A Case Study on Analytical Geometry and its Application in Real Life

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***Abstract*: The main objective of this paper is to use analytical geometry in our day to day life. Analytical Geometry has extensive range of applications in our life. Its uses are extensively spread among almost all the fields like trigonometry, calculus, dimensional geometry, etc. It's used in engineering fields, medicine, physics, and constructions.**

***Index Terms*: Geometry, Microphone, Hyperbola, Distance, Cooling Tower, Horizontal, Axis, Focus, Origin**

I. INTRODUCTION

The basic idea of analytic geometry is to represent the curves by equations, but this is not the whole idea. If it were, then the Greeks would have been considered the first analytic geometers. Analytic geometry is also called as coordinate geometry. The importance of analytic geometry is that it establishes a correspondence between algebraic equations and the geometric curves.

II. APPLICATION OF HYPERBOLA

a). HYPERBOLOID NUCLEAR COOLING TOWERS

Most cooling towers from coal-fired power plants are shaped like hyperbolas because it is an optimal shape for letting heat dissipates. The hyperbola accelerates upward air flow, pushing air higher and higher, losing heat as it goes. The hyperboloid shape also aids in their structural strength and minimum usage of material.

Fig1: Nuclear Cooling Tower

REASON BEHIND ITS HYPERBOLOID SHAPE

- The strength factor

First and foremost, the shape of the cooling tower impacts the strength of the entire structure. Since cooling towers are supposed to cool the working fluid down to a very low temperature, they release vapors through the opening at the top of the tower into the atmosphere. Therefore, these towers have to be necessarily tall, or else the released vapor may cause fogging or recirculation. To support such a high structure, it is very important that the base is spread over a large area and considerably consolidated so that it can support the tall, heavy structure above it. This is why cooling towers have a circular base that is large.

- Facilitating aerodynamic lift

As hot water evaporates and begins to rise in the concrete structure, the narrowing effect of the tower helps to boost the speed of parallel layers of vapor without any disruption (known as laminar flow). Since hot air is less dense than cool air, it easily rises above in the tower, particularly due to the narrowness of the tower at the middle.

-Faster and more efficient diffusion into the atmosphere

A wider top opening enhances the process of diffusion. The top of cooling tower is widened because it is the point where the hot air from inside the tower diffuses and mixes with atmospheric air. Therefore, we should maximize the area through which this diffusion takes place, so that the hotter vapor is quickly mixed and the entire process of cooling is done more efficiently.

There are also other reasons behind this shape. For example, a wide base will not only provide strength to the whole structure, but also offer ample space for the installation of machinery. From a logical standpoint, this shape is easier to build when compared to others, as it has a lattice of straight beams to erect the tower. Also, these types of structures are more resistant to external natural forces than straight buildings.

b). EXPLOSION RECORDED BY MICROPHONE

In a dynamic **microphone**, the variations in the sound pressure move the cone, which moves the attached coil of wire in a magnetic field, which therefore generates a voltage. The coil moves in response to the audio signal, moving the cone and producing sound in the air. Thus the location of explosion is determined using hyperbola with the help of the sound waves.

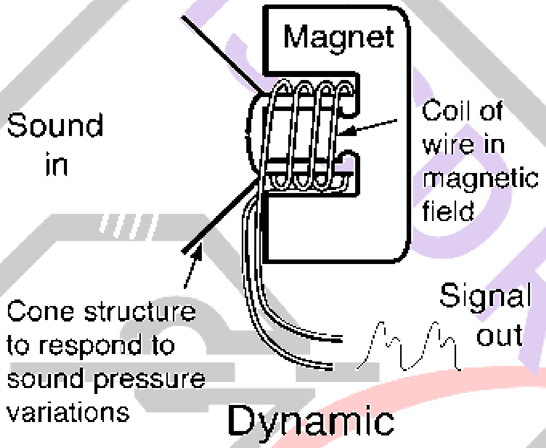

Fig2: Coil of Wire in a Microphone

III. PROBLEMS

1) An explosion is recorded by two microphones that are 2 miles apart. Microphone M_1 received the sound of the explosion 4 seconds before microphone M_2. Assuming sound travels at 100 feet per second, determine the possible locations of the microphone.
Solution

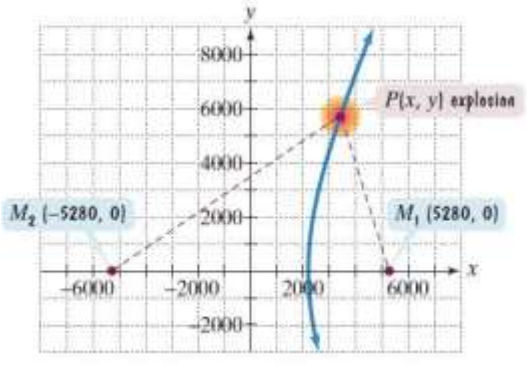

Fig3: Explosion recorded by microphones

Given

M_1 and M_2 are the two microphones.

The two microphones are 2 miles apart.

Sound travels at 100 feet per second.

To Find

The locations of the two microphones M_1 and M_2 .

