# Study of Ziegler-Nichols and Lambda (λ) Tuning methods for Tuning of PID Controllers

[1]Gyanesh Kumar Pathak, [2]Megha Gautam, [3]Sharad Tripathi

[1-3]Student
[1-3]Control &Instrumentation
[1]Indian Institute of Technology, Kanpur, India
[2]Rajasthan Technical University, Kota,India
[3]Indian Institute of Engineering Science and Technology, Kolkata,India

*Abstract*—**The ease implementation, the simple mechanism and robustness of PID controllers has attracted the use of these controllers in the process industries. There are numerous tuning techniques are available for tuning of PID controllers. In this research work, in the first part basically a first-order plus dead time (FOPDT) model is developed and tunning of PID controller is done by using the Ziegler -Nichols method and then in the second part tunning is done using Lambda (λ) tuning method. The comparative and quantitative analysis is also done here for these two methods by taking some suitable examples. Here a quantitative and comparative analysis of the original higher order systems and the reduced FOPDT systems (modelled by Ziegler-Nicholas), is done to compare the various parameters like, settling time, rise time, transient time, peak overshoot, peak time etc. and then the unit step response and overall response for both the systems has been plotted and analyzed by using both the techniques.**

***Index Terms-FOPDT, PID controller, Ziegler Nichols, Lambda-Tuning, settling time, tunning, rise time, peak overshoot etc.***

## I.INTRODUCTION:

PID controller as name suggests is a combination of 3 gain parameters, namely P-Proportional, I-Integral, Derivative gain parameters. These three elements control all the processes in the process industries [1]. A PID controller is an instrument/component/controller used in industrial control applications to regulate temperature, flow, pressure, speed and other process variables. It is also known as three-term controller.The PID controller as a whole change the dc gain, increases the order of the system, reduces the steady state error and improve the transient response of the system.

## II.OBJECTIVE AND MOTIVATION FOR TUNING OF PID CONTROLLERS:

Since the last few decades, a lot of research work has been done because of the growing popularity into process control industries. In very early nineties century Ziegler-Nichols [2] gave tuning procedure for PID controller, after that a large number of methods have been used and developed to tune PID controller for getting good results. Because of high requirement of best tuning procedures which tune the plant in such a way that could provide optimized solution, many tuning methods have been developed so far in which some methods give better response for speed of the system and some show good response for stability. Thus, maximum methods are application oriented. Some important methods of tunning of PID controller are: Tuning method of PID controllers for Desired Damping coefficient [3], Tuning of PID controller by D partition rule [4], Tuning of PID controller using immune algorithm [5], etc.Several methods have been developed and used to find out PID controller parameters for SISO (single input single output) and multiple input multiple output (MIMO) systems [6-7], but maximum of these tunning methods are developed for any specific applications [8-11] hence can be used only for the applications.

**III.Tuning of PID Controller for FOPDT process: Here** the tunning of PID controller is done by using Ziegler-Nichols's and Lambda- tunning method. In these methods the higher order system with delayed input is reduced to a FOPDT process by using Ziegler-Nichols's or Lambda tuning technique and consequently the tunning is done for PID controller.

3.1 Ziegler-Nichols's Method:This is the oldest and simplest method used for Tunning of FOPDT process. In this method firstly the higher order system is modeled into a FOPDT process and then the 3 parameters of PID controller are calculated by using some empirical formulas [2].

Let us consider the first-order plus delay time process with transfer function,

$$G(s)=\frac{ke^{-st_0}}{1+Ts} \qquad (1.1)$$

There are three unknown parameters in FOPDT process.These unknowns has been calculated using the stated method. The MATLAB tool is being used here to model a higher order system into FOPDT system. To model a higher order system into a FOPDT process, the following steps are followed-

**(i)** First the step response of the given higher order system is plotted using MATLAB tool.

**(ii)** A tangent to the curve (step response) is drawn and its cuts x-axis. The distance from origin to the point of intersection of x-axis and the tangent is measured, it is the time delay parameter $t_0$ of FOPDT process.

