcup \{ (L̃, T) : (L̃, T) \text{ is (i,j)*-soft open} \} \)

(ii) The (i,j)*-soft closure of (Q, T) is the soft set,

\( (i,j)*-\text{cl} (Q, T) = \bigcap \{ (L̃, T) : (L̃, T) \text{ is (i,j)*-soft closed} \} \).

ISSN: 2455-2631 © November 2018 IJSDR | Volume 3, Issue 11
The \((i,j)^*\)-soft closure of \((Q,T)\) expressed as \((i,j)^*\text{-cl}(Q,T)\) is the intersection of each \((i,j)^*\)-soft closed of \((Q,T)\). Obviously \((Q,T)\) is the lowest \((i,j)^*\)-soft closed set contained \((Q,T)\).

The \((i,j)^*\)-soft interior of \((Q,T)\) expressed as \((i,j)^*\text{-int}(Q,T)\) is the union of each \((i,j)^*\)-soft open of \((Q,T)\). Obviously \((Q,T)\) is the greatest \((i,j)^*\)-soft closed set contained \((Q,T)\).

2. Soft preopen sets

**Definition 2.1:** In a soft topological space \((A, \bar{\sigma}, \sigma)\), a soft set,

(i) \((Q,T)\) is known as \((i,j)^*\)-soft preopen set if \((Q,T) \subseteq -\text{int}(\bar{\sigma}_{ij}\text{-cl}(Q,T))\).

(ii) \((P,T)\) is known as \((i,j)^*\)-soft preclosed set if \((P,T) \supseteq \bar{\sigma}_{ij}\text{-cl}(\bar{\sigma}_{ij}\text{-int}(P,T))\).

**Theorem 2.2:** An arbitrary union of \((i,j)^*\)-soft preopen sets is a \((i,j)^*\)-soft preopen set.

**Proof:** We take the family of soft set \(\{(Q,T)_\gamma | \gamma \in \chi\}\) be a collection of \((i,j)^*\)-soft preopen sets in \((A, \bar{\sigma}, T)\). Then for each \(\gamma, \bar{\sigma}_{ij}\text{-int}(\bar{\sigma}_{ij}\text{-cl}(Q,T)_\gamma) \supseteq (Q,T)_\gamma\)

Claim: \(\cup \bar{\sigma}_{ij}\text{-int}(\bar{\sigma}_{ij}\text{-cl}(Q,T)_\gamma) \supseteq \cup (Q,T)_\gamma\)

From definition, \(\cup \bar{\sigma}_{ij}\text{-int}(\bar{\sigma}_{ij}\text{-cl}(Q,T)_\gamma) \supseteq \cup (Q,T)_\gamma\)

Hence \(\bar{\sigma}_{ij}\text{-int}(\bar{\sigma}_{ij}\text{-cl}(\cup (Q,T)_\gamma)) \supseteq \cup (Q,T)_\gamma\).

**Note:** An arbitrary intersection of \((i,j)^*\)-soft preopen sets is a \((i,j)^*\)-soft preopen set.

**Example 2.3:**

Let \(A = \{a_1, a_2, a_3, a_4\}, T = \{t_1, t_2, t_3\}\) and Let \(P_1, P_2, P_3, P_4, P_5, P_6\) the maps from \(T\) to \(P(A)\) is defined by

\[(P_1, T) = \{(t_1, \{a_1, a_2\}), (t_2, \{a_1, a_2\})\},\]
\[(P_2, T) = \{(t_1, \{a_2\}), (t_2, \{a_1, a_3\})\},\]
\[(P_3, T) = \{(t_1, \{a_1\}), (t_2, \{a_1\})\},\]
\[(P_4, T) = \{(t_1, \{a_2\}), (t_2, \{a_1\})\},\]
\[(P_5, T) = \{(t_1, \{a_1, a_2\}), (t_2, \{a_1, a_2\})\},\]
\[(P_6, T) = \{(t_1, \{a_1, a_2, a_3\}), (t_2, \{a_1, a_2\})\}\]

are soft sets in \(A\).

