A Study on Intuitionistic Fuzzy Soft Set Operations

[1]Mrs. A. Mohanaadevi, [2]Mrs. S.Thenmozhi

[1]Assistant Professor, [2]M.Phil Scholar
Department of Mathematics,
Navarasam Arts and Science College For Women, Erode, Tamilnadu, India.

Abstract: **Soft set theory is a mathematical tool for dealing with uncertainties. Its application has boomed in recent years. In this paper we extend few results in intuitionistic fuzzy soft set operations. Also derived some properties and give an examples.**

Keywords: **Soft Sets and Intuitionistic Fuzzy Soft Sets.**

1. INTRODUCTION

Fuzzy set theory is a branch of mathematics started by Lotfi A.Zadeh as appropriate on time as 1965. In 1983 Atanassov proposed the concept of intuitionistic fuzzy set. It is a generalized form of fuzzy set. In 1983 George Gargov defined two new operation (namely necessity operation and possibility operation) in intuitionistic fuzzy sets. In fuzzy set theory, setting the value of the membership function is most difficult to the real life problems. In 1999 Molodtsov introduced the concept of soft set theory. The problem of setting the membership function doesnot arise, which make the theory easily applied to many different fields such as the Operation research, smoothness of functions, Perron integration, Game theory, Probability theory, Topological space, medical diagnosis, Decision making problems etc., In 2001 Maji et al introduced the concept fuzzy soft sets. And also introduced the idea of intuitionistic fuzzy soft sets, which is a generalized form of fuzzy soft sets and soft sets. B.Chetia et al defined four new operations (ie., "!", "?", "⊕", " ⊗" operations) of intuitionistic fuzzy soft sets in 2013. This paper we extend the properties of intuitionistic fuzzy soft set operations.

PRELIMINARIES

DEFINITION 1.1: [5]

For $P \subset E$, a **soft set** $(\boldsymbol{f_P}, \boldsymbol{E})$ or F_P on the universe X is defined by the set of ordered pairs $(f_P, E) = F_P = \{ (t, f_P(t)) : t \in E, f_P(t) \in \mathcal{P}(X)\}$ where $f_P: E \to \mathcal{P}(X)$ such that $f_P(t) = \{ \}$ if t doesnot belongs to P

Here f_P is called an approximate function of the soft set F_P . The set $f_P(t)$ is called t-approximate value set or t-approximate set which Consists of related objects of the parameter t ∈ E . Let S(X) be the set of all soft sets over X.

EXAMPLE 1.2:

Assume X = {incandescent bulb, fluorescent bulb, led bulb} be the set of bulbs. ie, X=$\{s_1, s_2, s_3\}$ and let E = $\{lifespan(t_1), cost\ per\ bulb(t_2), brightness(t_3), low\ watt(t_4)\}$ be the set of parameters or attributes. Take P = $\{t_1, t_3, t_4\} \subset E$ then we write approximate value set $f_P(t_1)$= $\{s_2, s_3\}, f_P(t_3) = \{s_1, s_2, s_3\}$ and $f_P(t_4) = \{s_3\}$. Hence$(f_P, E) = F_P = \{(t_1, \{s_2, s_3\}), (t_3, \{s_1, s_2, s_3\}), (t_4, \{s_3\})\}$ over X which describes the 'quality of bulb'.

DEFINITION 1.3:[5]

An **Intutionistic Fuzzy Soft set** $(\boldsymbol{f_P}, \boldsymbol{E})$ or $\boldsymbol{IF_P}$ on the universe X is defined by the set of ordered pairs $(f_P, E) = IF_P = \{(t, f_P(t)) : t \in E, f_P(t) \in F(X)\}$ where $f_P: E \to F(X)$ such that $f_P(t)$ = null intutionistic fuzzy set of X, if t doesnot belongs to P.

EXAMPLE 1.4:

Let $X = \{s_1, s_2, s_3\}$ be a set of three chain and let $E = \{silver(t_1),\ gold(t_2),\ platinum(t_3)\}$ be a set of parameters showing the metals of the jewellers. Consider $P = \{t_1, t_2\} \subset E$ then
$f_P(t_1) = \{s_1/(.9,.1), s_2/(.5,.4), s_3/(0,.2)\}$ and $f_P(t_2) = \{s_1/(.3,.2), s_2/(.7,.1), s_3/(.4,.4)\}$.
Hence $IF_P = \{(t_1, f_P(t_1)), (t_2, f_P(t_2))\}$ is the Intuitionistic Fuzzy Soft set over X.

DEFINITION 1.5: [5]

If IF_P, IF_Q be two Intuitionistic Fuzzy Soft sets then IF_P and IF_Q are IFS equal denoted by $IF_P = IF_Q$ iff $f_P(t)$ = $f_Q(t)$ $\forall\ t \in E$.

DEFINITION 1.6: [5]

Let IF_P be an Intuitionistic Fuzzy Soft set, then the complement of IF_P , denoted by ${IF_P}^c$ is an Intuitionistic Fuzzy Soft set defined by the approximate function $f_{P^c}(t) = f_P^c(t)$ $\forall\ t \in E$, where $f_P^c(t)$ is complement of the intuitionistic fuzzy set $f_P(t)$.

