# Free Convection Flow of Newtonian Fluids in an Anisotropic Porous Medium

**Dr. G.Soudjada[1], Dr. S.Subbulashmi[2]**

[1]Assistant Professor, Department of Mathematics, Bharathidaan Government College for Women,
UT of Puducherry -605001
[2]Assistant Professor, Department of Mathematics, DGGA College (W), Mayiladuthurai. TN

***Abstract:*** **Convection in porous media plays an essential role in recent advancements. The application of porous media are found in different areas like Geophysics, Petroleum processes, and Air conditioning porosity. In this study, the anisotropic effects of porous medium are investigated for suitable range of parameters.**
**Free Convection flow of Newtonian liquids in an anisotropic porous medium is talked about by applying Galerkin technique. Direct strength is done for both stationary and oscillatory mode. Here the Anisotropic parameter ε increases, the critical magnetic heat Rayleigh number Nc is found to decrease. This indicated that, the system destabilizes with respect to anisotropic porous medium.**

***Keywords*: Newtonian Flow, Free Convection, Anisotropic Porous Medium, Rayleigh number, Galerkin technique.**

## I. INTRODUCTION

Free convection flow of Newtonian liquids in an anisotropic permeable medium utilizing Darcy model is concentrated on with computational routines. Galerkin technique is connected. Direct dependability examination is done for both stationary and oscillatory modes. The basic magnetic Rayleigh number is registered for different estimations of the parameters which describe the flow. It is found that the anisotropic parameter builds, the heat Rayleigh number is found to diminish. Numerical calculations are made and showed graphically.

## II. MATHEMATICAL FORMULATION

Consider an endlessly spread layer of Boussinesq ferro liquid of density 'd', of magnetic field heated from underneath is considered. The temperature at the base surface and at the upper surface are and individually. Further the framework is thought to be an anisotropic thickly stuffed permeable medium with anisotropy along the vertical course which is taken as the z-axis. The liquid is thought to be incompressible liquid having viscosity given by,

$$\mu = \mu_1 \qquad (1)$$

The controlling mathematical equations utilized are as below:
The continuity equation is

$$\nabla.\vec{q} = 0 \qquad (2)$$

The momentum equation for an incompressible Newtonian fluid with variable viscosity $\mu$ is

$$\rho_0 \frac{Dq}{Dt} = -\nabla p + \rho g + \nabla.(HB) - \frac{\mu}{k} q \qquad (3)$$

The temperature equation for an incompressible fluid which obeys the modified Fourier's law as given by Finlayson (1970) is

$$\left[\rho_0 C_{v,H} + \mu_0 H \left[\frac{\partial M}{\partial T}\right]\right] \frac{dT}{dt} + \mu_0 T \left[\frac{\partial M}{\partial T}\right] \frac{dH}{dt} = K_1 \nabla^2 T + \emptyset \qquad (4)$$

The density equation of state for Boussinesq magnetic fluid is

$$\rho = \rho_0[1 - \alpha(T - T_0)] \qquad (5)$$

Basic state is taken to be quiescent. A little perturbation has been imparted on all the dynamical variables and linear theory is utilised.
Modified Navier Stoked equations on linearization:

$$\rho_0 \frac{\partial u}{\partial t} = -\frac{\partial p}{\partial x} + \mu_0(M_0 + H_0)\frac{\partial H_1^1}{\partial z} - \frac{\mu_1}{\varepsilon k_1} u \qquad (6)$$

$$\rho_0 \frac{\partial u}{\partial t} = -\frac{\partial p}{\partial y} - \mu_0(M_0 + H_0)\frac{\partial H_2^1}{\partial z} - \frac{\mu_1}{\varepsilon k_1} v \qquad (7)$$

$$\rho_0 \frac{\partial w}{\partial t} = -\frac{\partial p}{\partial z} + \rho_0 g \alpha T^1 - \mu_0 K_2 \beta H_3^1 + \frac{\mu_0 K_2^2 \beta T^1}{(1+\chi)} + \mu_0 \ (M_0 + H_0)\frac{\partial H_3^1}{\partial z} - \mu_0 \ (M_0 + H_0)\frac{K_2 \, \beta}{(1+x)} - \frac{\mu_1}{k_2} w - \frac{\mu_1}{k_2}\delta\mu_0 \ (M_0 + H_0)w \qquad (8)$$