Now, we consider \(\bar{\sigma} = \{\emptyset, A, (P_1, T), (P_2, T), (P_3, T), (P_4, T), (P_5, T), (P_6, T)\}\) a soft topology in \(A\). Here, \((R, T) = \{(t_1, \{a_2\}), (t_2, \{a_2, a_3\})\}\) is \((i,j)^*\)-soft preopen set and also \((S, T) = \{(t_1, \{a_1, a_3\}), (t_2, \{a_1, a_3\})\}\) is \((i,j)^*\)-soft preclosed set. But \((R, T) \cap (S, T) = \{(t_1, \emptyset), (t_2, \{a_3\})\}\) is \((i,j)^*\)-soft preopen set.

**Theorem 2.4:** Let \((Q,T)\) be a \((i,j)^*\)-soft preopen set such that \((R, T) \subseteq (Q, T) \subseteq \bar{\sigma}_{ij}\text{-cl}(R, T)\). Then \((Q,T)\) is a \((i,j)^*\)-soft preopen set.

**Proof:** Let \((A, \bar{\sigma}, T)\) be soft bitopological space. The set \((Q,T)\) is \((i,j)^*\)-soft preopen set if \((Q,T) \subseteq \bar{\sigma}_{ij}\text{-int}(\bar{\sigma}_{ij}\text{-cl}(Q,T)) \Rightarrow (R,T) \subseteq \bar{\sigma}_{ij}\text{-cl}(\bar{\sigma}_{ij}\text{-cl}(R,T)) \Rightarrow (R,T) \subseteq (Q,T) \Rightarrow \bar{\sigma}_{ij}\text{-int}(\bar{\sigma}_{ij}\text{-cl}(Q,T)) \subseteq \bar{\sigma}_{ij}\text{-int}(\bar{\sigma}_{ij}\text{-cl}(R,T)) \Rightarrow (R,T) \subseteq (Q,T) \subseteq \bar{\sigma}_{ij}\text{-int}(\bar{\sigma}_{ij}\text{-cl}(R,T)) \Rightarrow (R,T) \subseteq (Q,T) \subseteq \bar{\sigma}_{ij}\text{-int}(\bar{\sigma}_{ij}\text{-cl}(R,T)) \Rightarrow (R,T) \subseteq (Q,T) \subseteq \bar{\sigma}_{ij}\text{-int}(\bar{\sigma}_{ij}\text{-cl}(R,T)) \Rightarrow (R,T) \subseteq (Q,T) \subseteq \bar{\sigma}_{ij}\text{-int}(\bar{\sigma}_{ij}\text{-cl}(R,T)). Therefore, \((R,T)\) is \((i,j)^*\)-soft preopen set.

**Note:** Let \((Q,T)\) be a \((i,j)^*\)-soft preclosed set such that \((R,T) \subseteq (Q,T) \subseteq \bar{\sigma}_{ij}\text{-int}(R,T)\). Then \((Q,T)\) is a \((i,j)^*\)-soft preclosed set.
**Theorem 2.5:** If the \((i,j)^*\)-soft preclosure of \((Q,T)\) is a soft set in soft bitopological space. Then,

(i) \((\delta_{i,j} \bullet \text{pcl}(Q,T))^c = \delta_{i,j} \bullet \text{pint}(Q^c,T)\).

(ii) \((\delta_{i,j} \bullet \text{pint}(Q,T))^c = \delta_{i,j} \bullet \text{pcl}(Q^c,T)\).

**Proof:** Let \((Q,T)\) be a soft set over \(A\).

(i) Let \((\delta_{i,j} \bullet \text{pcl}(Q,T))^c = (\cap ((Q,T) \subseteq L,T) \text{ and } (L,T) \in \text{PCSS}(A_T)))^c = \cup \{(L,T)^c \& (L,T)^c \subseteq (Q,T)^c \& (L,T)^c \subseteq \text{POSS}(A_T)\}\)

\[= \cup \{(L,T)^c \& (L,T)^c \subseteq (Q,T)^c \& (L,T)^c \subseteq \text{POSS}(A_T)\}\]

\[= \delta_{i,j} \bullet \text{pint}(Q^c,T)\).