**(**iii**)**A perpendicular line to x-axis and touching the tangent (at starting point of tangent) is drawn and measure the value from origin to the point where its cuts x-axis, let this distance be $T_1$. The time constant T of the FOPDT process is given by Eq.(1.2),

$$T= T_1 - t_0 \qquad (1.2)$$

**(**iv**)**The dc gain k is calculated using Eq. (1.3),

$$G(0)=k \qquad (1.3)$$

**(**v**)** The controller parameters are calculated using the following empirical formulae [2]**-**

Experimentally The tuned gain parameter 'a' [2], related to proportional gain parameter $K_p$ is given by,

$$\frac{kt_0}{T} = a \qquad (1.4)$$

And the value of $K_p$ is given by Eq. (1.5),

$$K_p = \frac{1.2}{a} \qquad (1.5)$$

Now let $\tau_i$ is the tuned time constant parameter related to integral gain parameter $K_I$and it is given by [2] Eq. (1.6),

$$\tau_i = 2t_0 \qquad (1.6)$$

And the integral gain parameter $K_I$ is given by [2] the Eq.(1.7),

$$K_I = \frac{K_p}{\tau_i} \qquad (1.7)$$

Now let $\tau_d$ is the tuned time constant parameter related to derivative gain parameter $K_D$ and it is given by [2] Eq. (1.8),

$$\tau_d = \frac{t_0}{2} \qquad (1.8)$$

And the derivative gain parameter $K_D$ is given by [2] the Eq. (1.9),

$$K_D=K_p\tau_d \qquad (1.9)$$

3.2 Lambda (λ) Tuning Method:This is one of the simplest methods used for tuning of FOPDT process. In this method firstly the higher order system is modeled into a FOPDT process and then the three parameters of PID controller are calculated by using some empirical formulas [12]. To model a higher order system into a FOPDT process, same steps and procedure are used as in Ziegler-Nichols's method**.**

Thetuning parameters of PID controller are calculated using following empirical formulas [12]**-**

The value of $K_p$ is given by [12] Eq. (1.10),

$$K_p =\frac{T+\frac{t_0}{2}}{k(\frac{t_0}{2}+\lambda)} \qquad (1.10)$$

Where $\lambda$ is a tuning parameter and its value is taken as integer multiple of time constant T, to plot the step response of combined closed loop system.

The integral gain parameter $K_I$ is given by [12] the Eq. (1.11),

$$K_I=T+\frac{t_0}{2} \qquad (1.11)$$

And the derivative gain parameter $K_D$ is given by [12] the Eq. (1.12),

$$K_D=\frac{Tt_0}{2T+t_0} \qquad (1.12)$$

## IV RESULTS AND DISCUSSION:

In this section some examples have been taken and will demonstrate how to use these both methods Ziegler-Nichols's and Lambda Tuning Method. Also, a comparative and quantitative analysis is done for original higher order system (for which PID tunning is done) and the reduced first order system in terms of different parameters like, settling time, peak time, stability, overshoot and undershoot etc., for both the techniques. These quantitative parameters are shown in table for a comparison. The simulation results of step response for both the system by using both methods are also plotted for a comparative study and a better understanding.

**4.1 Ziegler-Nichols's Method for FOPDT process:** In this section some examples are taken and the demonstration of Ziegler-Nichols's method for tunning of FOPDT model is done. Firstly, the original higher order system is modeled into a FOPDT process and then tunning is done by Ziegler-Nichols's technique.

**Example 4.1:** Let us consider a non-oscillatory system,

$$G(s) = \frac{1}{(s+1)^2(s+2)} e^{-0.3s}$$

DC gain k for modeled FOPDT process, k =G(0)=0.5

Fig. (1.1) shows the step response and the demonstration of the Ziegler-Nichols's method for the given higher order system,