Further research and methods has been carried out as used by Vaidyanathan et.al. (1995) which leads to the below vertical component of momentum equation:

$$\rho_0\left[\frac{\partial\nabla^2 w}{\partial t}\right] = -\frac{\mu_1}{k_1}\frac{\partial^2 w}{\partial z^2} + \rho_0 g\alpha\nabla_1^2 T^1 - \mu_0 K_2\beta\frac{\partial}{\partial z}(\nabla_1^2\phi^1) + \frac{\mu_0 K_2^2\beta}{(1+x)}\nabla_1^2 T^1 - \frac{\mu_1}{k_2}\nabla_1^2 w \quad (9)$$

## III. NORMAL MODE ANALYSIS

The normal mode solutions of all dynamical variables can be presented as

$$f(x,y,z,t) = f(z,t)exp\{i(k_x x k_y y)\} \quad (10)$$

$$\rho_0\frac{\partial}{\partial t}\left(\frac{\partial^2}{\partial z^2} - k^2\right)w = -\frac{\mu_1}{\varepsilon k_1}\frac{\partial^2 w}{\partial z^2} - \rho_0 g\alpha k^2\theta + \frac{\mu_0 K_2\beta}{(1+x)}\left[(1+x)\frac{\partial\phi}{\partial z} - K_2\theta\right]k^2 + \frac{\mu_1}{k_2}k^2 w \quad (11)$$

Equation (3.4) is linearised and the resulting equation upon utilising $H^1 = \nabla\phi^1$ yields

$$\rho c\frac{\partial\theta}{\partial t} - \mu_0 T_0 K_2\frac{\partial}{\partial t}\left(\frac{\partial\phi}{\partial z}\right) = K_1\left[\frac{\partial^2}{\partial z^2} - k^2\right]\theta\left(\rho c\beta - \frac{\mu_0 T_0 K_2^2\beta}{(1+x)}\right)w \quad (12)$$

$$where\ \rho c = \rho C_{v,H} + \mu_0 K_2 H_0 \quad (13)$$

On fine tuning,

$$(1+x)\frac{\partial^2\theta k^2}{\partial z^2 k}\left(1+\frac{M_0}{H_0}\right)k^2\theta - K_2\frac{\partial\theta}{\partial z} = 0 \quad (14)$$

Using appropriate non-dimensional terms,

$$\left(\frac{\partial}{\partial t^*}\right)(D^2 - a^2)w^* = aR^{\frac{1}{2}}(M_1 D\phi^* - (1+M_1)T^*) + \left(\frac{-D^2}{K_1^*} + \frac{a^2}{K_2^*}\right)w^* \quad (15)$$

$$P_r\frac{\partial T^*}{\partial t^*} - P_r M_2\frac{\partial}{\partial t^*}(D\phi^*) = (D^2 - a^2)T^* + (1-M_2)aR^{1/2}w^* \quad (16)$$

$$D^2\phi^* - a^2 M_3\phi^* - DT^* = 0 \quad (17)$$

where the below non dimensional parameters are used

$$k_1^* = \frac{k_1}{d^2},\ k_2^* = \frac{k_2}{d^2},\ M_1 = \left(\frac{\mu_0 K_2^2\beta}{(1+\chi)\alpha\rho_0 g}\right),\ M_2 = \left(\frac{\mu_0 T_0 K_2^2}{\rho_0 c(1+\chi)}\right)$$

$$M_3 = \left(\frac{1+\frac{M_0}{H_0}}{(1+\chi)}\right),\ P_r = \frac{\mu c}{K_1} \quad (18)$$

**Numerical Solution using Galerkin method**

The boundary conditions for stress free boundaries are

$$w^* = D^2 w^* = T^* = D\phi^* = 0 \text{ at } z = -\frac{1}{2} \text{ and } z = \frac{1}{2} \quad (19)$$

