Gauss Elimination Modified Method

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Abstract**: Gauss elimination method is one of the direct numerical method to solve a system of equations. In this research paper, Gauss elimination method is modified to solve a system of linear equations with any number of variables.**

Index Terms**: Gauss elimination method, Modified Gauss elimination method, Gauss Jordan method, Gauss Seidel method.**

I. INTRODUCTION

This document is a template. For questions on pap A linear system of equations we can be solved by varies methods. There are so many direct methods and indirect methods. Direct methods give accurate solution without any iteration. A system of 'n' equations with 'n' variables can be solved by Gauss elimination method and Gauss Jordan Method. These are direct methods. Gauss Seidal and Gauss Jacobi are indirect methods. Gauss elimination method requires the system of equations to be written as an augmented matrix and converting the matrix to upper / lower triangular matrix which requires backward substitution method to find the solution. Gauss Jordan method requires augmented matrix, which is converted to diagonal matrix and hence the solution is obtained directly. Numerical errors can be controlled easily in Gauss elimination method. Hence Gauss elimination is preferred when we are solving large system of equations in a computer.

Always direct methods for the solution of linear system are preferred but in case of matrices with a large number of zero elements, it will be advantageous to use iterative method which preserves these elements. Gauss Jacobi starts with arbitrary solution to the different variables, say '0' then proceeded as iterations to find the final solutions. Gauss Seidal method and Gauss Jacobi method are similar but only one difference that updated values are used to find the values of variables in next iteration while Gauss Jacobi uses only old values. Where the system is given by.

$$
\begin{array}{llllll}
a11x1 & a12x2 & \dots & \dots & + a1nxn & = b1 \\
a21x1 & a22x2 & \dots & \dots & + a2nxn & = b2 \\
\dots & \dots & \dots & \dots & \dots & = \dots \\
an1x1 & an2x2 & \dots & \dots & + annxn & = bn
\end{array} \qquad (1)
$$

In this paper, Gauss elimination method is modified and modified Gauss elimination method is introduced to solve a system of linear equations. Section ll discusses the method along with some examples and section lll concludes the paper.

II. HOW TO SOLVE GAUSS ELIMINATION METHOD

A. Procedure

Let us consider the system of linear equations (1) for solving. Then the augmented matrix of (1) is

$$
\begin{array}{cccccc}
a11 & a12 & \dots & \dots & a1n & b1 \\
a21 & a22 & \dots & \dots & a2n & b2 \\
\dots & \dots & \dots & \dots & \dots & \dots \\
an1 & an2 & \dots & \dots & ann & bn
\end{array} \qquad (2)
$$

Augmented matrix (2) is converted to upper triangular matrix. The nth row is made of '0' except ann by using some row operations. Hence (2) becomes

$$
\begin{array}{ccccccc}
a11' & a12' & \dots & \dots & a1(n-1)' & a1n' & b1 \\
a21' & a22' & \dots & \dots & a2(n-1)' & a2n' & b2 \\
\dots & \dots & \dots & \dots & \dots & \dots & \dots \\
0 & 0 & \dots & \dots & 0 & ann & bn
\end{array} \qquad (3)
$$

Similarly proceeding the elements above the diagonal are made of '0' and hence (2) is converted to upper triangular matrix obtained as

$$\begin{matrix} a11' & a12' & \ldots & \ldots & a1(n-1)' & a1n' & b1 \\ 0 & a22' & \ldots & \ldots & a2(n-1)' & a2n' & b2 \\ 0 & 0 & a33' & \ldots & a3(n-1)' & a3n' & b3 \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & \ldots & \ldots & 0 & ann' & bn \end{matrix} \qquad (4)$$

It is clear that forward substitution method can be used to find the values of x1, x2,...,xn. It is clear that this method will fail if one of the elements (A11, A22 or A33) will vanish. In such type of case, the method can be modified by rearranging the rows so that the pivot is non-zero. Therefore this procedure is called as partial pivoting and can be easily implemented on a computer. If this is impossible, then the matrix is singular and (1) has no solution.

Example :

Solve by Gauss- Elimination method

x1 + x2 + 2 = 0

x2 + x3 =1

x1 + x3 = 1

The augmented matrix will be

1	1	2	0
0	1	1	1
1	0	1	1

The upper triangular matrix will be

1	1	0	-2
0	1	1	1
0	0	1	2

Substitute 'backwards' (bottom to top) and get the solution i.e. x1 = -1, x2 = -1, x3 = 2.

Consider the system of equation

2x + 3y – z – t =0

x – y – 2z – 4t = 0

3x + y + 3z – 2t = 0

6y + 2y – 7t = 0

The augmented matrix will be,

2	3	-1	-1	0
1	-1	-2	-4	0
3	1	3	-2	0
0	6	2	-7	0

The upper triangular matrix will be,

1	-1	-2	-4	0
0	1	-6	-3	0
0	0	1	2/3	0
0	0	0	1	0

Substitute 'backwards' (bottom to top) and get the solution

i.e. x = 0, y = 0, z = 0, t = 0.

III.Conclusion

We can be solved a system of linear equations by many direct and indirect methods. Gauss elimination method is modified in this paper to solve a system of equations.

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