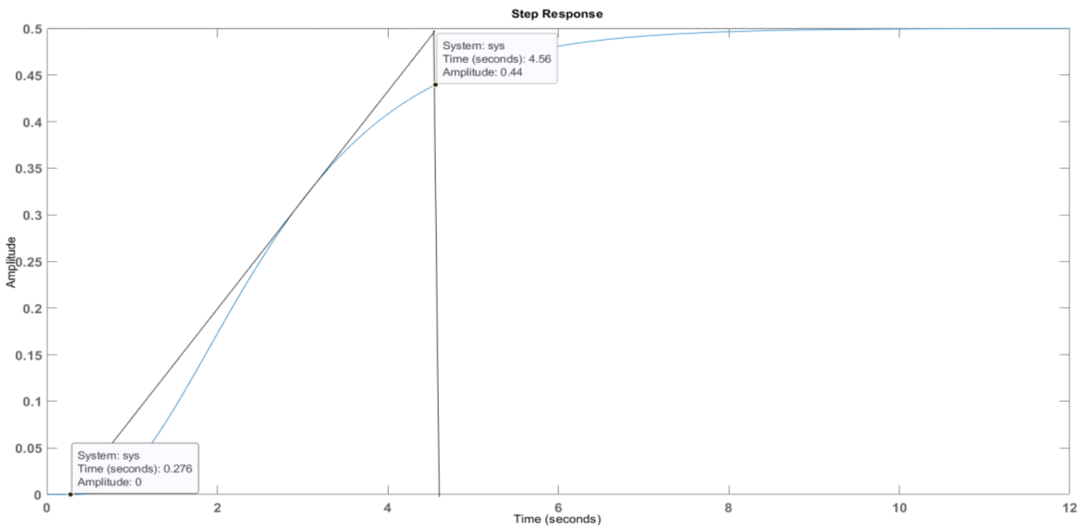


Fig. (1.1): Unit step response of the system $G(s) = \frac{1}{(s+1)^2(s+2)} e^{-0.3s}$

From Fig. (1.1) It is clear that, the time delay, $t_0$=0.276, The residential time, $T_1$=4.56, hence the time constant, T=$T_1$-$t_0$=4.284.

The modeled or reduced FOPDT system is given by Eq. (1.13),

$$G_1(s) = k\frac{e^{-st_0}}{(1+Ts)} = 0.5\frac{e^{-0.276s}}{(1+4.284s)} \qquad (1.13)$$

Now value of a can be calculated by Eq. (1.8), a=0.032

Value of $K_p$ is calculated by Eq. (1.9), $K_p$ =37.25

Value of $\tau_i$ is calculated by Eq. (1.10), $\tau_i$=0.552

Value of $K_I$ is calculated by Eq. (1.11), $K_I$ =67.02

Value of $\tau_d$ is calculated by Eq. (1.12), $\tau_d$=0.138

Value of $K_D$ is calculated by Eq. (1.13), $K_D$ =5.140

Now the Required PID controller is given by Eq. (1.14),

$$K(s)=37.25+\frac{67.02}{s}+5.140s \tag{1.14}$$

Cascading this controller with FOPDT process in forward path, the combined system is let denoted by C(s). Now the step response for this combined system C(s) with unity feedback is plotted in Fig. (1.2),