Using Galerkin technique force series expansion for the variables are indicated as

$$w(z,t) = Aw_1(z)e^{i\sigma t}$$
$$T(z,t) = BT_1(z)e^{i\sigma t}$$
$$\phi(z,t) = C\phi_1(z)e^{i\sigma t}$$

utilizing these, the equations become

$$\left[(\sigma + \frac{1}{\varepsilon k_1})D^2 w_1(z) - \sigma a^2 w_1(z) - \left(\frac{a^2}{k_2}\right)w_1(z)\right]A + aR^{\frac{1}{2}}T_1(z)B = 0 \quad (20)$$

$$\left[-(1-M_1)aR^{\frac{1}{2}}w_1(z)\right]A + [(P_r\sigma + a^2)T_1(z) - D^2T_1(z)]B - [P_r M_2\sigma D\phi_1(z)]C = 0 \quad (21)$$

$$D^2\phi_1(z)C - a^2 M_3\phi_1(z)C - DT_1(z)B = 0 \quad (22)$$

Taking $w_1(z) = \frac{z^4}{2} - \frac{3z^2}{4} + \frac{5}{32}$

$T_1(z) = z^4 + z^2 - \frac{5}{16}$

$\phi_1(z) = \frac{z^3}{3} - \frac{z}{4}$

So as to satisfy the boundary conditions

$$\left[\left(\sigma + \frac{1}{\varepsilon k_1}\right)(-0.1214285) - (0.01230158)\left[\sigma a^2 + \left(\frac{a^2+\delta M_3 a^2}{k_2}\right)\right]\right]A + \left[aR^{\frac{1}{2}}(1+M_1)(-0.0261367)\right]B - aR^{\frac{1}{2}}M_1(-0.02023809)C = 0 \quad (23)$$

$$\left[-(1-M_1)aR^{\frac{1}{2}}(-0.025992)\right]A + [(P_r\sigma + a^2)(0.0551873) - (-0.569047619)]B - [P_r M_2\sigma(0.04285714)]C = 0 \quad (24)$$

$$(0.042857)B + [(-0.03333) + a^2 M_3(0.003373)]C = 0 \quad (25)$$

For the existence of non trivial solutions for the above equations, the determinant of the coefficients of A,B and C in equation (23), (24) is equated to 0.

## IV. STABILITY ANALYSIS

**Stationary instability**

Taking $\sigma = 0$ in the above determinant

$$R = \frac{x_6 x_7\,[x_1 + x_2]}{x_3 x_5 x_7 + (0.042857) x_4 x_5}$$

Where, $x_1 = \frac{0.121428}{\varepsilon k_1}$

$$x_2 = \frac{(0.01230158)a^2(1+\delta M_3)}{\varepsilon k_1}$$

$$x_3 = a(1 - M_1)(0.0261367)$$

$$x_4 = aM_1(0.02023809)$$

$$x_5 = (1 - M_2)a(0.025992)$$

$$x_6 = a^2(0.0551587) + (0.569047)$$

$$x_7 = -[(0.03333)a^2 M_3(0.003373)].$$

**TABLE – 1**

**Marginal stability of Ferro fluid in an anisotropic porous medium with $M_1 = 1000, M_2 = 0$ , $k_1 = 0.0001$**

| $k_1$ | $\varepsilon$ | $M_3$ | $(a_c)$ | $N_C = (RM_1)_C$ |
|---|---|---|---|---|
| 0.0001 | 10 | 1 | 5.17 | 237643 |
| | | 3 | 5.17 | 195968 |
| | | 5 | 5.17 | 188211 |
| | | 7 | 5.17 | 185308 |
| | 30 | 1 | 5.17 | 203619 |
| | | 3 | 5.17 | 167477 |
| | | 5 | 5.17 | 160434 |
| | | 7 | 5.17 | 157555 |

