LRS Bianchi Type- ii Cosmological Model With Barotropic Perfect Fluid In C-Field Theory With Time-Dependent Term - Λ

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Abstract: **We have studied the Hoyle – Narlikar C- field cosmology for LRS Bianchi type ii with time varying cosmological constant $\Lambda(t)$ for barotropic perfect fluid distribution. To get deterministic solution we assume that $\Lambda=1/R^2$ as considered by Chen and Wu, where R is a scale factor and A = B where A and B are metric potentials. The conservation equation $T^j_{i;j} = 0$ and $T^i_j = T^j_{i(m)} + T^j_{i(c)}$, $T^j_{i(m)}$being energy-momentum tensor for matter and $T^j_{i(c)}$is the energy momentum tensor for C-field.**

We find that creation field (C) increase with time and $\Lambda\sim 1/t^2$which matches with the result obtained by Hoyle Narlikar theory. Above we have discussed special cases of model (28) like dust filled universe (γ= 0), stiff fluid universe (γ = 1) and radiation dominated universe (γ = 1/3). The physical and geometrical aspects for this model are also studied.

Keywords: LRS Bianchi type ii, creation field, barotropic perfect fluid, cosmological term.

Introduction:-

Big Bang theory gave the most prevailing cosmological model based on Albert Einstein's general relativity for the early development of universe. This model is popularly known as the big bang model of universe. According to this model of universe is continuously expanding from an extremely hot and dense state. This theory can explain the observed phenomenon including cosmic microwave background, large scale structure and abundance of light element and Hubble's law. It also helps in simplifying the assumption such as homogeneity and isotropy of space. This theory was unable to explain the initial condition of universe and this considered as a failure. The horizon problem, flatness problem and the magnetic monopole problem are considered are the three main problems related to the big bang theory. After that alternative theories were proposed from time to time.

Steady state theory:- it was given by Bondi and Gold. It state that the universe does not have singular beginning not an end on the cosmic time scale. According to this theory, the universe is always expanding but with this constant average density has maintained. It was also rejected as the observations shown clearly the big bang type cosmology and fine age universe contradicts to the fundamental laws of physics which state that matter and energy are interchangeable but the total amount of energy and matter in this universe remains constant but steady state theory requires the continuous creation of matter in violation of this law. For maintenance of consistency of mass density, they observed the very slow but continuous creation at t=o, however it faced the serious drawback of not giving any physical justification for continuous creation of matter.

C- field theory:- it was introduced by Hoyle and Narlikar (1-3). It bring to light the possibility of an ever existing expanding universe with constant density of matter and in this theory due to the presence of an appropriate creation field with negative energy the constancy of matter density is possible. Narlikar (4) also investigated that Singularity and matte creation is accomplished at the expense of negative energy C- field and that introduction of negative energy field has solved the problem of horizon and flatness faced by the former theory studied i.e. the big bang model. Bianchi type- I massive string cosmological model with magnetic field of barotropic perfect fluid distribution through the techniques used by Latelier and Stachel is investigated by Bali et.al. (5). Singh et.al. (6) have investigated Einstein field equation with variable gravitational and cosmological constant are considered in the presence of perfect fluid for the Bianchi type-III universe by assuming conservation law for the energy momentum tensor.

A cosmological model in creation field cosmology for dust distribution with varying Λ in the framework of FRW space time was investigated by Bali and Saraf (7). Singh and Ram (8) has studied Bianchi type – III model of universe filled with a magnetized perfect fluid together with a time – varying constant Λ is investigated in general relativity. Kaluza klein cosmological models with varying G in Hoyle Narlikar C-field theory of gravitation for barotropic fluid distribution have been investigated by Ghate and Mhaske (9).Parikh et. al. (10) has studied cosmological constant Λ in Bianchi type III string cosmological model is investigated for dust fluid. Time dependent Λ in Bianchi type IX cosmological model with barotropic perfect fluid in C-field theory have been studied by Parikh et. al. (11).Bianchi type – VI_0 cosmological model for barotropic fluid distribution in C-field cosmology with varying cosmological term Λ is investigated by Tyagi and Parikh (12). Bali and Tikekar (13) investigated C -field cosmological model for dust distribution with variable gravitational constant in framework of flat FRW space time.

