A STATISTICAL APPROACH TO FUZZY SHORTEST TRAVELLING PATH USING HEXAGONAL FUZZY NUMBERS

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***Abstract*: In the present paper fuzzy shortest travelling path is determined based on measure of central tendency. The uncertain parameters (ie) activity times are represented by Hexagonal fuzzy numbers, and these numbers converted into normal time (expected) for each activity by using statistical parameters such as mean and variance. Based on these fuzzy shortest travelling route is obtained and also a statistical analysis is made with the help of variability.**

***Keywords*: Fuzzy sets, Hexagonal fuzzy numbers, fuzzy network, Measures of Central Tendency.**

1. Introduction:

The fuzzy shortest path is a network based method planned for development and organization of complex projects in real life situations, Fuzzy shortest path is a network based optimization problem which has a wide range of applications in various fields

It is a classical network optimization problem which has a wide range of applications in various fields. It deals with finding the path which has minimum distance , time ,etc., Many real life situations dealing with the network requires the computation of the best shortest path from initial node to end node, when the activity duration times are uncertain.

The concept of fuzzy set theory was introduced by Zadeh[1] in 1965. The shortest path problems are very useful tool for complicated project and widely applied in science, engineering and management, e.g road network applications, transportation, routing in communication channels etc., The shortest path problems under an uncertain and imprecise environment was first analysed by Dubois and Prade.

Lee and Li proposed a comparison of fuzzy numbers by considering the mean and dispersion (variance) based on the concept of statistics.Chang proposed the Coefficient of variance (CV index) ie.CV= $\sigma / |\mu|$ where $\sigma > 0$,$\mu \neq 0$

In a classical network problem, weights of the edges are supposed to be real numbers, however, in most practical applications the parameters are not naturally precise in general, therefore, in real world situation they may be considered to be a fuzzy.Atanassov generalized the concept of fuzzy sets to interval –valued intuitionistic fuzzy sets.

The fuzzy shortest path length in a network by means of a fuzzy linear programing approach was proposed by Lin and Chen (1994). Again Lin depicted a new line of method to a fuzzy critical path method for activity network created on statistical buoyancy interval estimatesa and a ranking method for level (1-α) fuzzy numbers. His focus was to introduce an approach that combined fuzzy set theory with statistics that incorporates the signed distance ranking of level of (1-α) fuzzy number.

Measures of Central Tendency shows the tendency to some central value around which data tends to cluster. It is one of the powerful tools for analysis is to calculate a single average value that represents the entire mass of data. Such a value is neither the smallest nor the largest value, but is a number whose value is somewhere in the middle of the group. It describes the characteristics of entire data and to facilitate comparison.

This paper aims to use statistical parameters for defuzzyfying and ranking fuzzy numbers. Here we have proposed a new algorithm for finding shortest Travelling Path with the help of Hexagonal fuzzy numbers.

This paper is organized as follows. In section 2 some basic concepts on fuzzy sets, fuzzy numbers, Measures of central average, Coefficient of variation are discussed. In section 3

2. Preliminaries

In this section some basic definitions related to fuzzy set, fuzzy numbers are reviewed.

2.1 **Definitions: [FuzzySet]**

A Fuzzy set is characterized by a membership function mapping element of a domains, space, or the universe of discourse X to the unit interval [0,1].

2.2 Definition [Fuzzy numbers]

A fuzzy number is a generalization of a regular real number and which does not refer to a single value but rather to a connected a set of possible value, where each possible value has its weight between 0 and 1.

A Fuzzy number is a convex normalized fuzzy set on the real line R such that, There exist at least one i)$x \in X$ with $\mu A (x) = 1$,

ii)$\mu_A (x)$ is piece wise continuous.

2.3 Definition(Triangular fuzzy number)
A fuzzy number ***A(x),***it can be represented by ***A*(a,b,c;1)** with membership function μ(x) is given by

$$\mu(x) = \begin{cases} \frac{(x-a)}{(b-a)} & a \le x \le b \\ 1 & x=b \\ \frac{(c-x)}{(c-b)} & c \le x \le b \end{cases}$$

2.4.Definition(Trapezoidal fuzzy number)

A fuzzy number A defined on the universal set of real numbers R denoted by **Ã (a,b,c,d;1)** is said to be a Trapezoidal fuzzy number if its membership function μ_A (x) is given by

$$\mu_A(x) = \begin{cases} \frac{(x-a)}{(b-a)} & a \le x \le b \\ 1 & x=b \\ \frac{(d-x)}{(d-c)} & c \le x \le d \end{cases}$$

2.4.1 Arithmetic Operations of Trapezoidal Fuzzy Numbers
Let $Å_1$ and $Å_2$ be two trapezoidal fuzzy numbers where $Å_1$ – (a,b,c,d) and $Å_2$=(e,f,g,h) then Fuzzy numbers addition: $Å_1$ $Å_2$ = (a+e,b+f,c+g,d+h)