DEFINITION 1.7: [5]

Let $IF_P, IF_Q \in IF(X)$, then the operations ∪,∩, @, $, □,◊, !, ? of IF_P and IF_Q is an IFS set defined by the approximate function. $\forall\ t\ \in$ E

$(i) f_{P\tilde{\cup}Q}(t) =< \mu_{f_{P\tilde{\cup}Q}(t)}, \nu_{f_{P\tilde{\cup}Q}(t)} >= \max\{\mu_{f_P}(t), \mu_{f_Q}(t)\}, min\{\nu_{f_P}(t), \nu_{f_Q}(t)\} >$

$(ii) f_{P\tilde{\cap}Q}(t) =< \mu_{f_{P\tilde{\cap}Q}(t)}, \nu_{f_{P\tilde{\cup}Q},(t)} >$

$= < min\{\mu_{f_P}(t), \mu_{f_Q}(t)\}, \max\{\nu_{f_P}(t), \nu_{f_Q}(t)\} >$

$(iii) f_{P+Q}(t) =< \mu_{f_{P+Q}(t)}, \nu_{f_{P+Q}}(t) >$

$= < \mu_{f_P(t)} + \mu_{f_Q(t)} - \mu_{f_P(t)}.\mu_{f_Q(t)}, \nu_{f_P(t)}.\nu_{f_Q(t)} >$

$(iv) f_{P.Q}(t) =< \mu_{f_{P.Q}(t)}, \nu_{f_{P.Q}(t)} >$

$=< \mu_{f_P(t)}.\mu_{f_Q(t)}, \nu_{f_P(t)} + \nu_{f_Q(t)} - \nu_{f_P(t)}.\nu_{f_Q(t)} >$

(v)$f_{P@Q}(t) =< \mu_{f_{P@Q}(t)}, \nu_{f_{P@Q}}(t) > =< \frac{\mu_{f_P(t)}+\mu_{f_Q(t)}}{2}, \frac{\nu_{f_P(t)}+\nu_{f_Q(t)}}{2} >$

(vi) $f_{\square P}(t) =< \mu_{f_P(t)}, \nu_{f_P(t)} >=< \mu_{f_P(t)}, 1 - \mu_{f_P(t)} >$

(vii) $f_{\lozenge P}(t) =< \mu_{f_{\lozenge P}(t)}, \nu_{f_{\lozenge P}(t)} >=< 1 - \nu_{f_P(t)}, \nu_{f_P(t)} >$

(viii)$f_{!(P)}(t) =< \mu_{f_{!(P)}(t)}, \nu_{f_{!(P)}(t)} >=< \max\{\frac{1}{2}, \mu_{f_{(P)}(t)}\}, min\{\frac{1}{2}, \nu_{f_{(P)}(t)}\}>$

$(ix) f_{?(P)}(t) =< \mu_{f_{?(P)}(t)}, \nu_{f_{?(P)}(t)} >=< \min\{\frac{1}{2}, \mu_{f_{(P)}(t)}\}, max\{\frac{1}{2}, \nu_{f_{(P)}(t)}\}>$

$(x)\ f_{\oplus(P)}(t) =< \mu_{f_{\oplus(P)}(t)}, \nu_{f_{\oplus(P)}(t)} > = < \{\frac{\mu_{f_{(P)}(t)}}{2}, \frac{\nu_{f_{(P)}(t)}+1}{2}\}>$

$(xi) f_{\otimes(P)}(t) =< \mu_{f_{\otimes(P)}(t)}, \nu_{f_{\otimes(P)}(t)} > = < \{\frac{\mu_{f_{(P)}(t)}+1}{2}, \frac{\nu_{f_{(P)}(t)}}{2}\}>$

2. PROPERTIES ON INTUITIONISTIC FUZZY SOFT SET OPERATIONS

THEOREM 2.1:

If IF_P,$IF_Q \in IF(X)$, then

(i) $(!IF_P \tilde{\cup}\ !IF_Q)^{\tilde{c}} = ?IF_P^{\tilde{c}} \tilde{\cap}\ ?IF_Q^{\tilde{c}}$

(ii) $(?IF_P \tilde{\cup}\ ?IF_Q)^{\tilde{c}} = !IF_P^{\tilde{c}} \tilde{\cap}\ !IF_Q^{\tilde{c}}$

(iii) $(\oplus IF_P \tilde{\cup} \oplus IF_Q)^{\tilde{c}} = \otimes IF_P^{\tilde{c}} \tilde{\cap} \otimes IF_Q^{\tilde{c}}$

(iv) $(\otimes IF_P \tilde{\cup} \otimes IF_Q)^{\tilde{c}} = \oplus IF_P^{\tilde{c}} \tilde{\cap} \oplus IF_Q^{\tilde{c}}$

PROOF :(iii) $(\oplus \boldsymbol{IF_P} \tilde{\cup} \oplus \boldsymbol{IF_Q})^{\tilde{c}} = \otimes \boldsymbol{IF_P^{\tilde{c}}} \tilde{\cap} \otimes \boldsymbol{IF_Q^{\tilde{c}}}$

$f_{(\oplus P\ \tilde{\cup}\oplus Q)^{\tilde{c}}}(t) = <\mu_{f_{(\oplus P\ \tilde{\cup}\oplus Q)^{\tilde{c}}}(t)}, \nu_{f_{(\oplus P\ \tilde{\cup}\oplus Q)^{\tilde{c}}}(t)} = < 1- \mu_{f_{(\oplus P\ \tilde{\cup}\oplus Q)}(t)}, 1- \nu_{f_{(\oplus P\ \tilde{\cup}\oplus Q)}(t)}>$