Solutions of Einstein field equation admitting radiation with a negative energy massless scalar field C have been obtained by Narlikar and Padmnabhan (14). Chatterjee and Bannerjee (15) have studied C- field cosmology in higher dimensions. Singh and Chaubey (16) have investigated Bianchi type I, III, V, VI and Kantowski Sach universes in creation field cosmology. Adhav et al. (17) have obtained Kasner and Axially symmetric universes in C - field theory of gravitation. Adhav et al. (18) investigated stiff domain walls in creation field cosmology. Recently Ghate et al. (19) have studied the Kaluza-Klein Dust Filled Universe with Time Dependent Λ in Creation Field Cosmology. Bali and Kumawat (20) have studied cosmological model with variable G in C-field theory. Tyagi and Singh (21) have investigated time-dependent Λ in C-field theory with LRS Bianchi type III universe and barotropic perfect fluid. LRS Bianchi type V perfect fluid cosmological model in C-field theory with variable Λ is also investigated by Tyagi and Singh (22). Patil et al. (23) have obtained Bianchi type IX dust filled universe with ideal fluid distribution in creation field. Bianchi type-***IX*** string cosmological model in general relativity is also investigated by Bali and Dave (24).

In this paper, we have observed and investigated the LRS Bianchi type ii cosmological model for barotropic perfect fluid distribution in C-field cosmology with time dependent term Λ. For deterministic model, we assumed $\Lambda = 1/R^2$, where R is scale factor. We find that creation field (C) increase with time and $\Lambda \sim 1/t^2$.we have also studied and discussed the physical and geometrical parameters of the model.

2. The Metric and Field Equation

We have considered LRS Bianchi type ii of the form

$$ds^2 = -dt^2 + A^2(dx^2 + dy^2) + B^2(dy - x\,dz)^2 \qquad \text{....(1)}$$

in which, A and B are functions of t alone. Hoyle and Narlikar modify the Einstein's field equation by introducing C-field with time dependent cosmological term as:

$$R_i^j - \frac{1}{2}Rg_i^j = -8\pi G\left[T_{i(m)}^j + T_{i(c)}^j\right] + \Lambda g_i^j \qquad \text{....(2)}$$

The energy-momentum tensor $T_{i(m)}^j$ for perfect fluid and creation field $T_{i(c)}^j$ are given by

$$T_{i(m)}^j = (\rho + p)v_i v^j + p g_i^j \qquad \text{.....(3)}$$

$$T_{i(c)}^j = -f\left(c_i c^j - \frac{1}{2}g_i^j c_\alpha c^\alpha\right) \qquad \text{....(4)}$$

where f > 0 is coupling constant between the matter and creation field and $c^i = \frac{dc}{dx^i}$.

The co-moving coordinates are chosen such that v_i= (0, 0, 0, 1).

The non-vanishing components of energy-momentum tensor for matter are given by

$$T_{1(m)}^1 = T_{2(m)}^2 = T_{3(m)}^3 = p; \quad T_{4(m)}^4 = \rho \qquad \text{....(5)}$$

The non-vanishing components of energy-momentum tensor for creation field are given by

$$T_{1(c)}^1 = T_{2(c)}^2 = T_{3(c)}^3 = -\frac{1}{2}f\dot{c}^2; \quad T_{4(c)}^4 = \frac{1}{2}f\dot{c}^2 \qquad \text{....(6)}$$

Hence, the Einstein's field equation (2) for the metric (1) and energy-momentum tensor (5) and (6) takes the form

$$\frac{A_4^2}{A^2} + 2\frac{A_4B_4}{AB} - \frac{B^2}{4A^2} = 8\pi G\left(\rho - \frac{1}{2}f\dot{c}^2\right) + \Lambda \qquad \text{....(7)}$$

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{A_4B_4}{A} + \frac{B^2}{4A^4} = 8\pi G\left(-p + \frac{1}{2}f\dot{c}^2\right) + \Lambda \qquad \text{....(8)}$$

$$2\frac{A_{44}}{A} + \frac{A_4^2}{A^2} - \frac{3B^2}{4A^2} = 8\pi G\left(-p + \frac{1}{2}f\dot{c}^2\right) + \Lambda \quad \text{....(9)}$$

The suffix 4 by the symbols A and B denotes differentiation w.r.t. t.

