Critical Thinking and Creative Thinking as the Focus on Mathematics Education

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Abstract: The capacity to think critically and think creatively is progressively required in the life. The world community faces global problems such as population development, limited resources, changing modes of employment, climate change due to global warming, cultural change, and dynamic changes in the economy of society. Competition is getting tougher but on the other hand cooperation and flexibility in dealing with work and something is still needed. This condition is a challenge in the world of education as well as the obligation of teachers who directly face future generations. Teachers should be progressively mindful that the message of training isn’t only conveying material however teaching to manufacture critical and creative capacities. Educators need to pass on the culture to be careful, systematic, evaluative, analytical, flexible, and accept different ideas. So as to give these desires, particularly for mathematics educators, this paper will talk about the issue of critical reasoning aptitudes, creative reasoning aptitudes, and looking for these abilities to find.

Index Terms: Critical Thinking, Creative Thinking, Mathematics Education

1. INTRODUCTION

Mathematics education is not only a role in providing educational value which is intelligent for students, but also the value of education that helps shape the character of students including critical thinking and creative thinking. These capacities don't just show up normally however should be instructed and structured since at elementary and high school level. These capacities and aptitudes should be created in each subject including mathematics. Giving these aptitudes and capacities should be done in the classes when the learning procedure happens. The capacity to assume critically and creatively in real mathematics topics has long been the goal of learning clearly and inherently. Instructors may have endeavored to underline the capacity of critical and creative reasoning, yet the substance of such educational modules material causes them to organize different angles, for example, just getting ideas. Normally, mathematics learning in schools has not provided the chance for students to find answers or ways that are unique in relation to those taught by teachers. Class teachers do not allow learners to build up their own perspective or understanding of a mathematical concept. Critical and imaginative reasoning is once in a while stressed in school mathematics learning in light of the fact that the connected learning procedures will in general be situated towards creating logical reasoning with routine issues.

Thinking is a mental action experienced by someone if they are faced with a problem or situation that must be solved. Critical thinking (CT) is an ever changing process that can be described according to the procedure or path. The method of thinking essentially comprises of three stages, in particular the development of comprehension, opinion formation, and conclusion. This view shows if a person is faced with a situation, then in thinking, the person will arrange the relationship between the parts of information recorded as understanding. Then the person shapes opinions according to his knowledge. After that, he will make conclusions that are used to discuss or find solutions to the situation.

Thinking as an individual's psychological capacities can be isolated into several types, including logical, analytic, methodical, critical, and creative. Logical thinking can be interpreted as the capacity to think of students to draw legitimate conclusions according to the rules of logic and can prove that conclusions are true or valid in accordance with previous knowledge that is already known. Analytical thinking is the capacity to think of students to decipher, elaborate, and analyze information used to understand knowledge by using logical mind, not based on feelings or guesses. Systematic thinking is the capacity to think of students to work on or complete a task in accordance with the sequence, stages, steps, or planning that is appropriate and efficacious. All these three brands of thinking are interrelated. Someone can be said to think systematically, then he needs to think analytically to understand the information used. Then, to be able to think analytically, it requires the capacity to think logically in drawing conclusions from a situation.

1.1 Understanding Critical Thinking (CT)

Critical thinking is the embodiment of “higher order thinking”. This is because the capacity to think is the highest cognitive competence that learners need to ace in the class. Critical thinking may be viewed as the capacity to think of students to compare two or more information, for example information received from outside with information held. If there are differences or similarities, then he will ask questions or comments in order to get an explanation. Critical thinking is often associated with creative thinking.

As per Lewis & Smith [1] critical thinking has roots in two primary academic disciplines philosophy and psychology. Sternberg [2] has likewise noticed a third critical reasoning strand inside the field of instruction and training. These different scholastic strands have created various approaches to manage with characterizing critical reasoning that mirror their particular concerns. The
advantage of the instructive methodology is that it depends on long stretches of classroom experience and perceptions of understudy learning.