$= < 1- \max(\mu_{f_{\oplus P}(t)}, \mu_{f_{\oplus Q}(t)}), 1- \min(\nu_{f_{\oplus P}(t)}, \nu_{f_{\oplus Q}(t)}) >$

$= <1- \max\left(\frac{\mu_{f_P(t)}}{2}, \frac{\mu_{f_Q(t)}}{2}\right), 1- \min\left(\frac{\nu_{f_P(t)}+1}{2}, \frac{\nu_{f_Q(t)}+1}{2}\right)>$

$=<\min\left(1 - \frac{\mu_{f_P(t)}}{2}, 1 - \frac{\mu_{f_Q(t)}}{2}\right), \max\left(1 - \frac{\nu_{f_P(t)}+1}{2}, 1 - \frac{\nu_{f_Q(t)}+1}{2}\right)>$

$= < \min\left(\frac{2-\mu_{f_P(t)}}{2}, \frac{2-\mu_{f_Q(t)}}{2}\right), \max\left(\frac{2-\nu_{f_P(t)}-1}{2}, \frac{2-\nu_{f_Q(t)}-1}{2}\right)>$

$= <\min\left(\frac{1-\mu_{f_P(t)}+1}{2}, \frac{1-\mu_{f_Q(t)}+1}{2}\right), \max\left(\frac{1-\nu_{f_P(t)}}{2}, \frac{1-\nu_{f_Q(t)}}{2}\right)>$

$= <\min\left(\frac{\mu_{f_{P^{\tilde{c}}}(t)}+1}{2}, \frac{\mu_{f_{Q^{\tilde{c}}}(t)}+1}{2}\right), \max\left(\frac{\nu_{f_{P^{\tilde{c}}}(t)}}{2}, \frac{\nu_{f_{Q^{\tilde{c}}}(t)}}{2}\right)>$

$= <\min(\mu_{f_{\otimes(P)^{\tilde{c}}}(t)}, \mu_{f_{\otimes(Q)^{\tilde{c}}}(t)}), \max(\nu_{f_{\otimes(P)^{\tilde{c}}}(t)}, \nu_{f_{\otimes(Q)^{\tilde{c}}}(t)})>$

$=<\mu_{f_{\otimes(P)^{\tilde{c}}\tilde{\cap}\otimes(Q)^{\tilde{c}}}(t)}, \nu_{f_{\otimes(P)^{\tilde{c}}\tilde{\cap}\otimes(Q)^{\tilde{c}}}(t)}> = f_{\otimes(P)^{\tilde{c}}\tilde{\cap}\otimes(Q)^{\tilde{c}}}(t)$

THEOREM 2.2:

If $IF_P, IF_Q \in IF(X)$, then

(i) $(!\,IF_P \,\tilde{\cap}\, !\,IF_Q)^{\tilde{c}} = ?\,IF_P^{\tilde{c}} \,\tilde{\cup}\, ?\,IF_Q^{\tilde{c}}$

(ii) $(?\,IF_P \,\tilde{\cap}\, ?\,IF_Q)^{\tilde{c}} = !\,IF_P^{\tilde{c}} \,\tilde{\cup}\, !\,IF_Q^{\tilde{c}}$

(iii) $(\oplus\, IF_P \,\tilde{\cap}\oplus\, IF_Q)^{\tilde{c}} = \otimes\, IF_P^{\tilde{c}} \,\tilde{\cup}\, \otimes\, IF_Q^{\tilde{c}}$

(iv) $(\otimes\, IF_P \,\tilde{\cap}\otimes\, IF_Q)^{\tilde{c}} = \oplus\, IF_P^{\tilde{c}} \,\tilde{\cup}\, \oplus\, IF_Q^{\tilde{c}}$

(v) $(!\,IF_P @ !\,IF_Q)^{\tilde{c}} = ?\,IF_P^{\tilde{c}} @ ?\,IF_Q^{\tilde{c}}$

(vi) $(?\,IF_P @ ?\,IF_Q)^{\tilde{c}} = !\,IF_P^{\tilde{c}} @ !\,IF_Q^{\tilde{c}}$

(vii) $(\oplus IF_P @ \oplus IF_Q)^{\tilde{c}} = \otimes IF_P^{\tilde{c}} @ \otimes IF_Q^{\tilde{c}}$

(viii) $(\otimes IF_P @ \otimes IF_Q)^{\tilde{c}} = \oplus IF_P^{\tilde{c}} @ \oplus IF_Q^{\tilde{c}}$

PROOF: Proof of the Theorem 2.2 is same as theorem 2.1

THEOREM 2.3:

If $IF_P, IF_Q, IF_R \in IF(X)$, then

(i) $(\square IF_P \,\tilde{\cup}\, \square IF_Q)^{\tilde{c}} \neq \Diamond\, IF_P^{\tilde{c}} \,\tilde{\cap}\, \Diamond\, IF_Q^{\tilde{c}}$

(ii) $(\Diamond\, IF_P \,\tilde{\cup} \Diamond\, IF_Q)^{\tilde{c}} \neq \square IF_P^{\tilde{c}} \,\tilde{\cap}\, \square IF_Q^{\tilde{c}}$

