A case study on the effect of Thyroid Stimulating Hormone using fuzzy gamma distribution

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***Abstract*: Mathematical Modelling is a principled activity that has both principles and methods. The Gamma Process model is belongs to the class of Cumulative Damage model. Here we study the effect of TSH suffered women level and we calculate Fuzzy Mean and Fuzzy Variance using Gamma distribution.**

***Keywords*: Fuzzy mean, Fuzzy variance, Thyroid Stimulating Hormone.**

AMS Subject Classification: 94D05, 60A86, 62A86, 62E86

1. Introduction

Mathematical Model has a diversity of applications and thus a range of possible approaches. The use and versatility of such mathematical models has been heightened by the power of computers [1]. Such models result in an efficient and economical way of understanding, analyzing and designing processes. The gamma process method belongs to a general category of random processes, denoted to as the Markov process. In this process, increments are independent and non-negative random variables having a gamma distribution with an identical scale parameter and a time dependent shape function. The generalization of gamma distributions provides more flexible in fitting real world data. The probability density function of a gamma distributions changes quite a bit when one puts in different values of the parameters [2].

In first, fuzzy set theory was developed by Zadeh(1965) [5] and Goguen(1967,1969). One of the area, in which fuzzy sets have been applied most extensively in modeling. Thousands of articles have been published on fuzzy sets in various journals [6]. Successful real applications of fuzzy pure mathematics have conjointly exaggerated in range and in quality.

The thyroid hormones are essential for normal growth development and stimulate metabolism in most tissues. The most important regulator of thyroid homeostasis is Thyroid Stimulating Hormone (thyrotrophic; TSH). TSH is secreted from cells referred to us thyrotrophs in the anterior pituitary gland. The TSH stimulates a number of processes in the thyroid to lead thyroid hormone secretion [3].

The thyroid gland consists of two lobes (left and right) connected by a thin, median isthmus forming a "butterfly" shape. It is located in the neck, in front of the trachea, just below the larynx, weighs 15-20g in the adult. This is extremely vascularized organ with blood flow concerning 5mL/g/min of tissue. Although the thyroid represents about 0.4% of body weight it accounts for 2% of total blood flow. The gland receives fibers from both sympathetic and parasympathetic divisions of the autonomic nervous system. The sympathetic fibers are derived from the cervical ganglia and enter the gland along the blood vessels. The parasympathetic fibers are derived from the vagus and reach the gland by branches of the laryngeal nerves. The gland consists of thousands of follicles, each a spheroidal sac of epithelial cells (thyrocytes) surrounding a lumen containing colloid, a depot of thyroid hormone precursor, thyroglobulin. The average diameter of follicle is 300 microns. The epithelium of the normal gland is usually described as cuboidal, with the apical plasma membrane (facing the follicle lumen) and the basolateral plasma membrane on opposite site. The most conspicuous structural distinction between the apical and basolateral cell surface is that only former is equipped microvili (pseudopods). The thyroid gland secretes three hormones: thyroxine (T4) and triiodothyronine (T3), both of which are iodinated derivatives of tyrosine, and calcitonin, a polypeptide hormone.

2. Symbolization

λ	–	Scale parameter
ϕ	–	Shape parameter
$\bar{\lambda}[\alpha]$	–	Alpha cut of scale value
$\bar{\phi}[\alpha]$	–	Alpha cut of shape value
$E(X)$	–	Mean of the Gamma distribution
$VAR(X)$	–	Variance of the Gamma distribution
$E\left(\overline{X}\right)$	–	Fuzzy Mean of the Gamma distribution

