An application of Gauss Elimination technique to magic square

Dhanalakshmi. M, V.Jyothi, K. Anusha, K.N.V. Suhasini, P.Anusha

1,2Assistant Professor, 3,4,5Student
Department of Mathematics,
Sri Durga Malleswara Siddhartha Mahila Kalasala, Vijayawada, A.P, India,

Abstract: In this paper, we study a 3 by 3 magic square by using the Gauss elimination technique. May be you will change your mind after you look at the application.

INTRODUCTION:
A magic squares of size $n$ is an $n$ by $n$ square matrix whose entries consist of all integers. Between $1$ and $n^2$, with the property that the sum of the entries of each column, row or diagonal is the same. The sum of the entries of any row, column or diagonal of a magic squares of size $n$ is $n(n^2+1)/2$ for example $(1+2+…+k=k(k+1)/2)$.

Let us start with the easy case of a two by two magic square.

$$
\begin{array}{cc}
a & b \\
c & d \\
\end{array}
\quad
\begin{array}{ccc}
a & b & c \\
d & e & f \\
g & h & i \\
\end{array}
$$

In order to have a magic square, one would have a linear system of six equations and four unknowns;

$$
\begin{array}{c}
a + b = 5 \\
c + d = 5 \\
a + c = 5 \\
b + d = 5 \\
a + d = 5 \\
b + c = 5 \\
\end{array}
$$

One can use the Gaussian elimination to solve that system. A simpler way is to notice that the first and the third equation give that $b=c$; so the last equation becomes $2b=5$ which, of course, has no integer solution. So the system has no integer solution. In other words, there are no magic squares of size 2.
Fine, let us try now a magic square of size 3:

In this case, the sum of each row, column or diagonal must be 15. This gives the following system of equations:

\[
\begin{align*}
    a + b + c &= 15 \\
    d + e + f &= 15 \\
    g + h + i &= 15 \\
    a + d + g &= 15 \\
    b + e + h &= 15 \\
    c + f + i &= 15 \\
    a + e + i &= 15 \\
    c + e + g &= 15 \\
\end{align*}
\]

Using Gaussian elimination technique would give us the following reduced form of the above system:

\[
\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 10 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 10 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 5 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 20 \\
0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 1 & 15 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\]
So we have two free variables \( h \) and \( i \). Taking \( h = 9 \) and \( i = 2 \) would give the following solution:

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<td>4</td>
<td>9</td>
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References:

[1]. Wikipedia encyclopedia.