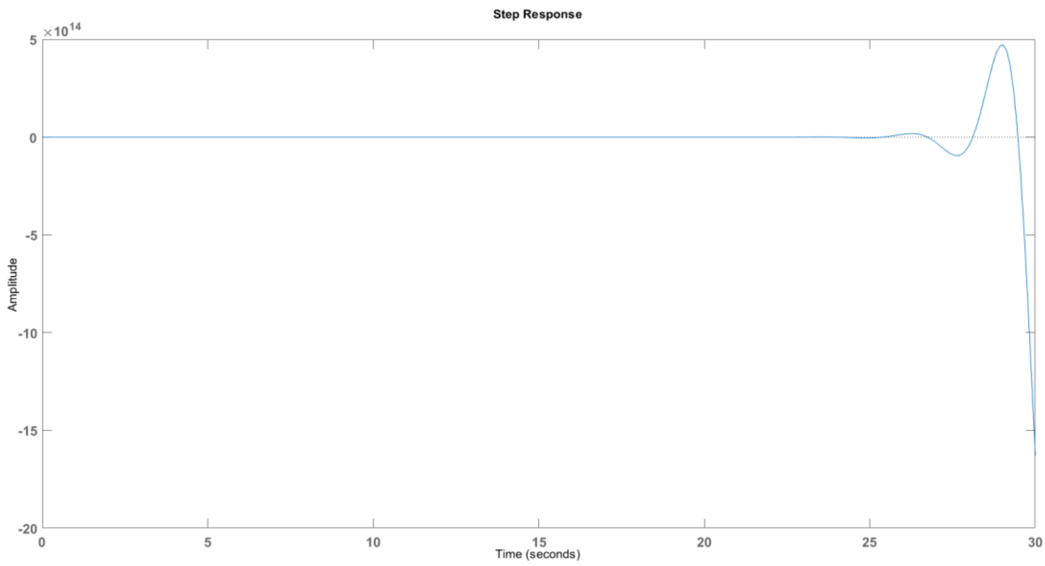


Fig. (1.2): Unit step response of the combined system $C(s)$

Clearly Fig. (1.2) shows the undesirable and impractical results, hence we shifted towards root locus technique for tunning of PID controllers.

Fig. (1.3) shows the step response of reduced or modeled FOPDT process,

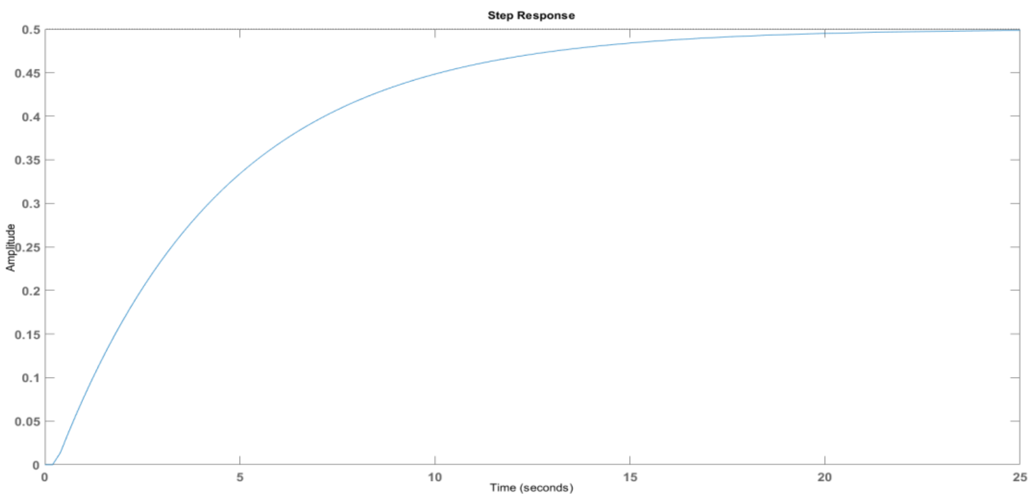


Fig. (1.3): Unit step response of the Reduced FOPDT system $G_1(s)$=0.5 $\frac{e^{-0.276s}}{(1+4.284s)}$

**Quantitative and comparative analysis**

The comparative analysis of both systems is shown below in table 1.1,

Table 1.1: Response values for unit step input

| Parameters | Response values for Higher order system G(s) | Response values for reduced FOPDT system $G_1$ (s) |
|---|---|---|
| Rise time | 3.6132 | 9.413 |
| Transient Time | 6.7730 | 17.03 |
| Settling Time | 6.7730 | 17.03 |
| Overshoot (%) | 11 | 0 |
| Undershoot (%) | 0 | 0 |
| Peak | 0.500 | 0.499 |
| Peak time | 12.8484 | 33.34 |

From the above table it is clear that the stability of the reduced 1st order system is decreases but the percentage overshoot is decreases (advantage). Also, for reduced system the rise time increases hence the transient response is not improved by use of this method.