3. Solution of Field Equations

The conservation equation of energy momentum tensor is

$$\left(8\pi G T_j^i + \Lambda g_j^i\right)_{;i} = 0 \quad \text{.....(10)}$$

which leads to

$$8\pi\dot{G}\left(\rho - \frac{1}{2}f\dot{c}^2\right) + 8\pi G\left[\dot{\rho} - f\dot{c}\ddot{c} + \rho\left(2\frac{\dot{A}}{A} + \frac{\dot{B}}{B}\right) + (f\dot{c}^2 - p)\left(2\frac{\dot{A}}{A} + \frac{\dot{B}}{B}\right)\right] + \dot{\Lambda} = 0 \quad \text{.....(11)}$$

Following Hoyle and Narlikar theory, the source equation of C-field i.e. $c_{;i}^i = \frac{n}{f}$ leads to c = t thus $\dot{c}$= 1.

To get determinate solution of equations (7) - (9), we assume condition between the metric potential i.e.

$$A = B \quad \text{... (12)}$$

Using (12), equations (7) to (9) becomes

$$3\frac{B_4^2}{B^2} - \frac{1}{4\boldsymbol{B}^2} = 8\pi G\left(\rho - \frac{1}{2}f\dot{c}^2\right) + \Lambda \quad \text{....(13)}$$

$$\frac{B_4^2}{B^2} + 2\frac{B_{44}}{B} - \frac{3}{4\boldsymbol{B}^2} = 8\pi G\left(-p + \frac{1}{2}f\dot{c}^2\right) + \Lambda \quad \text{....(14)}$$

$$\frac{B_4^2}{B^2} + 2\frac{B_{44}}{B} + \frac{1}{4\boldsymbol{B}^2} = 8\pi G\left(-p + \frac{1}{2}f\dot{c}^2\right) + \Lambda \quad \text{....(15)}$$

The barotropic perfect fluid condition leads to

$$p = \gamma\rho \quad ; \quad \text{where } 0 \le \gamma \le 1 \quad \text{... (16)}$$

Using (16) in equation (14) we have

$$2\frac{B_{44}}{B} + \frac{B_4^2}{B^2} - \frac{3}{4\boldsymbol{B}^2} = 8\pi G\left(-\gamma\rho + \frac{1}{2}f\dot{c}^2\right) + \Lambda \quad \text{....(17)}$$

Now equations (13) and (17) together with $\dot{c} = 1$ leads to

$$2\frac{B_{44}}{B} + (1+3\gamma)\frac{B_4^2}{B^2} - \frac{1}{4\boldsymbol{B}^2}(\gamma+3) = 4\pi f G(1-\gamma) + \Lambda(1+\gamma) \quad \text{....(18)}$$

To get solution of equation (18) we also assume that

$$\Lambda = \frac{1}{R^2} = \frac{1}{B^2} \quad \text{....(19)}$$

Using equation (19) in equation (18) we have

$$2B_{44} + (1+3\gamma)\frac{B_4^2}{B} = 4\pi f G(1-\gamma)B + \frac{(5\gamma+7)}{4B} \quad \text{....(20)}$$

Now put B_4 = f (B) which leads to B_{44} = ff'. ... (21)

Now equation (20) with the help of (21) becomes

$$\frac{df^2}{dB} + (3\gamma+1)\frac{f^2}{B} = 4\pi f G(1-\gamma)B + \frac{(5\gamma+7)}{4B} \quad \text{....(22)}$$

