New C_r Inequality $_mC_r$ with Unequal Weightage to the Random Variables

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Abstract: **In this $_mC_r$ inequality we can give unequal weightage to the random variables. As a special case we can get the Classic- C_r inequality if $m=r \geq 1$**

Index Terms: **Classic- C_r Inequality, Expectation, Random Variables.**

INTRODUCTION

The Classic- C_r inequality in ***lr*** space usually stated as follows:

X and **Y** are random variables,

If $E|X|^r$, $E|Y|^r$ is both finite then $E|X+Y|^r$ is also finite.

$$C_r[E|X|^r+E|Y|^r] \geq E|X+Y|^r$$

Where $C_r = 2^{r-1}$ if $r \geq 1$

$= 1$ if $0<r<1$

where $E|X|^r$ is Expectation of $|X|^r$

The above inequality is limited to give equal weightages to both the random variables as 2^{r-1} if $r \geq 1$. And is limited to give weightages as the powers of two only. In this paper I formulated an inequality to give unequal weightage to the random variables.

$_mC_r$ inequality in $l_{r,m}$ space stated as follows:

X and **Y** are random variables,

If $E|X|^r$, $E|Y|^r$ is both finite then $E|X+Y|^r$ is also finite.

$$(1+m/r)^{r-1} E|X|^r + (1+r/m)^{r-1} E|Y|^r \geq E|X+Y|^r \qquad m, r \geq 1$$

$$C_{1,r,m} E|X|^r + C_{2,r,m} E|Y|^r \geq E|X+Y|^r$$

Where, $C_{1,r,m} = (1+m/r)^{r-1}$

$C_{2,r,m} = (1+r/m)^{r-1}$

$$C_{1,r,m} / C_{2,r,m} = (m/r)^{r-1}$$

if $m/r < 1$ more weightage to random variable Y

$m/r = 1$ Same weightage to both the Random variables

$m/r > 1$ more weightage to random variable X

The proof follows similar way as **classic C_r inequality** derived but with some more generalized function.

PROOF

Let us consider a more generalized function

$$F(p)=[(mp)^r]/m + [(r(1-p))^r]/r \qquad (1)$$

where $m, r \geq 1$ and $0<p<1$

$F'(p)=r(mp)^{r-1} - r(r(1-p))^{r-1}$

Where F'(p) is the first derivative of F(p) with respect p

Let F'(p)=0

We get p*=r/(r+m)

After substituting p*= r/(r+m) in F''(p), We get F''(p*)>0 it means the point p*= r/(r+m) is an absolute minimum point of the given function. So, the minimum value of the function after substituting minimum point in (1) we get

$F(p^*) =[mr/(m+r)]^{r-1}$

So, $F(p)\geq[mr/(m+r)]^{r-1}$

Consider A=|X|, B=|Y|

p=|X|/(|X|+|Y|) Substitute in equation (1)

Then after doing a little bit algebraic manipulations equation (1) boils down to

$$\mathbf{m^{r-1}|X|^r + r^{r-1}|Y|^r \geq (|X|+|Y|)^r\,[mr/(m+r)]^{r-1}} \quad \textbf{(2)}$$

We can modify right side of inequality (2) by using Triangular Inequality of Modulus.

We know that $|X+Y|\leq|X|+|Y| \Rightarrow |X+Y|^r\leq (|X|+|Y|)^r$

Then the equation (2) will change into

$m^{r-1}|X|^r + r^{r-1}|Y|^r \geq |X+Y|^r\,[mr/(m+r)]^{r-1}$

Now take the Expectation on both sides of inequality then

$m^{r-1}E|X|^r + r^{r-1}E|Y|^r \geq [mr/(m+r)]^{r-1}E|X+Y|^r$

after rearranging coefficients of Expectations, we get the following form in $l_{r,m}$ space where m, r ≥1

$\mathbf{(1+m/r)^{r-1}E|X|^r + (1+r/m)^{r-1}E|Y|^r \geq E|X+Y|^r}$ **m, r≥1**

$C_{1,r,m}\,E|X|^r + C_{2,r,m}\,E|Y|^r \geq E|X+Y|^r$

Where, $C_{1,r,m} = (1+m/r)^{r-1}$ $C_{2,r,m} = (1+r/m)^{r-1}$

$C_{1,r,m}/C_{2,r,m} = (m/r)$^r-1

if m/r <1 more weightage to random variable Y

m/r =1 Same weightage to both the Random variables

m/r >1 more weightage to random variable X

The special case of ${}_mC_r$ inequality is Classic- C_r inequality if **m=r≥1**

$C_{1,r} = 2^{r-1} = C_{2,r}$

$\mathbf{2^{r-1}[E|X|^r + E|Y|^r] \geq E|X+Y|^r}$.

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