(iii) $(\square IF_P \,\tilde{\cap}\, \square IF_Q)^{\tilde{c}} \neq \Diamond\, IF_P^{\tilde{c}} \,\tilde{\cup}\, \Diamond\, IF_Q^{\tilde{c}}$

(iv) $(\Diamond\, IF_P \,\tilde{\cap} \Diamond\, IF_Q)^{\tilde{c}} \neq \square IF_P^{\tilde{c}} \,\tilde{\cup}\, \square IF_Q^{\tilde{c}}$

PROOF: Proof of the Theorem 2.3 is same as theorem 2.1

THEOREM 2.4:

If $IF_P, IF_Q, IF_R \in IF(X)$, then

(i) $((\,?\,IF_P)^{\tilde{c}})^{\tilde{c}} = ?\,IF_P$

(ii) $((!\,IF_P)^{\tilde{c}})^{\tilde{c}} = !\,IF_P$

(iii) $((\square\, IF_P)^{\tilde{c}})^{\tilde{c}} = \square IF_P$

(iv) $((\,\Diamond\, IF_P)^{\tilde{c}})^{\tilde{c}} = \Diamond\, IF_P$

(v) $((\,\otimes\, IF_P)^{\tilde{c}})^{\tilde{c}} = \otimes\, IF_P$

(vi) $((\oplus\, IF_P)^{\tilde{c}})^{\tilde{c}} = \oplus IF_P$

PROOF :(i) $((\,?\,\boldsymbol{IF_P})^{\tilde{c}})^{\tilde{c}} = ?\,\boldsymbol{IF_P}$

$f_{((?P)^{\tilde{c}})^{\tilde{c}}}(t) = <\mu_{f_{((?P)^{\tilde{c}})^{\tilde{c}}}(t)}, \nu_{f_{((?P)^{\tilde{c}})^{\tilde{c}}}(t)}> = < 1-(\mu_{f_{(?P)^{\tilde{c}}}(t)}), 1-(\nu_{f_{(?P)^{\tilde{c}}}(t)})>$

$= < 1-(1-(\min(\frac{1}{2}, \mu_{f_P(t)}))), 1-(1-(\max(\frac{1}{2}, \nu_{f_P(t)})))>$

$=< 1-(\max(1-\frac{1}{2}, 1-\mu_{f_P(t)})), 1-(\min(1-\frac{1}{2}, 1-\nu_{f_P(t)})) >$

$= < 1-(\max(\frac{1}{2}, 1-\mu_{f_P(t)})), 1-(\min(\frac{1}{2}, 1-\nu_{f_P(t)})) >$

$= < \min(1-\frac{1}{2}, 1-(1-\mu_{f_P(t)})), \max(1-\frac{1}{2}, 1-(1-\nu_{f_P(t)})) >$

$= < \min(\frac{1}{2}, \mu_{f_P(t)}), \max(\frac{1}{2}, \nu_{f_P(t)}) > \; = f_{?P}(t)$

EXAMPLE 2.5:

Let X = $\{s_1, s_2, s_3\}$ be a universal set and E = $\{t_1, t_2, t_3\}$ be a set of parameters. Let P = $\{t_1, t_3\}$, Q = $\{t_2\}$ and R = $\{t_1, t_2\}$. Suppose,

$IF_P = \{(t_1, \{s_1/(.9,.1), s_2/(.5,.4), s_3/(0,.2)\}),$
$(t_3, \{s_1/(.3,.2), s_2/(.7,.1), s_3/(.4,.4)\})\}.$
$IF_Q = \{(t_1, \{s_1/(.9,.1), s_2/(.5,.4), s_3/(0,.2)\})\}$
$IF_R = \{(t_1, \{s_1/(.9,.1), s_2/(.5,.4), s_3/(0,.2)\}),$
$(t_2, \{s_1/(.3,.2), s_2/(.7,.1), s_3/(.4,.4)\})\}.$

THEOREM 2.6:

If $IF_P, IF_Q, IF_R \in IF(X)$, then
(i) $\Box IF_P \tilde{\cup} (\Box IF_Q \tilde{\cap} \Box IF_R) = (\Box IF_P \tilde{\cup} \Box IF_Q) \tilde{\cap} (\Box IF_P \tilde{\cup} \Box IF_R)$
(ii) $\Diamond IF_P \tilde{\cup} (\Diamond IF_Q \tilde{\cap} \Diamond IF_R) = (\Diamond IF_P \tilde{\cup} \Diamond IF_Q) \tilde{\cap} (\Diamond IF_P \tilde{\cup} \Diamond IF_R)$.
(iii) $!\, IF_P \tilde{\cup} (!\, IF_Q \tilde{\cap} !\, IF_R) = (!\, IF_P \tilde{\cup} !\, IF_Q) \tilde{\cap} (!\, IF_P \tilde{\cup} !\, IF_R)$.
(iv) $?IF_P \tilde{\cup} (?IF_Q \tilde{\cap} ?IF_R) = (?IF_P \tilde{\cup} ?IF_Q) \tilde{\cap} (?IF_P \tilde{\cup} ?IF_R)$
(v) $\oplus IF_P \tilde{\cup} (\oplus IF_Q \tilde{\cap} \oplus IF_R) = (\oplus IF_P \tilde{\cup} \oplus IF_Q) \tilde{\cap} (\oplus IF_P \tilde{\cup} \oplus IF_R)$
(vi) $\otimes IF_P \tilde{\cup} (\otimes IF_Q \tilde{\cap} \otimes IF_R) = (\otimes IF_P \tilde{\cup} \otimes IF_Q) \tilde{\cap} (\otimes IF_P \tilde{\cup} \otimes IF_R)$