Analysts of critical thinking regularly concur on the particular capacities enveloped by the definition, which incorporate, breaking down contentions, cases, or proof [3-6]; making derivations utilizing inductive or deductive thinking [4-7]; settling on choices or taking care of issues [3][6][7] and, judging or assessing [5-10]. Different practices distinguished as applicable to critical thinking incorporate seeing the two sides of a challenge [7], thinking verbally, particularly in connection to ideas of feasibility and scruple [3], foreseeing [9], deciphering and clarifying [5], recognizing assumptions [4], posing and noting inquiries for elucidation and characterizing terms [6].

In spite of the fact that exact proof seems to affirm the idea that critical reasoning capacities and dispositions are separate elements [5], specialists tend to distinguish comparative arrangements of dispositions as pertinent to critical reasoning. The most generally referred to critical reasoning dispositions incorporate receptiveness [5][6][11]; impartiality [5][11]; the inclination to look for reason [4][11]; curiosity [5][11]; the longing to be well-educated [5]; adaptability [3]; and regard for others' perspectives [5][11].

Nickerson (as cited in [12]) an expert in critical thinking conveys the characteristics of people who think critically in terms of knowledge, abilities, attitudes, and habits in acting as follows:

i) Use facts skillfully and honestly.
ii) Organizing thoughts and articulating them clearly, logically or logically.
iii) Distinguish between conclusions based on valid logic and invalid logic.
iv) Identify data adequacy.
v) Understand the difference between reasoning and rationalization.
vi) Try to anticipate the possible consequences of various activities.
vii) Understand ideas according to their level of confidence.
viii) Seeing the similarities and analogy is not superficial.
ix) Can learn independently and have attention that never goes away in its work.
x) Implement problem solving techniques in other domains than they have learned.
xi) Being able to informally represent problems in formal ways such as mathematics can be used to solve problems.
xii) Can express a verbal argument that is irrelevant and express an essential argument.
xiii) Questioning a view and questioning the implications of a view.
xiv) Delicate to the distinction between the validity and intensity of a belief with the validity and intensity it holds.
xv) Recognizing that a person's facts and understanding are always limited, many facts must be explained by a non-inquiry attitude.
xvi) Recognize the possibility of mistaking an opinion, the possibility of bias in opinion, and recognize the danger of weighting facts according to personal choice.

1.2 Understanding Creative Thinking (CrT):

Creativity is a product of one's creative thinking. It is a process which is used when someone bring in or bring up a new idea. It combines ideas with previous thoughts that have not been considered. Creative thinking can also be elucidate as a fusion of logical thoughts and diverse thinking premised on intuition yet at the same time in cognizance [13]. When someone handles creative thinking during a problem solving practice, divergent thinking produces many ideas. This manner will be useful in finding a solution. In thinking creatively two components of the brain will be much needed. The harmony between rationality and creative thought is significant. If one puts it logical deduction is too much, then creativity will be ignored. Thus to bring up creativity requires freedom of thought not under control or pressure.

Weisberg [14] defines creative thinking as referring to processes to produce a creative product which is a new (innovative) work that is obtained from an activity that is directed according to purpose. Creative thinking involves intensive production that fulfills novelty, so that someone can be said to be creative by producing something that has been known before. If someone produce something new in his/her opinion, but someone else has already produced it, then he/she can still be said to be creative.

The traditional view of reviewing creativity refers to four P (4P), namely process, product, person (personal / individual), and place (context, situation). Kozbelt-A et al.[15] added 2P more, namely persuasion and potential. Creativity is the performance of an individual that produces something new and unpredictable. Creativity is a typical meeting point between 3 psychological attributes, namely intelligence, cognitive style, and personality / motivation. Intelligence includes verbal ability, fluent thinking, planning knowledge, problem formulation, strategy formulation, mental portrayal, basic leadership and balance skills, and intellectual integration in general. Cognitive or intellectual style shows leniency and attachment to conventions, creates its own rules, does things in its own way, likes problems that are not too structured, likes to write, design and interest in positions that demand creativity. Personality or motivation dimensions include flexibility, tolerance, encouragement for achievement and recognition, tenacity in facing obstacles and anticipated risk taking.