$VAR(\overline{X})$ – Fuzzy Variance of the Gamma distribution

t – Test termination time

3. Fuzzy Gamma Distribution

The probability density function of the Gamma Distribution is given by

$$f(x,\lambda,\phi)=\frac{1}{\phi^{\lambda}\Gamma(\lambda)}x^{\lambda-1}e^{-\frac{x}{\phi}},\lambda,\phi>0;x>0$$

where λ and ϕ are the scale and shape parameter of the gamma probability distribution respectively. Mean and Variance of the Gamma distribution are

$$E(X)=\frac{\lambda}{\phi}$$

$$VAR(X)=\frac{\lambda}{\phi^2}$$

Fuzzy Mean of the Gamma distribution is

$$E(\overline{X})=\frac{\overline{\lambda}}{\overline{\phi}}t$$

So that for $\alpha\in[0,1]$ the α cuts of Fuzzy Mean of the Gamma distribution is

$$\overline{R}[\alpha]=\{\overline{M_1}[\alpha],\overline{M_2}[\alpha]\}$$

where

$$\overline{M_1}[\alpha]=\min\left[\frac{\overline{\lambda}}{\overline{\phi}}t,\overline{\lambda}\in\overline{\lambda}[\alpha],\overline{\phi}\in\overline{\phi}[\alpha]\right]$$

$$\overline{M_2}[\alpha]=\max\left[\frac{\overline{\lambda}}{\overline{\phi}}t,\overline{\lambda}\in\overline{\lambda}[\alpha],\overline{\phi}\in\overline{\phi}[\alpha]\right]$$

where $\overline{M_1}[\alpha]$ and $\overline{M_2}[\alpha]$ is denoted by Lower Fuzzy Mean and Upper Fuzzy Mean respectively.

Fuzzy Variance of the Gamma distribution is

$$VAR(\overline{X})=\frac{\overline{\lambda}}{\overline{\phi}^2}t$$

So that for $\alpha\in[0,1]$ the α cuts of Fuzzy Variance of the Gamma distribution is

$$\overline{S}[\alpha]=\{\overline{M_3}[\alpha],\overline{M_4}[\alpha]\}$$

where

$$\overline{M_3}[\alpha]=\min\left[\frac{\overline{\lambda}}{\overline{\phi}^2}t,\overline{\lambda}\in\overline{\lambda}[\alpha],\overline{\phi}\in\overline{\phi}[\alpha]\right]$$

$$\overline{M_4}[\alpha]=\max\left[\frac{\overline{\lambda}}{\overline{\phi}^2}t,\overline{\lambda}\in\overline{\lambda}[\alpha],\overline{\phi}\in\overline{\phi}[\alpha]\right]$$

Where $\overline{M_3}[\alpha]$ and $\overline{M_4}[\alpha]$ is denoted by lower fuzzy variance and upper fuzzy variance respectively.

4. Case study with an example

The Gamma distribution is widely used as a conjugate prior in statistics. It is conjugate prior for the precision of a normal distribution. In wireless communication, the gamma distribution is used to model the multi-path fading of signal power. In the table: 4.1, the patient who suffered from the Thyroid Stimulating Hormone in the period of various month and year. Isolated High TSH especially in the range of 5.6 to 15 mIU/ml is commonly associated with physiological and Biological TSH variability. Isolated Low TSH especially in the range of 0.1 to 0.29 mIU/ml often seen in elderly and associated with Non-Thyroidal illness. The

treatment of the patient may result in lower TSH levels while thyroid hormone levels are normal. The TSH levels are higher in the patient by 15 days after the injection. The patient was started on 100 micrograms of Thyroxine/day and now, one year later, she is feeling well and she is clinically and biochemically euthroid.