PROOF : (i) $\Box IF_P \tilde{\cup} (\Box IF_Q \tilde{\cap} \Box IF_R) = (\Box IF_P \tilde{\cup} \Box IF_Q) \tilde{\cap} (\Box IF_P \tilde{\cup} \Box IF_R)$

$f_{\Box P \tilde{\cup} (\Box Q \tilde{\cap} \Box R)}(t) = < \mu_{f_{\Box P \tilde{\cup} (\Box Q \tilde{\cap} \Box R)}(t)}, \nu_{f_{\Box P \tilde{\cup} (\Box Q \tilde{\cap} \Box R)}(t)} >$

$= < \max(\mu_{f_{\Box P}(t)}, \mu_{f_{(\Box Q \tilde{\cap} \Box R)}(t)}), \min(\nu_{f_{\Box P}(t)}, \nu_{f_{(\Box Q \tilde{\cap} \Box R)}(t)}) >$

$= < \max(\mu_{f_{\Box P}(t)}, \min(\mu_{f_{\Box Q}(t)}, \mu_{f_{\Box R}(t)})), \min(\nu_{f_{\Box P}(t)}, \max(\nu_{f_{\Box Q}(t)}, \nu_{f_{\Box R}(t)})) >$

$= < \max(\mu_{f_P(t)}, \min(\mu_{f_Q(t)}, \mu_{f_R(t)})), \min(1-\mu_{f_P(t)}, \max(1-\mu_{f_Q(t)}, 1-\mu_{f_R(t)})) >$

$= < \min(\max(\mu_{f_P(t)}, \mu_{f_Q(t)}), \max(\mu_{f_P(t)}, \mu_{f_R(t)})), \max(\min(1-\mu_{f_P(t)}, 1-\mu_{f_Q(t)}), \min(1-\mu_{f_P(t)}, 1-\mu_{f_R(t)})) >$

$= < \min(\mu_{f_{(\Box P \tilde{\cup} \Box Q)}(t)}, \mu_{f_{(\Box P \tilde{\cup} \Box R)}(t)}), \max(\nu_{f_{(\Box P \tilde{\cup} \Box Q)}(t)}, \nu_{f_{(\Box P \tilde{\cup} \Box R)}(t)}) >$

$= < \mu_{f_{(\Box P \tilde{\cup} \Box Q) \tilde{\cap} (\Box P \tilde{\cup} \Box R)}(t)}, \nu_{f_{(\Box P \tilde{\cup} \Box Q) \tilde{\cap} (\Box P \tilde{\cup} \Box R)}(t)} > = f_{(\Box P \tilde{\cup} \Box Q) \tilde{\cap} (\Box P \tilde{\cup} \Box R)}(t)$

EXAMPLE 2.7:
Using example 2.5 we get

$\Box IF_P \tilde{\cup} (\Box IF_Q \tilde{\cap} \Box IF_R) = \{(t_1, \{s_1/(.5,.5), s_2/(.2,.8), s_3/(.3,.7)\}),$
$(t_2, \{s_1/(0,1), s_2/(.1,.9), s_3/(.6,.4)\}),$
$(t_3, \{s_1/(.8,.2), s_2/(.8,.2), s_3/(.4,.6)\})\}$------(I)

$(\Box IF_P \tilde{\cup} \Box IF_Q) \tilde{\cap} (\Box IF_P \tilde{\cup} \Box IF_R) = \{(t_1, \{s_1/(.5,.5), s_2/(.2,.8), s_3/(.3,.7)\}),$
$(t_2, \{s_1/(0,1), s_2/(.1,.9), s_3/(.6,.4)\}),$
$(t_3, \{s_1/(.8,.2), s_2/(.8,.2), s_3/(.4,.6)\})\}$-----(II)

From (I) and (II) we get

$\Box IF_P \tilde{\cup} (\Box IF_Q \tilde{\cap} \Box IF_R) = (\Box IF_P \tilde{\cup} \Box IF_Q) \tilde{\cap} (\Box IF_P \tilde{\cup} \Box IF_R)$

THEOREM 2.8:

If $IF_P, IF_Q, IF_R \in IF(X)$, then
(i) $(\Box IF_P @ \Box IF_Q) @ \Box IF_R = (\Box IF_P @ \Box IF_R) @ (\Box IF_Q @ \Box IF_R)$
(ii) $(\Diamond IF_P @ \Diamond IF_Q) @ \Diamond IF_R = (\Diamond IF_P @ \Diamond IF_R) @ (\Diamond IF_Q @ \Diamond IF_R)$
(iii) $(!\, IF_P @ !\, IF_Q) @ !\, IF_R = (!\, IF_P @ !\, IF_R) @ (!IF_Q @ !\, IF_R)$
(iv) $(?IF_P @ ?IF_Q) @ ?IF_R = (?IF_P @ ?IF_R) @ (?IF_Q @ ?IF_R)$
(v) $(\oplus IF_P @ \oplus IF_Q) @ \oplus IF_R = (\oplus IF_P @ \oplus IF_R) @ (\oplus IF_Q @ \oplus IF_R)$

(vi) $(\otimes IF_P @ \otimes IF_Q) @ \otimes IF_R = (\otimes IF_P @ \otimes IF_R) @ (\otimes IF_Q @ \otimes IF_R)$