Evans [16] explains that creative thinking is a mental activity to make connections that are continuous, so that a combination that is “right” is found or until someone gives up. Creative affiliations happen through something similar or across analogical thinking. Associations of ideas form new ideas. So, creative thinking ignores established relationships, and creates separate relationships. This understanding shows that creative thinking is a mental activity to find a combination that has not been known before.

Creative thinking is a series of actions carried out by an individual using his/her mind to create new thoughts from a collection of memories that contain various ideas, data, concepts, experiences, and knowledge. This understanding shows that creative thinking is characterized by the creation of something completely new from the results of various previously occupied ideas, old information, concepts, experiences, and knowledge that are in his/her mind.
As indicated by [17] creativity may communicate not simply by the way we individualize and join routines (along these lines making new ones) yet in addition, maybe for the most part, in how and when we choose to apply those patterns we as of now have. To put it plainly, the view as indicated by which routinization is contrary with creativeness isn’t simply mixed up—it is the accurate inverse of what we trust genuine: Routines are the very stuff wherein all our creativities takes its foundations, the medium where it discovers its expressions. Creativity in general is an idea that covers a wide area of style cognitive style, performance categories and various benefits or outcomes [18]. Creativity refers to a divergent thought and a product that is accepted as creative. Haylock [18] describes two main approaches to getting to know creative thinking:

i) Pay attention to subject responses to solve problems, where a cognitive process specifically, namely understanding the features of creative thinking that are expected to succeed. How to do it overcoming fixation, thinking out of the ordinary i.e. the breaking of a mentality set.

ii) Determine the criteria of a product which is an indicator of creative thinking. How to do it see divergent production which includes flexibility, authenticity and appropriateness.

1.3 Relationship between CT and CrT.
Numerous specialists [19],[20],[21],[23] have built up associations between critical reasoning and creativity. Critical reasoning and creativeness are not totally unrelated developments. As per Bailin [20] a specific measure of creativity is vital for critical idea. Paul & Elder [19] noted that both creativeness and critical reasoning are significant parts of intentional reasoning.

In looking at the connection between CT and CrT there are two views. First, looking at creative thinking is intuitive which is different from critical thinking (analytical) which is based on logic, and secondly looking at creative thinking is a combination of analytical as well as intuitive thinking. Intuitive thinking means thinking of getting something by using instincts or feelings that are suddenly without common facts. The first view tends to be influenced by the views on the dichotomy of the both side’s brains that have different functions, while the second view sees two hemispheres of the brain working synergistically together that are not separate.

Critical reasoning and creativeness might be considered like two peas in the same pod. Great reasoning requires the capacity to create scholarly items, which is related with creativeness. Critical reasoning without innovativeness diminishes to simple doubt and pessimism, and inventiveness without creative idea decreases to negligible curiosity. Paul and Elder [19] point out that the two ideas are inseparably connected and create in parallel. As needs be, the scholars accept both innovative and critical reasoning should be incorporated amid guidance.

Johnson [23] explains that critical thinking organizes processes utilized in mental exercises, for example, problem solving, decision making, convincing, and analyzing scientific assumptions and discoveries. Critical thinking is a capacity to motivation in an organized way. Critical thinking is also an ability to evaluate systematically the quality of thought of oneself and others. Creative thinking is a mental activity that pays attention to authenticity and insight (ideas).

CT and CrT allows students to study problems systematically, bring together many challenges in an organized way, form innovative questions and design original solutions. Creative thinking as opposed to destructive thinking, involves finding opportunities to change things for the better. Creative thinking does not explicitly organize processes, such as critical thinking. Creative thinking is a habit of sharp thinking with intuition, moving imagination, revealing new possibilities, unveiling amazing ideas and inspiring unexpected ideas. This understanding clearly distinguishes CT and CrT.

De Bono [24] distinguishes between two kinds of thinking, namely lateral thinking and vertical thinking. Lateral thinking refers to the discovery of new clues in searching for ideas, while thinking vertically facing the development of ideas and examining an objective criterion. Vertical thinking is selective and sequential which moves vertically if there is a clue in its movements. Lateral thinking is fertile which can jump and move in order to build a new clue. Lateral thinking does not need to be right at each progression and does not utilize fixed classifications arrangements or labels. Vertical thinking choosing approaches that are very promising for a problem during lateral thinking builds many alternative approaches. CrT is a synthesis of both these thinking that is complementary. This understanding states that in creative thinking involves critical thinking (logical and analytical) as well as intuitive, as in the second view in terms of creative thinking.