2.5 Definition (Hexagonal Fuzzy number)

A Fuzzy number A_H is a HFN [9]. denoted by A_H = (a_1, a_2, a_3, a_4, a_5, a_6). Where a_1, a_2, a_3, a_4, a_5, a_6 and real numbers. And it membership function is given below.

$$\mu_A(x) = \begin{cases} \frac{1}{2}\left[\frac{x-a_1}{a_2-a_1}\right] & \text{for } a_1 \le x \le a_2 \\ \frac{1}{2}+\frac{1}{2}\left[\frac{x-a_2}{a_3-a_2}\right] & \text{for } a_2 \le x \le a_3 \\ 1 & \text{for } a_3 \le x \le a_4 \\ 1-\frac{1}{2}\left[\frac{x-a_4}{a_5-a_4}\right] & \text{for } a_4 \le x \le a_5 \\ \frac{1}{2}\left[\frac{a_6-x}{a_6-a_5}\right] & \text{for } a_5 \le x \le a_6 \\ 0 & \text{otherwise} \end{cases}$$

3. Construction of Expected time (Normal) using Statistical Data(Ranking Function)
A Statistical parameters can be used in an unknown situation, there in inaccuracy network problems involving fuzzy numbers as activity time. Therefore the statistical parameters such as unbiased estimate of mean and variance are used .
Let D_{ij}a unbiased estimate mean value which is treated as the expected time for each activity which is computed from given Hexagonal fuzzy numbers x_1 ,x_2 ,x_3 ,x_4 ,x_5 ,x_6 as follows

D_{ij} =**Ĥ(1)**= **Σ**$\frac{xk}{n}$where xk denotes the HFN ,k =1,2,…6

Similarly let d_{ij} a unbiased estimate for variance which is treated as the expected time for each activity can be calculated from given HFN x_1 ,x_2 ,x_3 ,x_4 ,x_5 ,x_6 as follows,

d_{ij} = **Ĥ(2) =** nS^2 /(n – 1)where $S^2 = \Sigma(x_k - \dot{x})^2 /n$

4. Description of this paper
Hexagonal fuzzy numbers are converted into expected time (Normal time) for each activity by using**Ĥ(1)**and**Ĥ (2)** .These values treated as a normal time between the nodes and the shortest travelling path ,(ie.minimum distance) is determined by using the given algorithm.
4.1 Algorithm

Step: 1 Construct the network diagram according to the given activity.

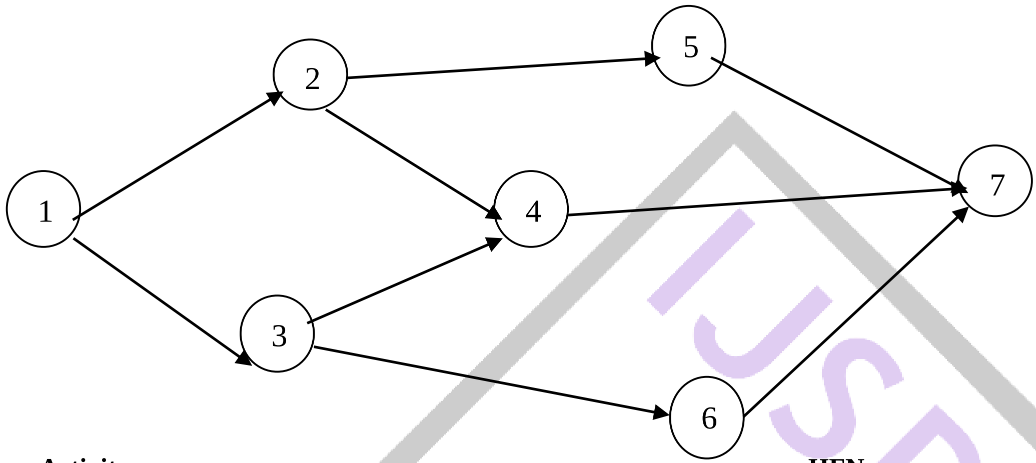
Step: 2 From given HFN find the Expected time for each activity (normal time) by using unbiased estimate mean $\hat{\mathbf{H}}\mathbf{(1)}$, as well as unbiased estimate variance $\hat{\mathbf{H}}\mathbf{(2)}$

Step: 3 Determine the Number of possible ways of i.e path from initial node to end node.for both mean value travelling route as well as variance value travelling route

Step: 4 From the all possible ways calculate the summation of expected time from that we can find the minimum Travelling time (i.e) shortest traveling path.