PROOF: Proof of the Theorem 2.8 is same as theorem 2.6

THEOREM 2.9:

If $IF_P, IF_Q, IF_R \in IF(X)$, then
(i) $(\Box IF_P + \Box IF_Q) + \Box IF_R = \Box IF_P + (\Box IF_Q + \Box IF_R)$
(ii) $(\lozenge IF_P + \lozenge IF_Q) + \lozenge IF_R = \lozenge IF_P + (\lozenge IF_Q + \lozenge IF_R)$
(iii) $(!IF_P + !IF_Q) + !IF_R = !IF_P + (!IF_Q + !IF_R)$
(iv) $(?IF_P + ?IF_Q) + ?IF_R = ?IF_P + (?IF_Q + ?IF_R)$
(v) $(\oplus IF_P + \oplus IF_Q) + \oplus IF_R = \oplus IF_P + (\oplus IF_Q + \oplus IF_R)$
(vi) $(\otimes IF_P + \otimes IF_Q) + \otimes IF_R = \otimes IF_P + (\otimes IF_Q + \otimes IF_R)$

PROOF: (ii) $(\lozenge \boldsymbol{IF_P} + \lozenge \boldsymbol{IF_Q}) + \lozenge \boldsymbol{IF_R} = \lozenge \boldsymbol{IF_P} + (\lozenge \boldsymbol{IF_Q} + \lozenge \boldsymbol{IF_R})$

$f_{(\lozenge P+\lozenge Q)+\lozenge R}(\mathrm{t}) = < \mu_{f_{(\lozenge P+\lozenge Q)+\lozenge R}(\mathrm{t})}, \nu_{f_{(\lozenge P+\lozenge Q)+\lozenge R}(\mathrm{t})} >$

$= < \mu_{f_{(\lozenge P+\lozenge Q)}(\mathrm{t})} + \mu_{f_{\lozenge R}(\mathrm{t})} - \mu_{f_{(\lozenge P+\lozenge Q)}(\mathrm{t})} \cdot \mu_{f_{\lozenge R}(\mathrm{t})}, \nu_{f_{(\lozenge P+\lozenge Q)}(\mathrm{t})} . \nu_{f_{\lozenge R}(\mathrm{t})} >$

$= < \mu_{f_{\lozenge P}(\mathrm{t})} + \mu_{f_{\lozenge Q}(\mathrm{t})} - \mu_{f_{\lozenge P}(\mathrm{t})} . \mu_{f_{\lozenge Q}(\mathrm{t})} + \mu_{f_{\lozenge R}(\mathrm{t})} - (\mu_{f_{\lozenge P}(\mathrm{t})} + \mu_{f_{\lozenge Q}(\mathrm{t})} - \mu_{f_{\lozenge P}(\mathrm{t})} . \mu_{f_{\lozenge Q}(\mathrm{t})}) \cdot \mu_{f_{\lozenge R}(\mathrm{t})}, (\nu_{f_{\lozenge P}(\mathrm{t})} . \nu_{f_{\lozenge Q}(\mathrm{t})}) \cdot \nu_{f_{\lozenge R}(\mathrm{t})} >$

$= < 1-\nu_{f_P(\mathrm{t})} + 1-\nu_{f_Q(\mathrm{t})} - (1-\nu_{f_P(\mathrm{t})})(1-\nu_{f_Q(\mathrm{t})}) + 1-\nu_{f_R(\mathrm{t})} - (1-\nu_{f_P(\mathrm{t})}) \cdot (1-\nu_{f_R(\mathrm{t})})$

$-(1-\nu_{f_Q(\mathrm{t})}) \cdot (1-\nu_{f_R(\mathrm{t})}) + (1-\nu_{f_P(\mathrm{t})}) \cdot (1-\nu_{f_Q(\mathrm{t})}) \cdot (1-\nu_{f_R(\mathrm{t})}), \nu_{f_P(\mathrm{t})} \cdot \nu_{f_Q(\mathrm{t})} \cdot \nu_{f_R(\mathrm{t})} >$

$= < 1-\nu_{f_P(\mathrm{t})} + 1-\nu_{f_Q(\mathrm{t})} + 1-\nu_{f_R(\mathrm{t})} - (1-\nu_{f_Q(\mathrm{t})}) \cdot (1-\nu_{f_R(\mathrm{t})}) - (1-\nu_{f_P(\mathrm{t})})(1-\nu_{f_Q(\mathrm{t})}$

$+1-\nu_{f_R(\mathrm{t})} - (1-\nu_{f_Q(\mathrm{t})}) \cdot (1-\nu_{f_R(\mathrm{t})})), \nu_{f_P(\mathrm{t})} \cdot \nu_{f_Q(\mathrm{t})} \cdot \nu_{f_R(\mathrm{t})} >$

$= < \mu_{f_{\lozenge P}(\mathrm{t})} + \mu_{f_{(\lozenge Q+\lozenge R)}(\mathrm{t})} - \mu_{f_{\lozenge P}(\mathrm{t})} \cdot \mu_{f_{(\lozenge Q+\lozenge R)}(\mathrm{t})}, \nu_{f_{\lozenge P}(\mathrm{t})} \cdot \nu_{f_{\lozenge Q}(\mathrm{t})} \cdot \nu_{f_{\lozenge R}(\mathrm{t})} >$