We begin by putting the microphones in a co-ordinate system. Because 1 mile = 5280 feet .

We place M_1 5280 feet on a horizontal axis to the right of the origin and M_2 5280 feet on a horizontal axis to the left of the origin.

In the above diagram, the two microphones are two miles apart.

We know that,

M_2 received the sound 4 seconds after M_1.Because the sound travels at 1100 feet per second, the difference between the distance from P to M_1 and the distance from P to M_2 is,

1100 * 4 = 4400 feet

The set of all the points P satisfying these conditions fist the definition of a hyperbola, with microphones M_1 and M_2 at the foci.

The standard form of hyperbola's equation;

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

By using this standard form, we must find a^2 and b^2.

P(x, y) is the explosion point, lies on the hyperbola. The difference between the distances is 2a which is 4400 feet.

Thus, 2a = 4400

a = 2200

Therefore Substitute a = 2200 in 1;

$$\frac{x^2}{2200} - \frac{y^2}{b2} = 1$$

$$\frac{x^2}{484000} - \frac{y^2}{b2} = 1$$

Now to find $b^{2,}$

We know that, a = 2200

The distance from the center to either focus is;

Center: (0, 0) => (h, k)

Focus: ($\pm$ae, 0) => ($\pm$ 52800)

Thus, distance c = 5280

Using $C^2 = a^2 + b^2$

We have;

$(5280)^2 = (2200)^2 + b^2$

$b^2 = (5280)^2 - (2200)^2$

b^2 = 2, 30, 38,400

The equation of hyperbola at both the focus which has each a microphone is;

Thus, substitute b^2 in second equation;

$$\frac{x^2}{484000} - \frac{y^2}{23038400} = 1$$

Result

We can conclude that the explosion occurred somewhere on the right branch (i.e. the branch that is closer to M_1 of the hyperbola. The purpose of microphone is to find the location of the explosion.

2) A cooling tower stands 179.6 meters tall. The diameter of the top is 72 meters. At their narrowest part, the sides of the tower are 60 meters apart. Find the equation of the hyperbola that obtains the sides of the cooling tower. Assume that the center of the hyperbola – indicated by the intersection of dashed perpendicular lines in the diagram below is the origin of the co-ordinate plane.

Solution

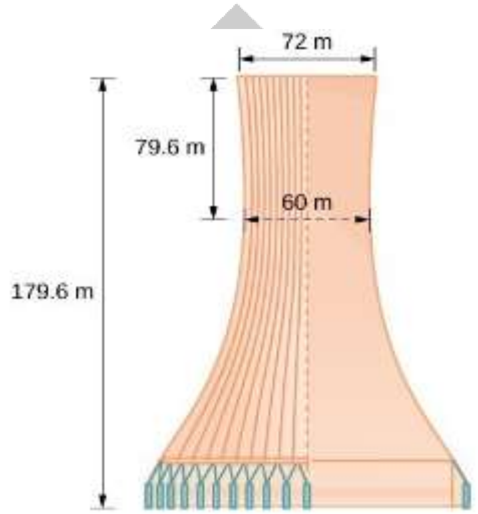

Fig4: A cooling tower

Given

The design layout of a cooling tower is shown below. The tower stands 179.6 meters tall. The diameter of the top is 72 meters. At their narrowest part, the sides of the tower are 60 meters apart.

To Find

The equation of the hyperbola that models the sides of the cooling tower

Let us assume that the center of the tower is at the origin. Hence we use the standard form of a horizontal hyperbola centered at the origin, where the branches of the hyperbola form the sides of the cooling tower.

$$\frac{x^2}{a^2} - \frac{y^2}{b2} = 1$$

To find values of a^2 and $b^{2;}$

We know that the length of the transverse axis of a hyperbola is 2a.

$$2a = 60$$

$$a = 30$$

Therefore,

$$a^2 = 900$$

To solve for b^2 ,

Substitute x and y in our equation using a known point;

Now to find some point (x, y) that lies on the hyperbola we can use the dimensions of the tower. Consider the top right corner of the tower to represent that point.

Since the y-axis bisects the tower, our x-values can be represented by the radius of the top or 36 meters. The distance from the origin to the top is represented as the y-value which is given as 79.6 meters.

Therefore, standard form of horizontal hyperbola;

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ Standard form of horizontal hyperbola

$b^2 = \frac{y^2}{\frac{x^2}{a^2} - 1}$ Isolate b^2;

Substitute the values for a^2, x and y;

$b^2 = \frac{(79.6)^2}{\frac{(36)^2}{900} - 1}$

$= 14400.3636$

Therefore the sides of the tower can be modeled by the hyperbolic equation,

$$\frac{x^2}{900} - \frac{y^2}{14400.3636} = 1 \text{ or } \frac{x^2}{30^2} - \frac{y^2}{120.0015^2} = 1$$

Result

In brief, purpose of a cooling tower is to cool down the water that gets heated up by industrial equipments and processes. Water comes in the cooling tower at a high temperature (by the industrial process) and goes out of the cooling tower at a cold temperature (back into the industrial process).

IV. CONCLUSION

From the above case study, it is observed that the concept of hyperbola from analytical geometry has many real life applications.

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