An application of Gauss Elimination technique to magic square

[1]Dhanalakshmi. M, [2]V.Jyothi, [3]K. Anusha, [4]K.N.V. Suhasini, [5]P.Anusha

[1,2]Assistant Professor, [3,4,5]Student
Department of Mathematics,
Sri Durga Malleswara Siddhartha Mahila Kalasala, Vijayawada, A.P, India,

***Abstract*: In this paper, we study a 3 by 3 magic square by using the Gauss elimination technique. May be you will change your mind after you look at the application.**

INTRODUCTION:

A magic squares of size n is an n by n square matrix whose entries consist of all integers. Between $1$and n^2,with the property that the sum of the entries of each column, row ordiagonal is the same. The sum of the entries of any row, column or diagonal of a magic Squares of size n is $n(n^2+1)/2$for example (1+2+…+k=k(k+1)/2).

Let us start with the easy case of a two by two magic square.

a	*b*
c	*d*

a	*b*	*c*
d	*e*	*f*
g	*h*	*i*

In order to have a magic square, one would have a linear system of six equations and four unknowns;

a	+	*b*	=	*5*
c	+	*d*	=	*5*
a	+	*c*	=	*5*
b	+	*d*	=	*5*
a	+	*d*	=	*5*
b	+	*c*	=	*5*

One can use the Gaussian elimination to solve that system. A simpler way is to notice that the first and the third equation give that $b=c$; so the last equation becomes $2b=5$ which, of course, has no integer solution. So the system has no integer solution. In other words, there are no magic squares of size 2.

Fine, let us try now a magic square of size 3:

In this case, the sum of each row, column or diagonal must be15. This gives the following system of equations:

a	+	*b*	+	*c*	=	*15*
d	+	*e*	+	*f*	=	*15*
g	+	*h*	+	*i*	=	*15*
a	+	*d*	+	*g*	=	*15*
b	+	*e*	+	*h*	=	*15*
c	+	*f*	+	*i*	=	*15*
a	+	*e*	+	*i*	=	*15*
c	+	*e*	+	*g*	=	*15*

$$\begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 15 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 15 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 15 \\ 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 15 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 15 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 15 \\ 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 15 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 15 \end{bmatrix}$$

Using Gaussian elimination technique would give us the following reduced form of the above System:

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 10 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 10 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & -1 & -1 & -5 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & -2 & -10 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 5 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 2 & 20 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 15 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

So we have two free variables *h* and *i*. Taking *h= 9* and *i =2* would give the following solution:

8	1	6
3	5	7
4	9	2

References:

[1]. Wikipedia encyclopedia.

[2]. G.William, linear algebra with application, 7th edition, Jones and Barnetlett publisher, LLC, UK(2011).

[3]. A text book of B. Sc mathematics volume (III), S. Chand Publications.