(ii) Let \((\delta_{i,j} \bullet \text{pint}(Q,T))^c = (\cup \{(L,T) \subseteq (Q,T) \& (L,T) \in \text{POSS}(A_T)\})^c = \cap \{(L,T)^c \& (Q,T)^c \subseteq (L,T)^c \& (L,T)^c \subseteq \text{PCSS}(A_T)\}

\[= \cap \{(L,T)^c \& (Q,T)^c \subseteq (L,T)^c \& (L,T)^c \subseteq \text{PCSS}(A_T)\}

\[= \delta_{i,j} \bullet \text{pcl}(Q^c,T)\).

**Theorem 2.6:** Let \((A, \delta_i, \delta_j)\) be a soft bitopological space and \((Q,T)\) be a soft set in \(A\).

(i) \(\delta_{i,j} \bullet \text{pcl}((Q,T) \cup (M,T)) = \delta_{i,j} \bullet \text{pcl}(Q,T) \cup \delta_{i,j} \bullet \text{pcl}(M,T)\)

(ii) \(\delta_{i,j} \bullet \text{pint}(Q,T) \cap (M,T) = \delta_{i,j} \bullet \text{pint}(Q,T) \cap \delta_{i,j} \bullet \text{pint}(M,T)\)

**Proof:** We take, \((Q,T) \cup (M,T) \supset (Q,T) \& (Q,T) \cup (M,T) \supset (M,T)\)

We have, \((Q,T) \subseteq (M,T), \Rightarrow \delta_{i,j} \bullet \text{pcl}((Q,T) \cup (M,T)) \supset \delta_{i,j} \bullet \text{pcl}(Q,T) \& \delta_{i,j} \bullet \text{pcl}((Q,T) \cup (M,T)) \supset \delta_{i,j} \bullet \text{pcl}(M,T) \Rightarrow \delta_{i,j} \bullet \text{pcl}(Q,T) \supset \delta_{i,j} \bullet \text{pcl}(M,T).\) \hspace{1cm} (1)

Since, \((i,j)^*\)-soft pre-closure \((Q,T), (i,j)^*\)-soft pre-closure \((M,T) \epsilon \) soft pre-closed set \((A_T)\) and \((i,j)^*\)-soft pre-closure \((Q,T) \cup (i,j)^*\)-soft pre-closed \((A_T)\). Now, \((Q,T) \subset \delta_{i,j} \bullet \text{pcl}(Q,T)\) and \((P,T) \subset \delta_{i,j} \bullet \text{pcl}(P,T)\). Implies, \(\delta_{i,j} \bullet \text{pcl}(Q,T) \cup \delta_{i,j} \bullet \text{pcl}(M,T) \supset ((Q,T) \cup (M,T))\). That is, \(\delta_{i,j} \bullet \text{pcl}((Q,T) \cup (M,T))\) is the lowest \((i,j)^*\)-soft pre-closed set containing \((Q,T) \cup (M,T)\) and \(\delta_{i,j} \bullet \text{pcl}(Q,T) \cup \delta_{i,j} \bullet \text{pcl}(M,T)\) is an \((i,j)^*\)-soft pre-closed set containing \((Q,T) \cup (M,T)\). Therefore, \(\delta_{i,j} \bullet \text{pcl}(Q,T) \cup (M,T) \subset \delta_{i,j} \bullet \text{pcl}(Q,T) \& \delta_{i,j} \bullet \text{pcl}(M,T)\).

From (1) and (2), \(\delta_{i,j} \bullet \text{pcl}((Q,T) \cup (M,T)) \subset \delta_{i,j} \bullet \text{pcI}(Q,T) \cup \delta_{i,j} \bullet \text{pcl}(M,T)\).

(ii) Similar to (i).

**References:**