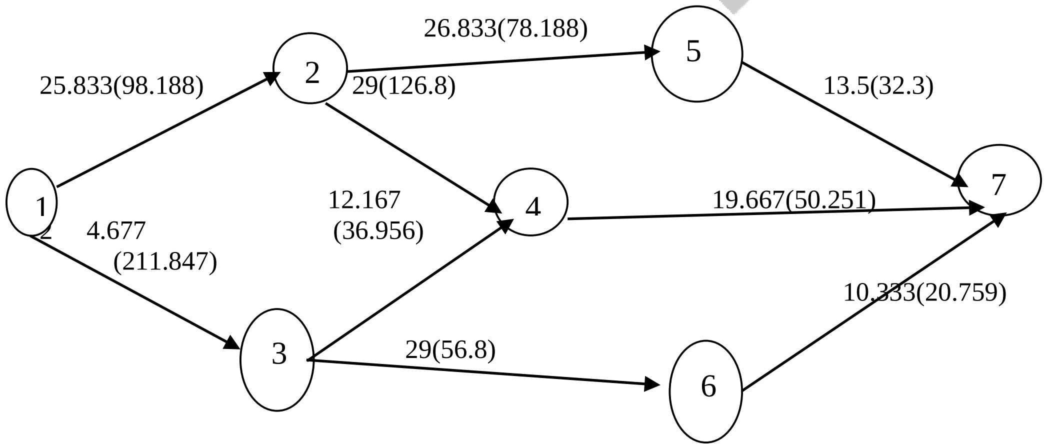
5. Numerical examples.

Consider a project network diagram with nine activities . The distance between them is represented by HFN



Activity	**HFN**
1-2	(13,18,23,28,33,40)
1-3	(6,13,20, 28,36,45)
2-4	(15,20,25,31,38,45)
2-5	(16,19, 24,29,34,39)
3-4	(5,7,10,13,17,21)
3-6	(20,23,26,30,35,40)
4-7	(11, 14, 17,21,25,30)
5-7	(7,9,11 ,14, 18,22)
6-7	(5,7,8,11,14, 17)

Step :1 after converting HFN as mean and variance as expected time (normal) by using $\hat{\mathbf{H}}\mathbf{(1)}$, **and** $\hat{\mathbf{H}}\mathbf{(2)}$for each activity the network given below becomes , where values outside indicate mean and inside the bracket indicate variance



Step 2 : The possible Number of ways are listedbelowwith minimum distance correspondingmean as well as variance and also C.V index isgivenalong the path

Route	Total time(Mean)	Total time(Variance)	Coefficient of Variation
1-2-5-7	66.166	208.676	21.832
1-2-4-7	74.5	275.239	22.269
1-3-4-7	56.501	299.054	30.607
1-3-6-7	64	289.506	26.586

6. Discussions and Comparison

1)The fuzzy hexagonal numbers are taken as time taken to traverse the distance between nodes.
2) The numbers can be considered as 6 sample travelling times between nodes.
3) The least number corresponds when there is less traffic and the largest number corresponds to heavy traffic.
4) The objective now becomes to find the " Shortest Travelling Time" between the start node and end node.
5) The methods adopted are (i) shortest travelling time based on an unbiased estimate of mean , without taking into account the corresponding variability. (ii) minimum variance time path based on the unbiased estimate of variance which minimizes the variability in travelling time and hence more stable. (iii) The most consistent path based on coefficient of Variation.The results are compared and conclusions are listed

7. Advantages of statistical study

The focus of this paper was to introduce an approach that combined fuzzy set theory with statistics .When ever uncertainty arises in complex project we use statistical concept to arrive some better conclusion. Since statistics parameters are important tool for real life situations for solving many problems. The following significant results obtained based on characteristics of statistical parameters .(i) The activity fuzzy network with fuzzy data is same as fuzzy project network based on statistical parameters.(ii) some relation exist between notion of fuzzy data.(iii) It can be noted that this approach is used to solve practical problems in unknown or vague situations , without any assumptions , using fuzzy numbers.

8. Conclusion

1.In this paper a new approach is developed on the newly constructed statistical parameters such as mean and variance to calculate shortest travelling route of a fuzzy project network using hexagonal fuzzy numbers which is treated as normal time for each activity
2. Least Travelling time occurs along the route 1-3-4-7 (time 56.501) and with maximum variance 299.054, with CV index30.607.
3.We notice that as the travelling time increases the corresponding path becomes more consistent. (ie) shortest path is not the safest path in terms of consistency and reliability.
4. This analysis throws light on the fact that people more often seek the shortest path and cram up the route with traffic congestion . In such cases " take diversion" is the only option.
5.Further the Paths 1-2-5-7 and 1-2-4-7 are more consistent since CV index be almost 22%.
6. A Combination of fuzzy logic statistics and operations research throw more light in such kind of complex traffic problems which can be solved by a) New bye-pass route b) new over bridges and subways.
7. . Hence this method provides new tool and ideas for the project researchers on how to approach fuzzy shortest travelling route on the basis of consistency in network problems using statistical data.

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