RP-95: Formulation of Solutions of a Special Standard Cubic Congruence of Prime-power Modulus

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***Abstract*: In this paper, a special type of standard cubic congruence of prime-power modulus is considered for formulation. The author formulated the solutions of the said congruence successfully. First time the said congruence is formulated. It is not formulated earlier. The established formula is tested and found true. Formulation of the congruence is the merit of the paper.**

***Keywords*: Binomial cubic expansion, Cubic congruence, Formulation, Prime-power- modulus.**

INTRODUCTION

In this paper, the author considered a special type of standard cubic congruence of prime-power modulus for his study. It was not studied earlier.

First time the author has taken a bold attempt to study and formulate the solutions of the congruence under consideration.

Peeping through different books on Number Theory, the author found no discussion on this standard cubic congruence of composite modulus. But some recent papers of the author on formulation of standard cubic congruence of composite modulus are found [1], [2], [3], [4], [5].

PROBLEM-STATEMENT

The problem is-

"To formulate the cubic congruence: $x^3 \equiv a^3 \pmod{p^n}$ in two different cases for $n > 3; n \leq 3$ with $a = p$.

ANALYSIS & RESULTS

Consider the congruence: $x^3 \equiv a^3 \pmod{p^n}$.

CASE-I: If $a = p$, then the congruence under consideration becomes:

$x^3 \equiv p^3 \pmod{p^n}$ with $n > 3$.

For its formulation, let us consider that: $x \equiv p^{n-1}k + p \pmod{p^n}$.

Then using binomial cubic expansion,

$$x^3 \equiv (p^{n-1}k + p\ )^3 \pmod{p^n} \pmod{p^n}$$

$$\equiv (p^{n-1}k)^3 + 3.(p^{n-1}k)^2.p + 3.p^{n-1}k.p^2 + p^3 \pmod{p^n}$$

$$\equiv p^3 + p^{n-1}k\{3p^2 + 3p.p^{n-1}k + (p^{n-1}k)^2\} \pmod{p^n}$$

$$\equiv p^3 + p^{n-1}k.\{pm\} \pmod{p^n}$$

$$\equiv p^3 + p^n k.m \pmod{p^n}$$

$$\equiv p^3 \pmod{p^n}.$$

Thus, it can be said that

$x \equiv p^{n-1}k + p \pmod{p^n}; k = 0, 1, 2, \dots (p-1), p, \dots \dots$

gives all the solutions of the congruence.

But if $k = p, p+1, \dots \dots$, it can be easily seen that the solution of the congruence is the same as for $k = 0, 1, \dots\dots$

Thus, the congruence has p incongruent solutions for $k = 0, 1, 2, \dots, (p-1)$.

CASE-II: Let us consider that $n \leq 3$.

Then the congruence reduces to $x^3 \equiv 0 \pmod{p^n}$.

In such cases, the solutions are the positive integers multiple of p

$\text{i.e.}\quad x \equiv mp \pmod{p^n};\ m = 1, 2, 3, \ldots \ldots$

ILLUSTRATIONS

Consider the congruence $x^3 \equiv 125 \pmod{625}$.

It can be written as $x^3 \equiv 5^3 \pmod{5^4}$.

It is of the type: $x^3 \equiv p^3 \pmod{p^n}$ with $p = 5, n = 4$,

having five solutions.

The solutions are then given by

$$x \equiv p^{n-1}k + p \pmod{p^n} \text{ with } k = 0,1,2, \ldots \ldots (p-1).$$

$$\text{i.e.}\quad x \equiv 5^{4-1}k + 5 \pmod{5^4}$$

$$\text{i.e.}\quad x \equiv 125k + 5 \pmod{625};\ k = 0, 1, 2, 3, 4.$$

$$\text{i.e.}\ x \equiv 5, 130, 255, 380, 505 \pmod{625}.$$

Consider one more example: $x^3 \equiv 343 \pmod{16807}$.

It can be written as $x^3 \equiv 7^3 \pmod{7^5}$.

It is of the type: $x^3 \equiv p^3 \pmod{p^n}$ with $p = 7, n = 5$,

having seven solutions.

The solutions are then given by

$$x \equiv p^{n-1}k + p \pmod{p^n} \text{ with } k = 0,1,2, \ldots \ldots (p-1)$$

$\text{i.e.}\quad x \equiv 7^{5-1}k + 7 \pmod{7^5}$

$\text{i.e.}\quad x \equiv 2401k + 7 \pmod{16807};\ k = 0, 1, 2, 3, 4, 5, 6.$

$\text{i.e.}\ x \equiv 7, 2408, 4809, 7280, 9611, 12012, 14413 \pmod{16807}.$

Now consider the congruence: $x^3 \equiv 125 \pmod{125}$.

It can be written as: $x^3 \equiv 5^3 \pmod{5^3}$.

Its solutions are the multiples of five $\text{i.e.}\ x \equiv 5, 10, 15, 20, 25, \ldots \ldots \ldots$

CONCLUSION

Therefore, it can be concluded that the standard cubic congruence of prime-power modulus $x^3 \equiv p^3 \pmod{p^n};\ n \geq 4$ is formulated for its solutions. The solutions are given by $x \equiv p^{n-1}k + p \pmod{p^n};\ k = 0, 1, 2, \ldots \ldots, (p-1)$.

It has exactly p-incongruent solutions.

But if $n \leq 3$, then the solutions are all multiples of p

$\text{i.e.}\quad x \equiv mp \pmod{p^n};\ m = 1, 2, 3, \ldots \ldots$

MERIT OF THE PAPER

The standard cubic congruence under consideration is formulated successfully and tested true for the solutions. First time the congruence is formulated. Formulation is the merit of the paper.

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