Table: 4.1. TSH level for suffered patient during various month

Time (months)	1	2	3	4	5
TSH level (mIU/ml)	0.01	14.05	50.43	3.45	18.40

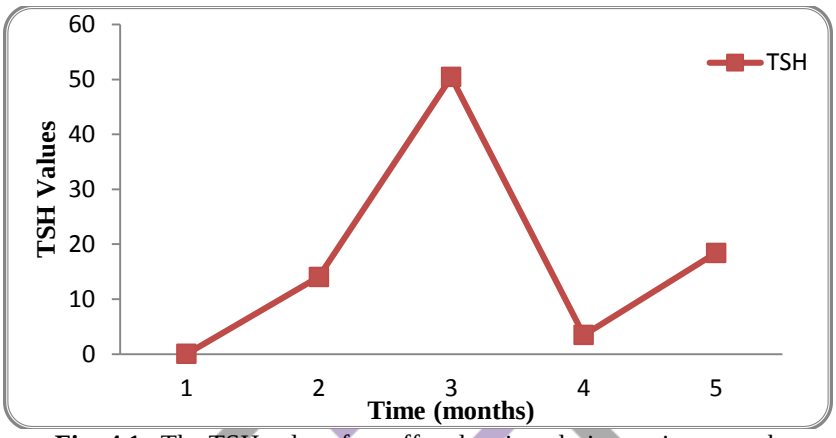

Fig: 4.1. The TSH values for suffered patient during various month

In many situations the value of the scale parameters and shape parameters of the two parameter Gamma distribution are not known precisely. We consider the triangular numbers for the scale parameter and shape parameter. The triangular fuzzy number of the scale and the shape parameters respectively are

$\bar{\lambda}$ = [0.17163 , 0.29173 , 0.50483] and $\bar{\phi}$ = [10.109 , 11.323 , 12.377]

The alpha cut of scale and shape parameters respectively are

$\bar{\lambda}[\alpha]$ = [0.17163 + 0.1201 α , 0.50483 - 0.2131 α] and $\bar{\phi}[\alpha]$ = [10.109 + 1.214 α , 12.377 - 1.054 α]

Table: 4.2. Lower and Upper α -cut of Fuzzy Mean

α	t =1		t =2		t =3		t =4		t =5	
	$\bar{M}_1[\alpha]$	$\bar{M}_2[\alpha]$	$\bar{M}_1[\alpha]$	$\bar{M}_2[\alpha]$	$\bar{M}_1[\alpha]$	$\bar{M}_2[\alpha]$	$\bar{M}_1[\alpha]$	$\bar{M}_2[\alpha]$	$\bar{M}_1[\alpha]$	$\bar{M}_2[\alpha]$
0	0.017	0.041	0.034	0.082	0.051	0.122	0.068	0.163	0.085	0.204
0.1	0.018	0.039	0.036	0.079	0.054	0.118	0.072	0.158	0.090	0.197
0.2	0.019	0.038	0.038	0.076	0.057	0.114	0.076	0.152	0.095	0.190
0.3	0.020	0.037	0.040	0.073	0.059	0.110	0.079	0.146	0.099	0.183
0.4	0.021	0.035	0.041	0.070	0.062	0.105	0.083	0.140	0.104	0.175
0.5	0.022	0.034	0.043	0.067	0.065	0.101	0.086	0.134	0.108	0.168
0.6	0.022	0.032	0.045	0.064	0.067	0.096	0.090	0.128	0.112	0.160
0.7	0.023	0.031	0.047	0.061	0.070	0.092	0.093	0.122	0.117	0.153
0.8	0.024	0.029	0.048	0.058	0.072	0.087	0.097	0.116	0.121	0.145
0.9	0.025	0.027	0.050	0.055	0.075	0.082	0.100	0.110	0.125	0.137
1	0.026	0.026	0.052	0.052	0.077	0.077	0.103	0.103	0.129	0.129