Equation (22) leads to

$$f^2.B^{3\gamma+1} = 4\pi f G(1-\gamma).\frac{B^{3\gamma+3}}{3(\gamma+1)} + \frac{(5\gamma+7)}{4}\frac{B^{3\gamma+1}}{3\gamma+1} \qquad \text{....(23)}$$

which gives

$$f^2 = \alpha B^2 + \beta \qquad \text{....(24)}$$

Where $\alpha=\frac{4\pi f G(1-\gamma)}{3(\gamma+1)}$, $\beta = \frac{(5\gamma+7)}{4(3\gamma+1)}$ ....(25)

Equation (24) leads to

$$\frac{dB}{\sqrt{\alpha B^2+\beta}} = dt \qquad \text{....(26)}$$

Hence, equation (26) gives

$$B = \sqrt{\frac{\beta}{\alpha}}\sin h\sqrt{\alpha}t \qquad \text{....(27)}$$

Also the metric (1) reduces to

$$ds^2 = -dt^2 + \frac{\beta}{\alpha}\sin h^2\sqrt{\alpha}t(dx^2 + dz^2 + \{dy - xdz\}^2) \qquad \text{....(28)}$$

From equation (19) and (15) we have

$$\Lambda = \frac{1}{R^2} = \frac{1}{B^2} = \frac{\alpha}{\beta}cosech^2\sqrt{\alpha}t \qquad \text{....(29)}$$

and

$$8\pi G\rho = 3\alpha + cosech^2\sqrt{\alpha}t\left(3\alpha - \frac{5}{4}\frac{\alpha}{\beta}\right) + 4\pi f G \qquad \text{....(30)}$$

Now using p = $\gamma\rho$ and equation (12) in equation (11) we have

$$8\pi G\left[\dot{\rho} - f\dot{c}\ddot{c} + \{(1-\gamma)\rho + f\dot{c}^2\}3\frac{B_4}{B}\right] + \dot{\Lambda} = 0 \qquad \text{....(31)}$$

Equation (31) leads to

$$4\pi f G\frac{d\dot{c}^2}{dt} + 24\pi f G\sqrt{\alpha}\,coth\sqrt{\alpha}t\dot{c}^2 =$$
$$\left(3\alpha - \frac{5\alpha}{4\beta}\right)2\sqrt{\alpha}cosech^2\sqrt{\alpha}t\,coth\sqrt{\alpha}t + (1+\gamma)\,9\alpha\sqrt{\alpha}$$
$$coth\sqrt{\alpha}t + 4\pi f G(1+\gamma)3\sqrt{\alpha}\,coth\sqrt{\alpha}t + \frac{2\alpha\sqrt{\alpha}}{\beta}cosech^2\sqrt{\alpha}t\,coth\sqrt{\alpha}t$$
$$-(1+\gamma)3\sqrt{\alpha}\left(3\alpha - \frac{5\alpha}{4\beta}\right)cosech^2\sqrt{\alpha}t\,coth\sqrt{\alpha}t \qquad \text{......(32)}$$

To obtain the solution of equation (32) we assume that $4\pi f G = 3$ and $\alpha = 1$, so equation (32) leads to

$$\frac{d\dot{c}^2}{dt^2} + (6\coth t)\dot{c}^2 = 6(\coth t) \qquad \text{.....(33)}$$

Equation (33) gives $\dot{c}^2 = 1$ .....(34)

So, we have $\dot{c} = 1$ .....(35)

which agrees with the value used in source equation. Thus, creation field is proportional to time t.