**Example 1.2:** Let us consider a system,

$$G(s) = \frac{1}{(s+2)^2(s+1)} e^{-0.5s}$$

Dc gain k for modeled FOPDT process, k =G(0)=0.25

Fig. (1.4) shows the step response and the demonstration of the Ziegler-Nichols's method for the given higher order system,

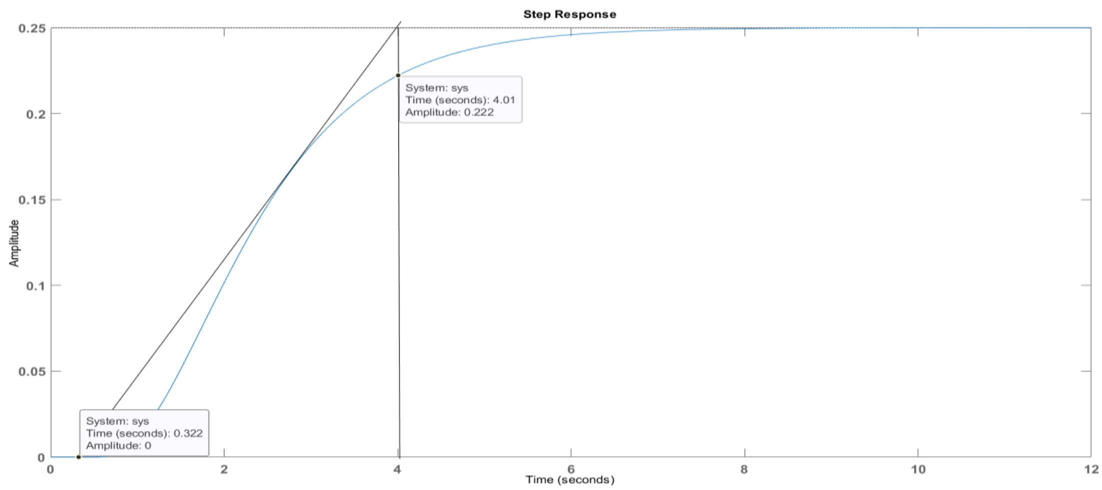


Fig. (1.4): Unit step response of the system $G(s) = \frac{1}{(s+2)^2(s+1)} e^{-0.5s}$

From Fig. (1.4) It is clear that, the time delay, $t_0$=0.322, The residential time, $T_1$=4.01, hence the time constant, T=$T_1$-$t_0$=3.688.

The modeled or reduced FOPDT system is given by Eq. (1.15),

$$G_1(s) = k\frac{e^{-st_0}}{(1+Ts)}=0.25\frac{e^{-0.322s}}{(1+3.688s)} \quad (1.15)$$