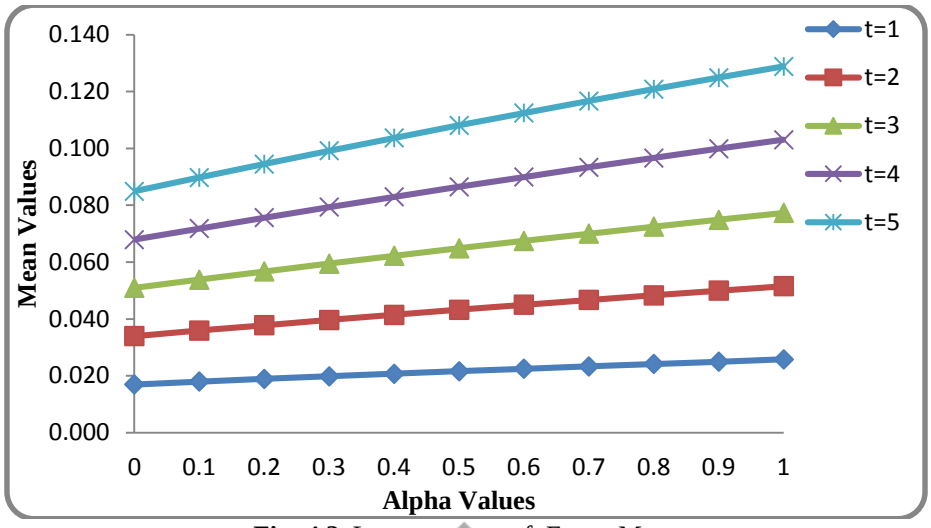

Fig: 4.2. Lower α -cut of Fuzzy Mean

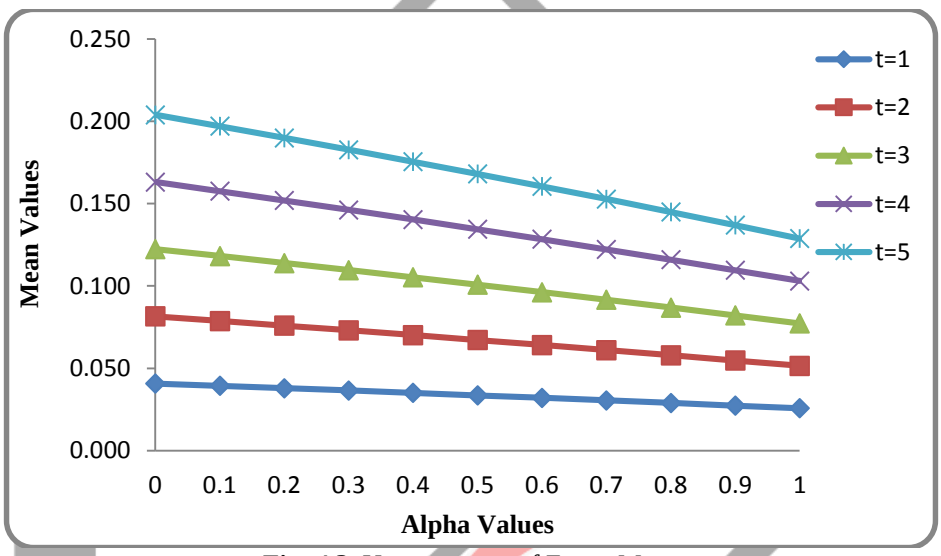

Fig: 4.3. Upper α -cut of Fuzzy Mean

Table: 4.3. Lower and Upper α -cut of Fuzzy Variance

A	t =1		t =2		t =3		t =4		t =5	
	$\overline{M}_3[\alpha]$	$\overline{M}_4[\alpha]$	$\overline{M}_3[\alpha]$	$\overline{M}_4[\alpha]$	$\overline{M}_3[\alpha]$	$\overline{M}_4[\alpha]$	$\overline{M}_3[\alpha]$	$\overline{M}_4[\alpha]$	$\overline{M}_3[\alpha]$	$\overline{M}_4[\alpha]$
0	0.0017	0.0033	0.0034	0.0066	0.0050	0.0099	0.0067	0.0132	0.0084	0.0165
0.1	0.0018	0.0032	0.0035	0.0064	0.0053	0.0096	0.0070	0.0128	0.0088	0.0161
0.2	0.0018	0.0031	0.0037	0.0062	0.0055	0.0094	0.0073	0.0125	0.0091	0.0156
0.3	0.0019	0.0030	0.0038	0.0061	0.0057	0.0091	0.0076	0.0121	0.0095	0.0152
0.4	0.0020	0.0029	0.0039	0.0059	0.0059	0.0088	0.0078	0.0117	0.0098	0.0147
0.5	0.0020	0.0028	0.0040	0.0057	0.0061	0.0085	0.0081	0.0113	0.0101	0.0142
0.6	0.0021	0.0027	0.0041	0.0055	0.0062	0.0082	0.0083	0.0109	0.0104	0.0137
0.7	0.0021	0.0026	0.0043	0.0053	0.0064	0.0079	0.0085	0.0105	0.0106	0.0131
0.8	0.0022	0.0025	0.0044	0.0050	0.0065	0.0075	0.0087	0.0101	0.0109	0.0126
0.9	0.0022	0.0024	0.0045	0.0048	0.0067	0.0072	0.0089	0.0096	0.0111	0.0120
1	0.0023	0.0023	0.0046	0.0046	0.0068	0.0068	0.0091	0.0091	0.0114	0.0114