4. Physical and Geometrical Properties

For the model (28), the mass density ρ is given by

$$8\pi G\rho = 3\alpha + cosech^2\sqrt{\alpha}t\left(3\alpha - \frac{5\alpha}{4\beta}\right) + 4\pi f G \quad .....(36)$$

The scale factor R is

$$R = B = \sqrt{\frac{\beta}{\alpha}}\sin h\sqrt{\alpha}t \quad ....(37)$$

The cosmological constant (Λ) is

$$\Lambda = \frac{\alpha}{\beta} cosech^2\sqrt{\alpha}t \quad ....(38)$$

and the deceleration parameter (q) is

$$q = -\tan h^2\sqrt{\alpha t} \quad ....(39)$$

5. Special Cases

Case I: Dust Filled Universe (γ = 0)

From equation (26), we have

$$\frac{dB}{\sqrt{B^2+\frac{7}{4}}} = dt \quad ....(40)$$

The metric (28) for the dust filled universe is given by

$$ds^2 = -dt^2 + \frac{7}{4}\sin h^2(t+t_0)(dx^2 + dz^2 + \{dy - xdz\}^2) \quad ....(41)$$

So, the mass density (ρ), scale factor (R), cosmological constant (Λ) and the decelerating parameter (q) for the model (41) are given by

$$8\pi G\rho = 6 + \frac{16}{7}\ cosech^2(t+t_0) \quad ....(42)$$

$$R = \sqrt{\frac{7}{4}}\sinh(t+t_0) \quad ....(43)$$

$$\Lambda = \frac{1}{B^2} = \frac{4}{7}\ cosech^2(t+t_0) \quad ....(44)$$

and $q = -tanh^2(t+t_0)$ ....(45)

Case II: Stiff Fluid University (γ = 1)

From equation (26), we have

$$\frac{dB}{\sqrt{\frac{3}{4}}} = dt \quad ....(46)$$

The metric (28) for stiff fluid universe is given by

$$ds^2 = -dt^2 + \frac{3}{4}(t+t_0)^2(dx^2 + dz^2 + \{dy - xdz\}^2) \quad ....(47)$$

Also the mass density (ρ), scale factor (R), cosmological constant () and declaration parameter (q) for the model (47) are given by

$$8\pi G\rho = 3 \quad ....(48)$$

$$R = \sqrt{\frac{3}{4}}\ (t+t_0) \quad ....(49)$$

$$\Lambda = \frac{1}{B^2} = \frac{4}{3(t+t_0)^2} \quad ....(50)$$

and $$q = 0 \quad ....(51)$$

Case III: Radiation Dominated Universe (γ = 1/3)

From equation (26), we have

$$\frac{dB}{\sqrt{\frac{1}{2}B^2+\frac{13}{12}}} = dt \quad ....(53)$$

The metric (28) for radiation dominated universe becomes

$$ds^2 = dt^2 + \frac{13}{6} sinh^2 \frac{(t+t_0)}{\sqrt{2}} (dx^2 + dz^2 + \{dy - xdz\}^2) \quad ....(54)$$

Also, the mass density (ρ), scale factor (R), cosmological constant (Λ) and deceleration parameter (q) for the model (52) are given by

$$8\pi G\rho = \frac{9}{2} + \frac{12}{13} cosech^2 \frac{(t+t_0)}{\sqrt{2}} \quad ...(55)$$

$$R = \sqrt{\frac{13}{6}}\, sinh \frac{(t+t_0)}{\sqrt{2}} \quad ...(56)$$

$$\Lambda = \frac{1}{B^2} = \frac{6}{13} cosech^2 \frac{(t+t_0)}{\sqrt{2}} \quad ...(57)$$

and $$q = -tanh^2 \frac{(t+t_0)}{\sqrt{2}} \quad ....(58)$$

6. Conclusion

The creation field C increases with time and $\dot{c}$=1 which agrees with the value taken in source equation. The scale factor R for the model (28) increases with time and the cosmological term Λ decreases as time increases. The decelerating parameter from equation (39) i.e. q < 0 which represent that universe is accelerating.

Further for all special cases i.e. dust filled universe, stiff fluid universe and radiation dominated universe, scale factor R increases and cosmological term Λ decreases with time. In case of stiff fluid, model has uniform motion (51) and the universe is accelerating for dust filled and radiation in dominated cases. Our model resembles exactly same with the model investigated by Parikh, S., Tyagi, A. &Tripathi, B. R., (2016) for bianchi type ix.

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