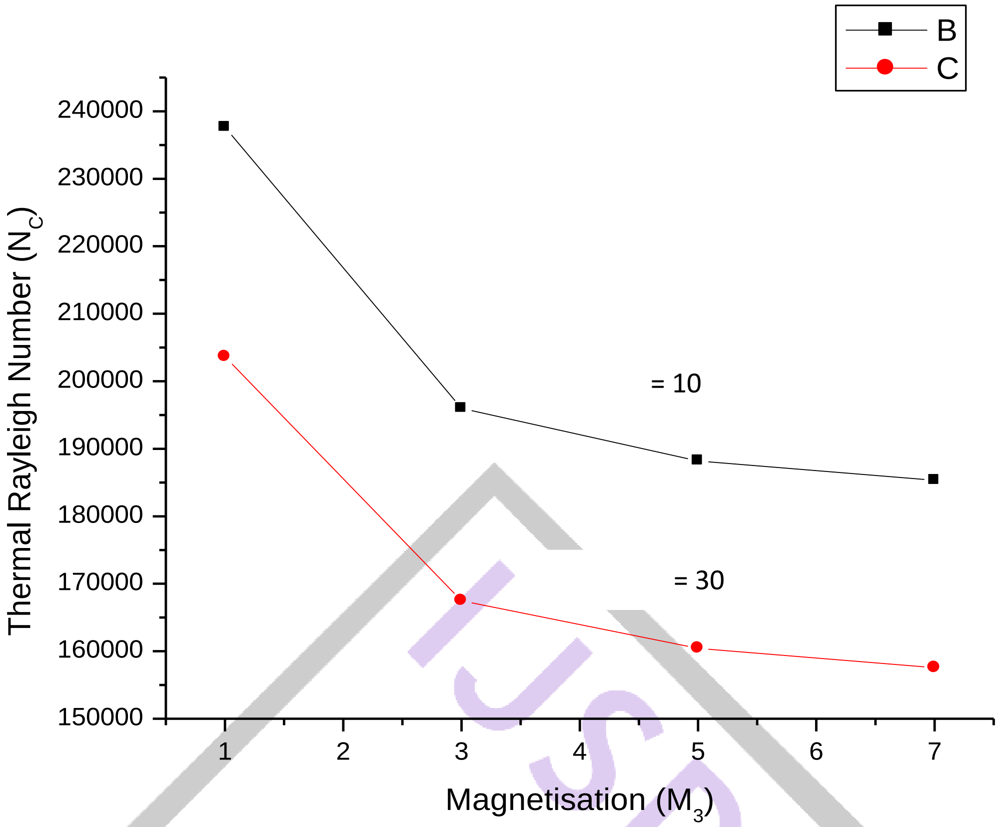


**Fig: 1: Marginal stability of Ferro fluid in an anisotropic porous medium with $M_1 = 1000, M_2 = 0$ , $k_1 = 0.0001$**

**TABLE - 2**

**Marginal stability of Ferro fluid in an anisotropic porous medium with $M_1 = 1000, M_2 = 0$ and $k_1 = 0.0001$.**

| $k_1$ | $\varepsilon$ | $M_3$ | $(a_c)$ | $N_C$= $(RM_1)_C$ |
|---|---|---|---|---|
| 0.0001 | 10 | 1 | 5.17 | 23764 |
| | | 3 | 5.17 | 19597 |
| | | 5 | 5.17 | 18821 |
| | | 7 | 5.17 | 18531 |
| | 30 | 1 | 5.17 | 20362 |
| | | 3 | 5.17 | 16748 |
| | | 5 | 5.17 | 16043 |
| | | 7 | 5.17 | 15756 |