Now a can be calculated by Eq.(1.8), a=0.021

Value of $K_p$ is calculated by Eq.(1.9), $K_p$ =54.97

Value of $\tau_i$ is calculated by Eq.(1.10), $\tau_i$=0.644

Value of $K_I$ is calculated by Eq.(1.11), $K_I$ =85.36

Value of $\tau_d$ is calculated by Eq.(1.12), $\tau_d$=0.161

Value of $K_D$ is calculated by Eq.(1.13), $K_D$ =8.85

Now the Required PID controller is given by Eq. (1.16),

$$K(s)=54.97+\frac{85.36}{s}+8.85s \tag{1.16}$$

Fig. (1.5) shows the step response of reduced or modeled FOPDT process,

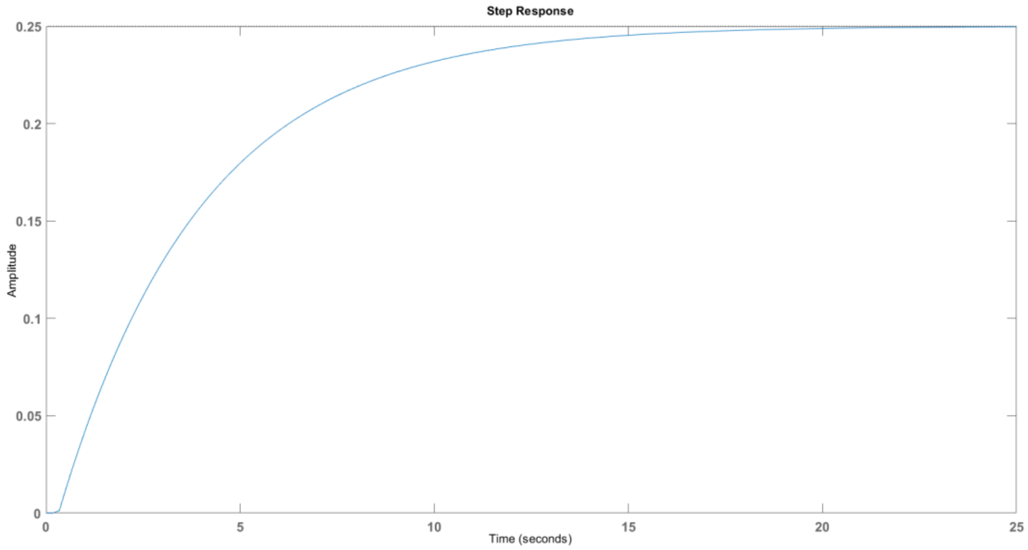


Fig. (1.5): Unit step response of the Reduced FOPDT system $G_1(s)$=0.25 $\frac{e^{-0.322s}}{(1+3.688s)}$

**Quantitative and comparative analysis:**

The comparative analysis of both systems is shown below in table 1.2,

Table 1.2: Response values for unit step input

| **Parameters** | **Response values for Higher order system G(s)** | **Response values for reduced FOPDT system $G_1$ (s)** |
|---|---|---|
| Rise time | 2.9139 | 8.10 |
| Transient Time | 5.7812 | 14.75 |
| Settling Time | 5.7812 | 14.75 |
| Overshoot (%) | 10 | 0 |
| Undershoot (%) | 0 | 0 |
| Peak | 0.250 | 0.249 |
| Peak time | 13.7695 | 28.70 |

From the above table it is clear that the stability of the reduced 1st order system is decreases but the percentage overshoot is decreases (advantage). Also, for reduced system the rise time increases hence the transient response is not improved by use of this method. The peak value remained same.

**Example 1.3:** Let us consider a higher order system,

$$G(s) = \frac{1}{(s^2+s+1)(s+1)}e^{-0.3s}$$

Dc gain k for modeled FOPDT process, k =G(0)=1

Fig. (1.6) shows the step response and the demonstration of the Ziegler-Nichols's method for the given higher order system,

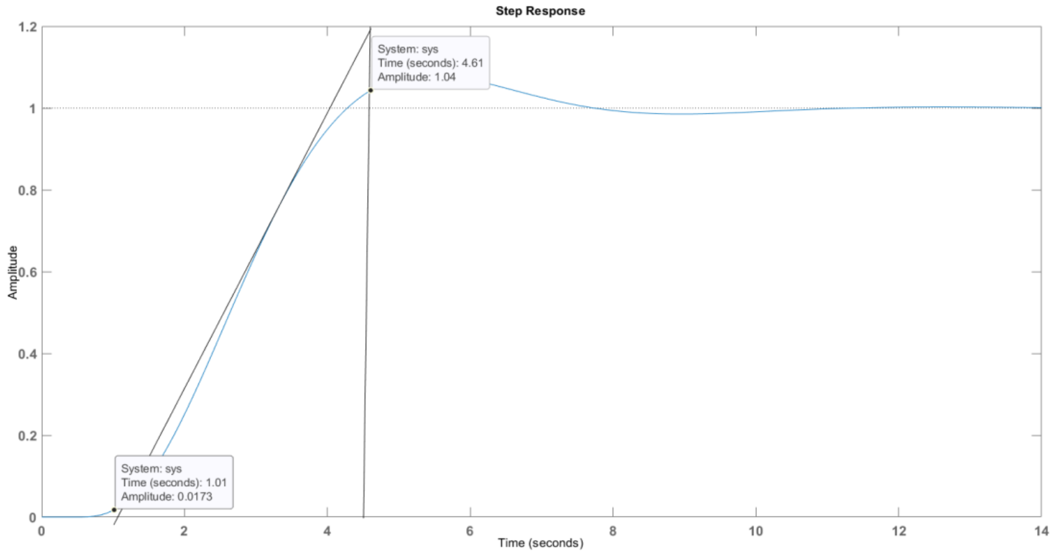


Fig. (1.6): Unit step response of the system $G(s) = \frac{1}{(s^2+s+1)(s+1)}e^{-0.3s}$

From Fig. (1.6) It is clear that, the time delay**,** $t_0$=1.01, The residential time, $T_1$=4.61, hence the time constant, T=$T_1$-$t_0$=3.60.

