3-Types of Products in Ψ-Polar Fuzzy Graph Network

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***Abstract*: The scope of this paper to familiarize that the concept strong product, Cartesian product ,Cross products in *Ψ* -Polar fuzzy graph and also we prove that the theorems and examples.**

***Index Terms*: Strong product, Cartesian product, Cross product, Fuzzy graph,$\Psi - Polar\ Fuzzy\ Graph$**

I. INTRODUCTION

Chenet al, established the concept of m -polar fuzzy set as an expansion of bipolar fuzzy sets in 2014. This concept beyond is that "m -polar information". An Ψ-polar fuzzy graph is a team of G=(M,N), wherever M:V→$[0,1]^m$ is an m-polar fuzzy set in V and N:V× V→$[0,1]^m$ is an m-polar fuzzy relation on V ∋ $p_i oN\ (x,y) \leq min\{p_i oM\ (x),\ p_i oM(y)\}$, ∀ $x,y\ \epsilon\ V$. An Ψ-polar fuzzy set or a $[0,1]^m$ or m-th power of [0,1] set on X is strictly a mapping A:X→$[0,1]^m$. The set $\Psi(x)$ is expressed by the set of all Ψ -polar fuzzy sets on X.

2. Ψ – Polar Fuzzy Graph Network

Definition 2.1 : Consider a graph structure G=(U,E_1,E_2,........,E_n) and let M and N_i are Ψ-polar fuzzy set on U and E_i individually, ∋ $p_i o$N$(xy) \leq min\{p_i o$M(x),$p_i o$M$(y)\}$∀ $x,y\ \epsilon U,\ i\ \epsilon\ n, j\ \epsilon\ m$.If $p_i o$N$(xy) = 0\ \forall\ x,y\ \epsilon U \times U \backslash E_j\ \forall\ j$.Then the graph $G_{(m)} = (M,N_1,N_2\ ,........,\ N_n)$ is called the Ψ -polar fuzzy graph structure on G. Assume the graph structure G =(U, E_1,E_2) ∋ $U = \{c_1, c_2, c_3, c_4\}$, $E_1 = \{c_1, c_2\}$ & $E_2 = \{c_3 c_2, c_2 c_4\}$.

If $G_{(m)} = (M, N_1,N_2)$ is a four polar fuzzy graph structure of G = (U, E_1, E_2,….., E_n).

Theorem 2.1: Cartesian product of two Ψ-polar fuzzy graph network is an Ψ-polar fuzzy graph network.

Proof:Consider the two graph structure are G_1=(U_1, E_{11}, E_{12},………,E_{1n}) & G_1 = (U_2,E_{21}, E_{22}, ………,E_{2n}) be the Cartesian product is G = ($U_1 \times U_2$,$E_{11} \times E_{21}, E_{12} \times E_{22}$,….,$E_{1n} \times E_{2n}$).Suppose that $G^1_{(m)}$=(M_1, N_{11}, N_{12},..,N_{1n}) be the Ψ -polar fuzzy graph network of G_1 & $G^2_{(m)}$=(M_2, N_{21}, N_{22},..,N_{2n}) be the fuzzy graph network of G_2.Subsequently Ψ -polar fuzzy graph network of G is ($M_1 \times M_2$,$N_{11} \times N_{21}, N_{12} \times N_{22}$, ………….,$N_{1n} \times N_{2n}$). We know that,$M_1 \times M_2$ & $N_{1i} \times N_{2i}$ are the Ψ -polar fuzzy set of $U_1 \times U_2$ & $E_{1i} \times E_{2i}$.Next we have to prove that $N_{1i} \times N_{2i}$ is an Ψ -polar fuzzy relation on $M_1 \times M_2\ \forall\ i$.