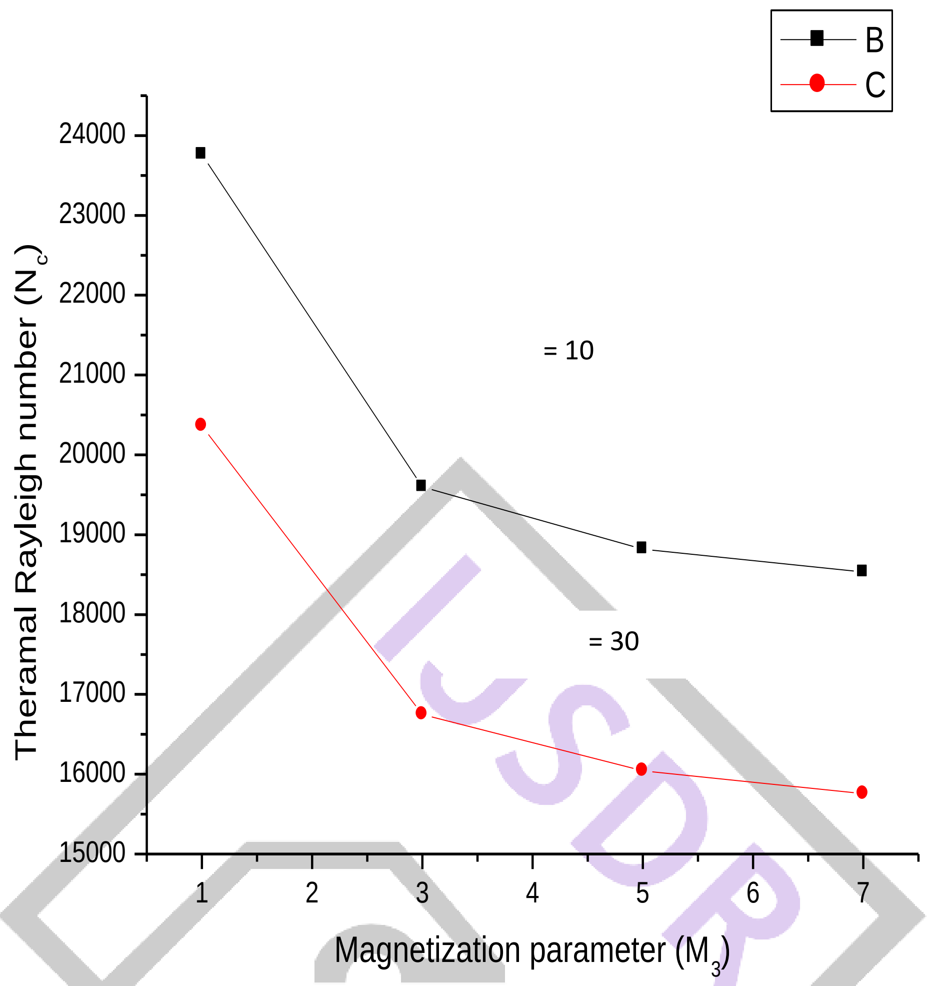


**Fig:2: Marginal stability of Ferro fluid in an anisotropic porous medium with $M_1 = 1000, M_2 = 0$ and $k_1 = 0.0001$.**

## V. RESULT AND DISCUSSION

The impact of magnetic field on free convection flow of Newtonian liquids in an anisotropic thickly stuffed permeable medium has been broke down utilizing Darcy model. The permeability value of the porous medium has been taken utilizing the qualities proposed by Walker and Homsy (1997). For these liquids M2 is accepted to have irrelevant quality and thus taken to be zero. The impact of anisotropy is considered by taking the anisotropic parameter which is the proportion of vertical to level penetrability and is fluctuated from 1 to 70 (Goel and Agrawal(1998). The porousness of the permeable medium is fluctuated from 0.0001 to 0.001. The discriminating magnetic heat Rayleigh number NC is gotten for distinctive estimations of penetrability k0, anisotropic parameter, subordinate viscosity. The normal estimation of $M_2$ is $10^{-6}$ Finlayson (1970) and henceforth it is accepted unimportant. It can be appeared taking after the investigation of Ramanathan and Suresh (2004), that oscillatory precariousness not happen for the issue under thought. Accordingly we restrain our thought to stationary instability. One can likewise see from the figures as the coefficient of magnetic field $\delta$ is expanded, the basic magnetic heat Rayleigh number $N_C$ diminishes, this would suggest that the magnetic field balances out through varieties as for magnetic field.

It is clear from the table that as the anisotropic parameter expands, the basic magnetic heat Rayleigh number $N_C$ is found to diminish. This demonstrates that the framework destabilizes. Comparative results were additionally found for diverse estimations of the porousness parameter. The increment in magnetization has a tendency to destabilize the framework. The vicinity of anisotropic thickly stuffed permeable medium destabilizes the framework. On examination with hypothetical results acquired by Ramanathan and Suresh (2004), the present computational results are observed to be completely in consent to the conceivable degree of exactness.