$= < \mu_{f_{\lozenge P+(\lozenge Q+\lozenge R)}(\mathrm{t})}, \nu_{f_{\lozenge P+(\lozenge Q+\lozenge R)}(\mathrm{t})} > = f_{\lozenge P+(\lozenge Q+\lozenge R)}(\mathrm{t})$

EXAMPLE 2.10:

Using example 2.5 we get
$(\lozenge IF_P + \lozenge IF_Q) + \lozenge IF_R = \{(t_1, \{s_1/(1,0), s_2/(1,0), s_3/(1,0)\}),$
$(t_2, \{s_1/(1,0), s_2/(1,0), s_3/(1,0)\}),$
$(t_3, \{s_1(1,0), s_2/(1,0), s_3/(1,0)\})\}$------------(I)
$\lozenge IF_P + (\lozenge IF_Q + \lozenge IF_R) = \{(t_1, \{s_1/(1,0), s_2/(1,0), s_3/(1,0)\}),$
$(t_2, \{s_1/(1,0), s_2/(1,0), s_3/(1,0)\}),$
$(t_3, \{s_1(1,0), s_2/(1,0), s_3/(1,0)\})\}$-----------(II)
From (I) and (II) we get
$(\lozenge IF_P + \lozenge IF_Q) + \lozenge IF_R = \lozenge IF_P + (\lozenge IF_Q + \lozenge IF_R)$

THEOREM 2.11:

If $IF_P, IF_Q, IF_R \in IF(X)$, then
(i) $(\Box IF_P \cdot \Box IF_Q) \cdot \Box IF_R = \Box IF_P \cdot (\Box IF_Q \cdot \Box IF_R)$
(ii) $(\lozenge IF_P \cdot \lozenge IF_Q) \cdot \lozenge IF_R = \lozenge IF_P \cdot (\lozenge IF_Q \cdot \lozenge IF_R)$
(iii) $(!IF_P \cdot !IF_Q) \cdot !IF_R = !IF_P \cdot (!IF_Q \cdot !IF_R)$
(iv) $(?IF_P \cdot ?IF_Q) \cdot ?IF_R = ?IF_P \cdot (?IF_Q \cdot ?IF_R)$
(v) $(\oplus IF_P \cdot \oplus IF_Q) \cdot \oplus IF_R = \oplus IF_P \cdot (\oplus IF_Q \cdot \oplus IF_R)$
(vi) $(\otimes IF_P \cdot \otimes IF_Q) \cdot \otimes IF_R = \otimes IF_P \cdot (\otimes IF_Q \cdot \otimes IF_R)$
PROOF: Proof of the Theorem 2.11 is same as theorem 2.9

THEOREM 2.12:
If $IF_P, IF_Q, IF_R \in IF(X)$, then
(i) $(\Box IF_P \tilde{\cup} \Box IF_Q) + \Box IF_R = (\Box IF_P + \Box IF_R) \tilde{\cup} (\Box IF_Q + \Box IF_R)$

(ii) $(\lozenge IF_P \tilde{\cup} \lozenge IF_Q) + \lozenge IF_R = (\lozenge IF_P + \lozenge IF_R) \tilde{\cup} (\lozenge IF_Q + \lozenge IF_R)$
(iii) $(! IF_P \tilde{\cup} ! IF_Q) + ! IF_R = (! IF_P + ! IF_R) \tilde{\cup} (! IF_Q + ! IF_R)$
(iv) $(? IF_P \tilde{\cup} ? IF_Q) + ? IF_R = (? IF_P + ? IF_R) \tilde{\cup} (? IF_Q + ? IF_R)$
(v) $(\oplus IF_P \tilde{\cup} \oplus IF_Q) + \oplus IF_R = (\oplus IF_P + \oplus IF_R) \tilde{\cup} (\oplus IF_Q + \oplus IF_R)$
(vi) $(\otimes IF_P \tilde{\cup} \otimes IF_Q) + \otimes IF_R = (\otimes IF_P + \otimes IF_R) \tilde{\cup} (\otimes IF_Q + \otimes IF_R)$
(vii) $(\Box IF_P \tilde{\cup} \Box IF_Q) . \Box IF_R = (\Box IF_P . \Box IF_R) \tilde{\cup} (\Box IF_Q . \Box IF_R)$
(viii) $(\lozenge IF_P \tilde{\cup} \lozenge IF_Q) . \lozenge IF_R = (\lozenge IF_P . \lozenge IF_R) \tilde{\cup} (\lozenge IF_Q . \lozenge IF_R)$
(ix) $(! IF_P \tilde{\cup} ! IF_Q) .! IF_R = (! IF_P .! IF_R) \tilde{\cup} (! IF_Q . ! IF_R)$
(x) $(? IF_P \tilde{\cup} ? IF_Q) .? IF_R = (? IF_P . ? IF_R) \tilde{\cup} (? IF_Q .? IF_R)$
(xi) $(\oplus IF_P \tilde{\cup} \oplus IF_Q) . \oplus IF_R = (\oplus IF_P . \oplus IF_R) \tilde{\cup} (\oplus IF_Q . \oplus IF_R)$
(xii) $(\otimes IF_P \tilde{\cup} \otimes IF_Q) . \otimes IF_R = (\otimes IF_P . \otimes IF_R) \tilde{\cup} (\otimes IF_Q . \otimes IF_R)$