Case (i):$x\epsilon\ U_1$ & $x_2\ y_2 \epsilon\ E_{2i}$ .We know that,

$p_j o(N_{1i} \times N_{2i})((xx_2)(xy_2))$ = $p_j oM_1\ (x)$^$p_j oN_{2i}(x_2\ y_2)$ ⟶ (1)

By the definition ,$p_i oN\ (x,y) \leq min\{p_i oM\ (x),\ p_i oM\ (y)\}$ ⟶ (2)

By applying (2) in (1), $p_j o(N_{1i} \times N_{2i})((xx_2)(xy_2)) \leq p_j oM_1(x)$^$[min\{p_j oM_2(x_2),\ p_j oM_2(y_2)\}]$= $min\{p_j oM_1(x)$^$p_j oM_2(x_2)$, $p_j oM_1(x)$^$p_j oM_2(y_2)\}]$ ⟶ (3)

By the Cartesian product, $p_j o(M_1 \times M_2)(x_1 x_2)$ = $p_j oM_1(x_1)$^$p_j oM_2(x_2)$

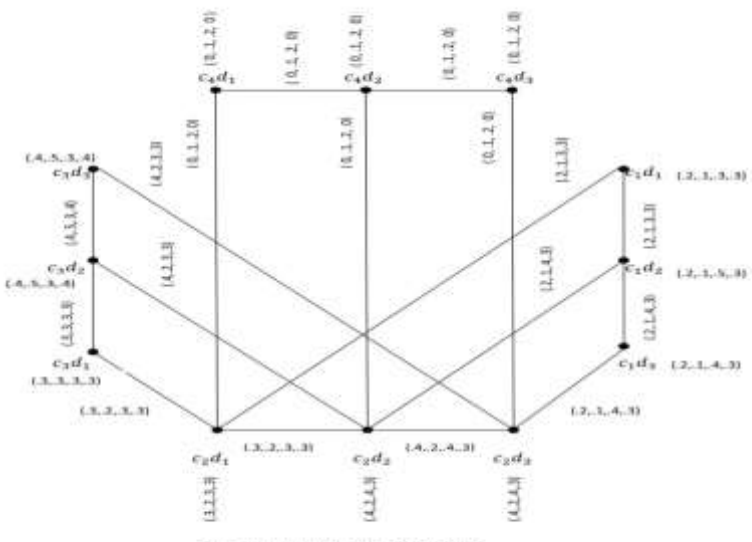
By (3),$p_j o(N_{1i} \times N_{2i})((xx_2)(xy_2))$=$min\{p_j o(M_1 \times M_2)(xx_2)$,$p_j o(M_1 \times M_2)(xy_2)\ \}$

Case (ii): If $x\epsilon\ U_2$ & $x,y\epsilon\ E_{1i}$ .We know that,$p_j o(N_{1i} \times N_{2i})((x_1 y)(y_1 y))$ = $p_j oM_2(y)$^$p_j oN_{1i}(x_1\ y_1)$

Byapplying(2)in(1),$p_j o(N_{1i} \times N_{2i})((x_1 y)\ (y_1 y)) \leq p_j o\ M_2(y)$^$[min\{\ p_j oM_1(x_1)$,$p_j oM_1(y_1)\}]$
= $min\{p_j oM_2(y)$^$p_j oM_1(x_1)$,$p_j oM_2(y)$^$p_j oM_1\ (y_1)\}]$.

By applying(3),$p_j o(N_{1i} \times N_{2i})\big((x_1 y)(y_1 y)\big) = min\{p_j o(M_1 \times M_2)(x_1 y)$,$p_j o(M_1 \times M_2)(y_1 y)\ \}$∀$j\ \in m$.In the case (i) and case (ii) takes ∀ $i\ \in n$. Hence $N_{1i} \times N_{2i}$ is an Ψ-polar fuzzy relation on $M_1 \times M_2\ \forall\ i\ \in n$. Hence the proved.