## VI. CONCLUSION

In this study, the impacts of porosity on the free convection flow of Newtonian liquids in an anisotropic permeable medium were explored. At the point when the anisotropic parameter builds, the heat Rayleigh number is found to diminish. This demonstrated that, the framework destabilizes as for Anisotropic permeable medium. In this manner the impact of both magnetization and additionally the vicinity of anisotropic thickly pressed permeable medium is to destabilizes the system.(for the picked values of the parameters )

**REFERENCES**

**Journal Article**

[1]. Benjan. A. K.R. Khair, "Heat and Mass transfer by natural convectional in a porous medium," International Journal of Heat and mass Transfer 28(5) (1985) 909 -918.

[2].Berkovskii, B.M. and Bashtovoi, V.G. (1971)."Gravitational convection in ferromagnetic liquid." Magnitnaya Gidrodinamika, Vol. 7, pp. 24–28.

[3].Deiwert.G.S. 'Numerial simulation of high Reynolds number transonic flows." AIAA journal, Vol. 13, pp, 1354-1360, 1975.

[4]. Dharmaadhikari, R.V and Kale, D.D. 'Flow of Non-Newtonian Fluids Through pours Media," Chem. Eng. Sci., Vol.40,No.3pp.527-529, 1985.

[5].Han – Taw Chen, Cha'o – Kuang, Chen 'Free convection flow of non-Newtonian fluids along a vertical plate embedded in a porous medium' Journal of Heat and Mass Transfer, 39(2) (2001), 1 – 106.

[6]Kim.E, 'Natural convection along a wavy vertical plate to non-Newtonian fluids', In: Heat and Mass Transfer, 40(13) (1997), PP. 3069 – 3078.

[7]Lalas, D.P. and Carmi, S. (1971). 'Thermoconvective stability of ferrofluids' .Phys. Fluids, Vol. 14, pp. 436–438.

[8]Sekar. R. Vaidanathan. G.Ramanathan. A, 'Ferro Convectioon in an anisotropic porous medium'. Int .J .Engng. Sci, Vol 34(4) (1996), PP. 399-405.

[9].Sekhar, G.N. (1991). 'Convection in magnetic fluids with internal heat generation' .Trans. ASME : J. Heat Trans., Vol. 113, 122–127.

[10].Sparrow, E.M., Goldstein, R.J. and Jonsson, V.K. (1964). 'Thermal instability in a horizontal fluid layer: effect of boundary conditions and nonlinear temperature profile'. J. Fluid Mech., Vol. 18, pp. 513–528.

**Book**

[1]. Anderson.D.A, J.C. Tannehill and R.H. Pletcher, 'computational Fuild Mechanics and Heat Transfer," Hemisphere Publishing Corporation', New York, 1984.

[2].Biswas," Mac and simple algorithms," In: 'Computational Fluid Flow and Heat Transfer' Ed.K .Muralidhar and T.Sundarajan, 2nd ed., Narosa Publishing House, New Delhi, 2003.

[3].Benjan. A., 'Convection Heat Transfer," A Wiley-interscience Publication, New York, 1984.

[4] Courant. R and D.Hilbert ," 'Methods of Mathematical Physics," Vol.II, Wiley, New York, 1953.

[5]Emmons. H.W. "The numerical solution of compressible fluid flow problems," NACA TN 932, 1999.

[6]Golub. G.H, and C.F. Van Loan," Matrix Computations ," 2nd Edn., John Hopkins University Press, Baltimore, 1989.

[7]Hall M.G. 'Cell Vertex Multigrid Scheme for on Numerical Methods for Fluid Dynamics', University of Reading UK, 1985, IBA Conf. Series Oxford University Press, 1985.

[8]Sengupta. T.K. 2004. 'Foundation of computational fluid dynamics' (Hyderabad University press)

[9]Shliomis, M.I. (1973). 'Convective instability of a ferrofluid'. Fluid Dynamics - Sov. Res., Vol. 6, pp. 957–962.

[10]Turner, J.S. (1973). Buoyancy effects in fluids. Cambridge University Press, Cambridge.