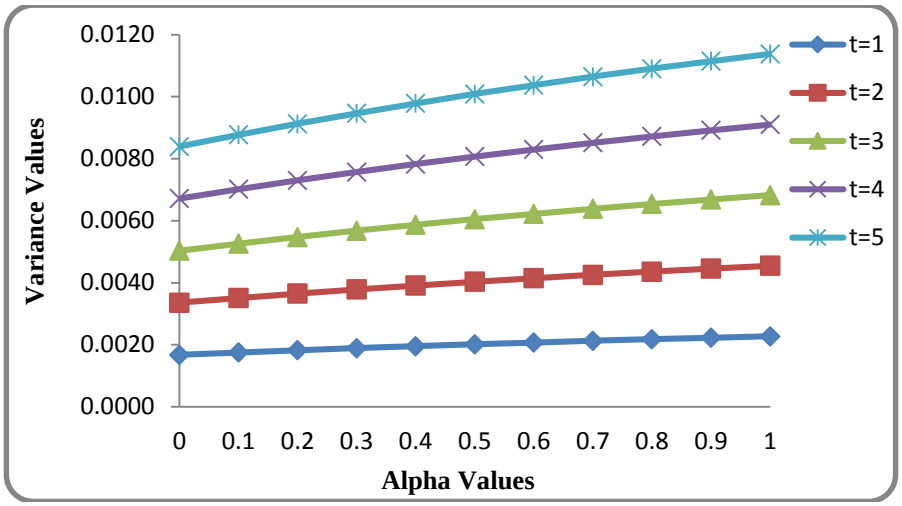

Fig: 4.4. Lower α -cut of Fuzzy Variance

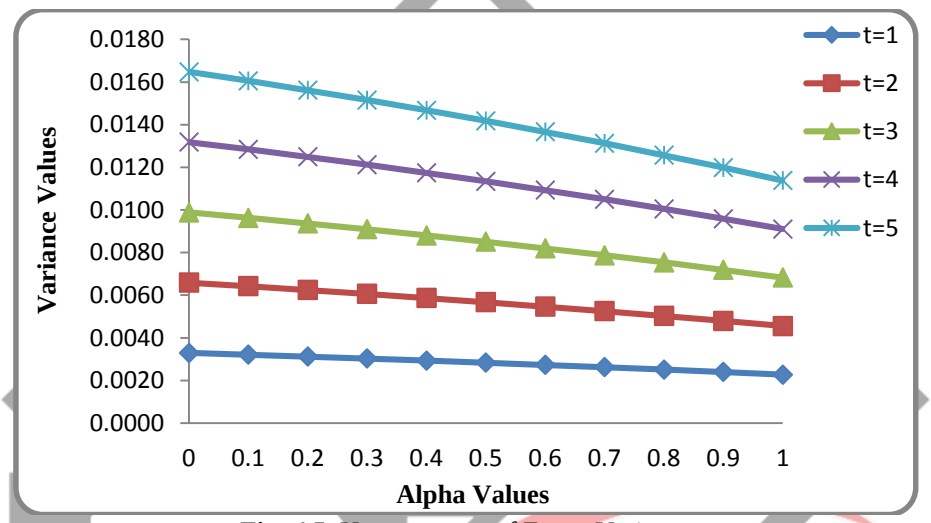

Fig: 4.5. Upper α -cut of Fuzzy Variance

5. Conclusion

In this case study, we have calculated the fuzzy mean and fuzzy variance for the TSH suffered patient with different period. Here, we used Gamma Distribution with two parameter. The α - cut for the lower fuzzy mean and lower fuzzy variance increases when time t increases. Similarly, the α - cut for the upper fuzzy mean and upper fuzzy variance decreases when time t increases. From the result, if the time increases, the fuzzy mean and fuzzy variance increases for the TSH levels by gamma distributions.

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