Example:



Cartesian product of Ψ -Polar Fuzzy Graph

Theorem 2.2: Cross Product of two Ψ -polar fuzzy graph network is an Ψ -polar fuzzy graph network

Proof: Consider the two graph structure are G_1=(U_1,E_{11}, E_{12}, …,E_{1n}) & G_1=(U_2,E_{21}, E_{22}, …,E_{2n}) be the Cross product is G=($U_1 * U_2$,$E_{11} * E_{21}$, $E_{12} * E_{22}$,….,$E_{1n} * E_{2n}$).Suppose that $G^1_{(m)}$=(M_1, N_{11}, N_{12},..,N_{1n}) be the Ψ-polar fuzzy graph network of G_1 & $G^2_{(m)}$=(M_2, N_{21}, N_{22},..,N_{2n}) be the fuzzy graph network of G_2.Subsequently Ψ-polar fuzzy graph structure of the G is ($M_1 * M_2$,$N_{11} * N_{21}$, $N_{12} * N_{22}$, ….,$N_{1n} * N_{2n}$). $M_1 * M_2$ & $N_{1i} * N_{2i}$ are the Ψ -polar fuzzy set of $U_1 * U_2$ & $E_{1i} * E_{2i}$.Next we have to prove that $N_{1i} * N_{2i}$ is an Ψ -polar fuzzy relation on $M_1 * M_2$ $\forall\, i$.Whenever $x_1\, y_1 \epsilon\ E_{1i}$ & $x_2\, y_2 \epsilon E_{2i}$,$p_j o(N_{1i} * N_{2i})((x_1x_2)(y_1y_2))$=$p_j oN_{1i}(x_1y_1)$^$p_j oN_{2i}(x_2\, y_2)$ ⟶(1) We know that,$p_i oN\ (x, y) \leq min\{p_i oM\ (x),\ p_i oM\ (y)\}$ .The equation (1) becomes,

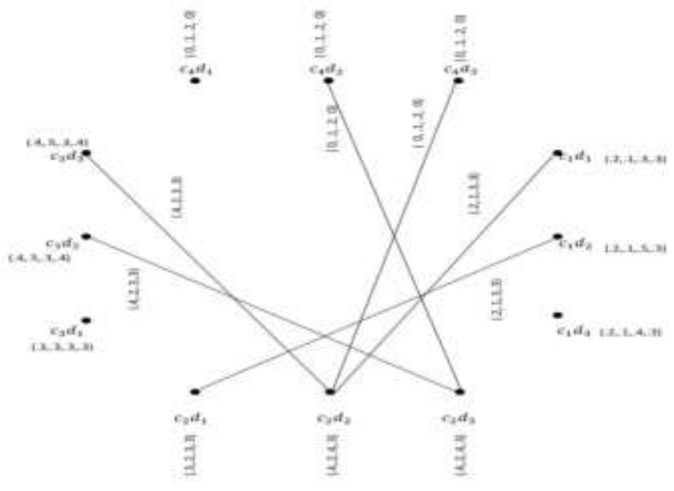
$p_j o(N_{1i} oN_{2i})((x_1x_2)(y_1y_2)) \leq [min\{p_i oM_1(x_1), p_i oM_1(y_1)\}]$^ $[min\{p_i o\ M_2(x_2), p_i oM_2(y_2)\}]$

$= min\{p_i oM_1(x_1) \wedge p_i oM_2(x_2),\ p_i oM_1(y_1) \wedge p_i oM_2(y_2)$⟶(2)

We know that the definition, the Cross product

$p_j o(M_1 * M_2)(x_1x_2) = p_j oM_1(x_1) \wedge p_j oM_2(x_2)$⟶ (3)

By,$p_j o(N_{1i} * N_{2i})\big((x_1x_2)(y_1y_2)\big)\ =\ min\{p_i o(M_1 * M_2)(x_1x_2), p_i o(M_1 * M_2)(y_1y_2)\}$ This is takes $\forall\ i \in n$. Hence $N_{1i} * N_{2i}$ is an Ψ-polar fuzzy relation on $M_1 * M_2$ $\forall\ i \in n$.



Cross product of Ψ-polar fuzzy graph

Corollary 2.3: Strong product Ψ -polar fuzzy graph network is an Ψ -polar fuzzy graph network.

Conclusion: In this paper we discussed the concept three types of three types Ψ -polar fuzzy graph network is an Ψ -polar fuzzy graph network.

References:

[1**]** Chei..J,Li.S,Ma.S,Wang.X(2014) m-polar fuzzy sets:extension of bipolar fuzzy sets.sci World J 2014
[2] Dinesh .T,Ramakrishna.T.V(2011) On Generalised fuzzy Structure.Appl Math Sci 5(4):173-180