2. OBJECTIVE OF THE STUDY:
The objective of the present article is to discuss:

i) The notion of critical mathematical thinking (CMT),

ii) The notion of creative mathematical thinking (CrMT),

iii) The relationship between CMT and CrMT,

iv) Mathematics learning model that improve learners’ CMT and CrMT.

3. METHODOLOGY:
Research methodology demonstrates the rationale of advancement of the procedure used to create hypothesis that is procedural structure inside which the exploration is directed. The methodology of this investigation is to examine various aspects of CMT, CrMT, and some related subjects in certain subtleties. The information was gathered to accomplish the outcome for the reason and extent of this investigation. In this investigation secondary information are utilized to enhance the article. For the aggregation of secondary information the researchers have utilized distributed information sources. The distributed information are gathered from: I) different productions of governments or non government bodies and their backup associations, ii) different research reports are set up by researchers, universities and so forth., in the field of CMT and CrMT, iii) books of different writers, hand books,
developed in Japan. The teacher may become a facilitator, not a dispenser of knowledge, break down algorithms in taking care of issues, and problem solving skill. Krulik and Rudnick [26] says that what includes critical thinking in mathematics is thinking that tests, questions, connects, evaluates all aspects in a situation or problem. Glazer [27] characterizes that CMT include capacity and temperament which consolidate past learning, mathematical thinking, and cognitive methodology to generalize, prove, survey mathematical circumstance brilliantly.

Creativity in mathematics or mathematical creativity may be defined as students’ abilities that are related to independent creative mastery mathematics under the teaching of mathematics, independent formulation of problems uncomplicated mathematically, finding ways and means of completion problem, finding proof of theorem, and independent deduction of formulas and discovery methods original method of resolving non standard problems. The mathematical creativity in this study emphasizes on problem solving and submission of mathematical problems.

Problem solving is one way to encourage creativity or students’ creative thinking skills. However, that is not the only way because there are still other approaches. The method uses the “open-ended” problem in pushing class discussions, also known as the open-approach method have developed in Japan [13]. The use of investigation is a kind of open ended method developing in England. Many developing methods include the method known as realistic mathematics in the Netherlands.

3.2 Reasons for the need to be stressed on CMT and CrMT.

Some of the reasons for the need to be stressed on CMT and CrMT, including:

i) Mathematics is a complex and extensive knowledge that is not sufficiently taught by memorization,

ii) Students have the potential to think critically and creatively in all things, including mathematics which is the science of human activities,

iii) Students can find original solutions when solving problems, so that they satisfy themselves (trigger internal motivation),

iv) Educators can see the original contributions and amazing ideas of students, so as to provide opportunities to share ideas and learn from each other,

v) Improve students’ mathematical abilities and skills,

vi) Give experience that finding something original / original requires process, deep thinking and critical, perseverance, and never give up, such as making proof of finding theorems,

vii) Daily real life requires mathematics, while everyday problems are not routine, so it requires critical and creative thinking in solving them.

3.3 Constraints in developing critical thinking and creativity

Previously numerous researchers have studied the constraints in developing creativity including critical thinking in the classroom. Especially for creative thinking, Beghetto [28] writes that researchers have identified constraints in developing creativity including critical thinking in the classroom, namely convergent teaching practices, teacher attitudes and beliefs towards creativity, environmental motivation, and students’ own beliefs towards creativity. Convergent teaching tends to be dominated by teachers to talk or more than 70% of the lesson time is used to transfer information. The teacher does not accept ideas or input from students, if students throw ideas they are considered destructive or disturbing. This shows that critical thinking skills are not fully accepted in learning activities. Such practices are often influenced by the teacher’s own attitudes and beliefs. These attitudes and beliefs are built up during school time and also the environmental situation that builds up his experience. There are still many views that creativity and academic knowledge are separate things. Learning to develop students’ critical and creative potential is different for academic knowledge. Beghetto [28] said that teachers can develop students' creative potential (including critical thinking) as well as their academic knowledge.