PROOF: Proof of the Theorem 2.12 is same as theorem 2.6

THEOREM 2.13:

If $IF_P, IF_Q, IF_R \in IF(X)$, then
(i) $(\Box IF_P \tilde{\cap} \Box IF_Q) + \Box IF_R = (\Box IF_P + \Box IF_R) \tilde{\cap} (\Box IF_Q + \Box IF_R)$
(ii) $(\lozenge IF_P \tilde{\cap} \lozenge IF_Q) + \lozenge IF_R = (\lozenge IF_P + \lozenge IF_R) \tilde{\cap} (\lozenge IF_Q + \lozenge IF_R)$
(iii) $(! IF_P \tilde{\cap} ! IF_Q) + ! IF_R = (! IF_P + ! IF_R) \tilde{\cap} (! IF_Q + ! IF_R)$
(iv) $(? IF_P \tilde{\cap} ? IF_Q) + ? IF_R = (? IF_P + ? IF_R) \tilde{\cap} (? IF_Q + ? IF_R)$
(v) $(\oplus IF_P \tilde{\cap} \oplus IF_Q) + \oplus IF_R = (\oplus IF_P + \oplus IF_R) \tilde{\cap} (\oplus IF_Q + \oplus IF_R)$
(vi) $(\otimes IF_P \tilde{\cap} \otimes IF_Q) + \otimes IF_R = (\otimes IF_P + \otimes IF_R) \tilde{\cap} (\otimes IF_Q + \otimes IF_R)$
(vii) $(\Box IF_P \tilde{\cap} \Box IF_Q) . \Box IF_R = (\Box IF_P . \Box IF_R) \tilde{\cap} (\Box IF_Q . \Box IF_R)$
(viii) $(\lozenge IF_P \tilde{\cap} \lozenge IF_Q) . \lozenge IF_R = (\lozenge IF_P . \lozenge IF_R) \tilde{\cap} (\lozenge IF_Q . \lozenge IF_R)$
(ix) $(! IF_P \tilde{\cap} ! IF_Q) .! IF_R = (! IF_P .! IF_R) \tilde{\cap} (! IF_Q . ! IF_R)$
(x) $(? IF_P \tilde{\cap} ? IF_Q) .? IF_R = (? IF_P . ? IF_R) \tilde{\cap} (? IF_Q .? IF_R)$
(xi) $(\oplus IF_P \tilde{\cap} \oplus IF_Q) . \oplus IF_R = (\oplus IF_P . \oplus IF_R) \tilde{\cap} (\oplus IF_Q . \oplus IF_R)$
(xii) $(\otimes IF_P \tilde{\cap} \otimes IF_Q) . \otimes IF_R = (\otimes IF_P . \otimes IF_R) \tilde{\cap} (\otimes IF_Q . \otimes IF_R)$

PROOF: Proof of the Theorem 2.13 is same as theorem 2.6

THEOREM 2.14:

If $IF_P, IF_Q, IF_R \in IF(X)$, then

(i) $(\Box IF_P @ \Box IF_Q) + \Box IF_R = (\Box IF_P + \Box IF_R) @ (\Box IF_Q + \Box IF_R)$
(ii) $(\lozenge IF_P @ \lozenge IF_Q) + \lozenge IF_R = (\lozenge IF_P + \lozenge IF_R) @ (\lozenge IF_Q + \lozenge IF_R)$
(iii) $(! IF_P @ ! IF_Q) + ! IF_R = (! IF_P + ! IF_R) @ (! IF_Q + ! IF_R)$
(iv) $(? IF_P @ ? IF_Q) + ? IF_R = (? IF_P + ? IF_R) @ (? IF_Q + ? IF_R)$
(v) $(\oplus IF_P @ \oplus IF_Q) + \oplus IF_R = (\oplus IF_P + \oplus IF_R) @ (\oplus IF_Q + \oplus IF_R)$
(vi) $(\otimes IF_P @ \otimes IF_Q) + \otimes IF_R = (\otimes IF_P + \otimes IF_R) @ (\otimes IF_Q + \otimes IF_R)$
(vii) $(\Box IF_P @ \Box IF_Q) . \Box IF_R = (\Box IF_P . \Box IF_R) @ (\Box IF_Q . \Box IF_R)$
(viii) $(\lozenge IF_P @ \lozenge IF_Q) . \lozenge IF_R = (\lozenge IF_P . \lozenge IF_R) @ (\lozenge IF_Q . \lozenge IF_R)$
(ix) $(! IF_P @ ! IF_Q) . ! IF_R = (! IF_P . ! IF_R) @ (! IF_Q . ! IF_R)$
(x) $(? IF_P @ ? IF_Q) . ? IF_R = (? IF_P . ? IF_R) @ (? IF_Q . ? IF_R)$
(xi) $(\oplus IF_P @ \oplus IF_Q) . \oplus IF_R = (\oplus IF_P . \oplus IF_R) @ (\oplus IF_Q . \oplus IF_R)$
(xii) $(\otimes IF_P @ \otimes IF_Q) . \otimes IF_R = (\otimes IF_P . \otimes IF_R) @ (\otimes IF_Q . \otimes IF_R)$

PROOF: Proof of the Theorem 2.14 is same as theorem 2.6

CONCLUSION: We collected basic idea of soft sets and intuitionistic fuzzy soft sets. This paper deals with some properties of intuitionistic fuzzy soft set operations and few properties are derived.

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