4. MATHEMATICAL LEARNING THAT ENHANCES CT AND CrT ABILITY:

Generally problem-oriented learning can improve the capacity of learners’ CT and CrT. Such learning is like problem solving, problem submission, or problem based learning. Especially for learning to enhance creative thinking that can be employed to critical
thinking is the mathematical learning model which is oriented towards solving and submitting mathematical problems as the focus of learning and emphasizes mentally active learning with the objective of advancing the ability to think creatively.

4.1 Learning Model Based on Problem Posing and Problem Solving (LM-PPPS):

This problem solving and Posing learning model is based on five main theories, namely: Piaget's Theory, Vygotski's Theory, Bruner's Theory, Theories about Solving and Presenting Problems, and Theories of Creative Thinking.

Piaget's theory explains that knowledge comes from the adaptation of individuals to their environment. Scholarly advancement happens through the active construction of the knowledge controlled by individuals. Based on this view, learning should give students the opportunity to develop their own knowledge based on knowledge derived from their adaptation to the environment. Submission of a problem gives an open door for learners to develop their own knowledge based on their knowledge.

Vygotski's theory broadly explains that the development of knowledge requires adult intervention in children's thinking [29]. Youngsters are allowed the chance to build up the "Zone of proximal Development" (ZPD), which is a zone between the genuine dimension of improvement as managed by free issue solving and the element of potential progression chose through issue solving under cautious adult guidance.

The outline of Bruner's Theory explains that intellectual development is characterized by an increase in the ability of an individual to separate responses from close and specific stimuli [30]. Based on this view, it means that solving and presenting problems can be said to be stimuli that will trigger individual intellectual development. The potentiality to solve mathematical problems and submit these problems is influenced by the environment. Mathematical problem solving and submission can be an approach to develop the ability of students to think creatively.

Problem solving is a procedure or exertion of a person to react to or defeat obstructions or deterrents when an answer or technique for answer has not been clear. In solving issues, the accompanying aptitudes are required to be acquired by learners:

- i) Empirical skills to calculate and measure.
- ii) Application skills to deal with common situation
- iii) Thinking skills to work in an unfamiliar situation.

The reasons for teaching problem solving (PS) in mathematics classes are as (i) to develop cognitive skills in general, (ii) to encourage creativity in mathematics, (iii) to make a part of the mathematical application process, (iv) to persuade learners to learn mathematics.

Problem posing involves the generation of a new issue or reformulation of an issue from given circumstances or issues ([31]; [32]; [33]; [34]). The approach to presenting problems can help students develop beliefs and preferences for mathematics [35]. Therefore, presentation of problems (problem posing) occupies an important role during the time spent learning mathematics. Silver [39] explained the steps of presenting problems in three different forms of mathematical cognitive activity, namely: (i) pre-solution presentation in which a student makes a question of the situation held, (ii) within solution presentation in which a student reformulate the question, (iii) post solution presentation in which students modify the condition of the problem that has been completed to make a new question.

4.2 Learning syntax of LM-PPPS:

Table 1: Learning syntax of LM-PPPS

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<th>Sl. No.</th>
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| 1      | Conveying objectives and getting ready learners | i) Clarify objectives.  
|        |                                            | ii) Material preconditions.  
|        |                                            | iii) Motivate learners, and  
|        |                                            | iv) Relate topic to the setting of regular day to day life.                     |
| 2      | Orientating learners to issues and lineup them to learn. | i) Give issues that are suitable to the youngster's improvement level to be understood  
|        |                                            | ii) Ask learners to submit issues dependent on data or beginning issues  
|        |                                            | iii) Request that learners work in groups or separately and direct them to help and share with assembled individuals or different companions.  
| 3      | Guide settlement independently or group   | Instructors manage and direct students in a effective and proficient way.       |
| 4      | Display the results of a solution to the system | Teachers help learners in arranging and establishing up a group or a student in displaying the aftereffects of their assignments. |
| 5      | Check understanding and give feedback as an assessment. | Look at learners' capacities and give input to apply issues learned in a further developed material and in the real setting of everyday issues. |

A number of researchers ([36];[37]; [38]) who advocates for issue posing regularly contend that involvement with mathematical issue presenting can advance learners' commitment in real mathematical action; enable them to experience numerous issues, strategies and arrangements, and advance learners' creativeness – a disposition to search for new issues, exchange techniques, and novel arrangements. At the point when issue presenting has been efficiently fused into learners' mathematics guidance, notwithstanding something as straightforward as having students produce story issues, the detailed outcomes have commonly been
sue, including a beneficial outcome on learners’ critical thinking accomplishment and additionally their demeanors toward mathematics. According to Silver [32] engaging students in problem posing can foster their mathematical creativity.

Learners’ Critical Thinking skills are improved by issue posing [35]. Learners’ do their best to deliver unique thoughts amid issue posing exercises, accordingly upgrading their creativity. At that point, learners start to focus on intelligent connections and question sentence arrangements as they begin posing issues. Their critical thinking limits develop progressively proficient as they question whether solutions exist for issues they create ([32]; [40]).

Issue posing is a viable mathematical work that can help individuals to build mathematical comprehension through coordinating their current structures of knowledge [41] and it creates and reinforces learners’ critical thinking abilities [42].

5. Discussion and Conclusion

From the study we understood that some thinking skills affiliated to critical thinking are comparing, distinguishing, estimating, drawing conclusions, influencing, generalizing, specializing, classifying, grouping, sorting, predicting, validating, proving, connecting, analyzing, evaluating and making patterns.

A student is said to be able to think mathematics critically if he/she has the ability to: (i) Choose important calculations and theorems in problem solving and can be able to interpret carefully, (ii) Requires confidence to support a conclusion when he is forced to accept it, (iii) Analyze that belief and distinguish a fact from assumptions, (iv) Determine important assumptions for these conclusions, (v) Evaluate these assumptions, accept only a few and reject others, (vi) Evaluate opinions, accept or reject conclusions, (vii) Continually re-examine assumptions that have been made and believed before.

Pointer of creative thinking in mathematics are capacity to (i) define mathematical speculations concerning circumstances and logical results in mathematical circumstances; (ii) decide designs in mathematical circumstances; (iii) break into sets of arrangements in a mathematical circumstance; (iv) consider and assess bizarre numerical thoughts, to thoroughly consider potential ramifications for a mathematical circumstance; (v) detect what is absent from a given mathematical circumstance that will empower one to fill in the missing mathematical data; (vi) General mathematical issues into explicit sub issues.

Through the learning model based on problem posing and problem solving (LM-PPPS) students can understand the mathematical problem better and through problem formation exercises, students are stimulated to bring up creative ideas in solving mathematical problems.

Application of the LM-PPPS enables students to improve mathematics learning achievement. LM-PPPS stimulates students to be active in the learning process. The learning process is influenced by internal factors, one of which is student learning creativity. In decoding mathematical problems, creativity is needed to find solutions. Students with high learning creativity tend to be imaginative, have new ideas, and dare to try something new. So that when the process of learning mathematics, students with high creativity are able to link the concepts that have been obtained to obtain a better understanding. So, when faced with a mathematical problem, students with high creativity are thought to be able to decode mathematics problems better than students with moderate or low creativity. In this case, student learning creativity is thought to be a factor that influences mathematics learning achievement.

Critical and creative thinking are two sides that cannot be separated and become educational goals everywhere. Both are skills that are needed in everyday life. To be applied depending on the teacher’s ability and confidence to apply in classroom practice rooms. It doesn’t depend on difficult issues or difficult material. Simple problems can lead students to achieve that goal. The culture of learning in the classroom that needs to be changed by giving students the freedom of opinion and argumentation can create a critical attitude. By giving the opportunity to give other ideas or other strategies, although not the same as the habit, it can create creative thinking students. Then appreciating every result of wrong or right assignments can make learning fun. The hope is how all students will inherit the pleasure of mathematics teachers towards mathematics itself.

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