The modeled or reduced FOPDT system is given by Eq. (1.17),

$G_1(s) = k\frac{e^{-st_0}}{(1+Ts)}$= $\frac{e^{-1.01s}}{(1+3.60s)}$(1.17)

Now a can be calculated by Eq.(1.8), a=0.28

Value of $K_p$ is calculated by Eq.(1.9), $K_p$ =4.27

Value of $\tau_i$is calculated by Eq.(1.10), $\tau_i$=2.02

Value of $K_I$ is calculated by Eq.(1.11), $K_I$ =2.11

Value of $\tau_d$is calculated by Eq.(1.12), $\tau_d$=0.505

Value of $K_D$ is calculated by Eq.(1.13), $K_D$ =2.156

Now the Required PID controller is given by Eq. (1.18),

K(s)=4027+$\frac{2.11}{s} + 2.156s$ (1.18)

Fig. (1.7) shows the step response of reduced or modeled FOPDT process,

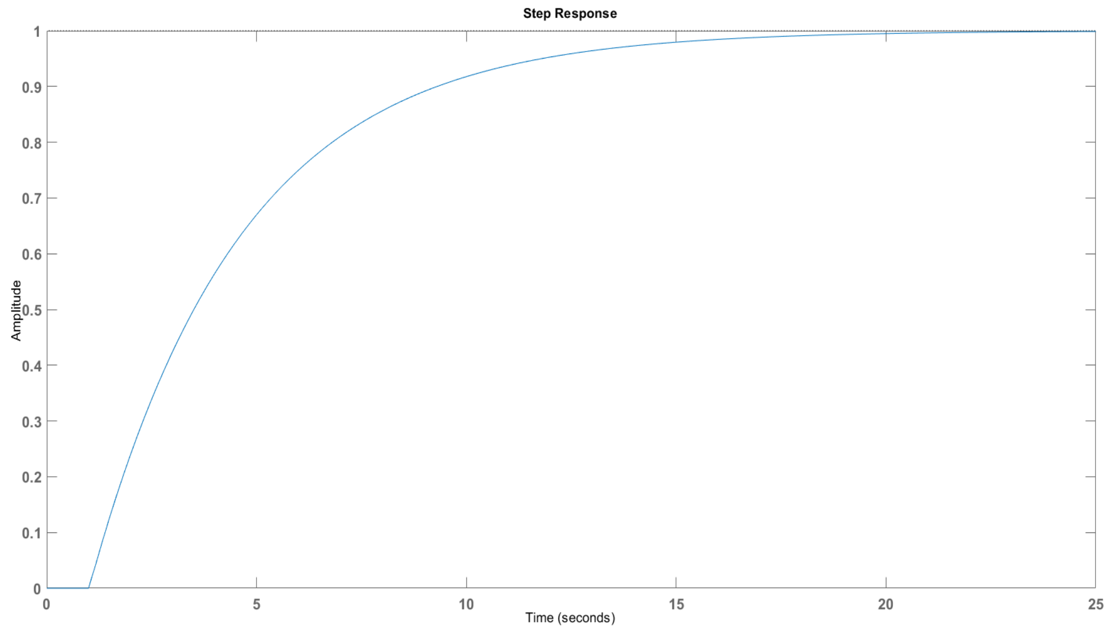


Fig. (1.7): Unit step response of the Reduced FOPDT system $G_1(s)=\frac{e^{-1.01s}}{(1+3.60s)}$

**Quantitative and comparative analysis**

The comparative analysis of both system is shown below in table 1.3,

Table 1.3: Response values for unit step input

| Parameters | Response values for Higher order system G(s) | Response values for reduced FOPDT system $G_1$ (s) |
|---|---|---|
| Rise time | 2.2910 | 7.90 |
| Transient Time | 6.9379 | 15.09 |
| Settling Time | 6.9379 | 15.09 |
| Overshoot (%) | 8.1432 | 0 |
| Undershoot (%) | 0 | 0 |
| Peak | 1.081 | 0.99 |
| Peak time | 5.249 | 31.33 |

From the above table it is clear that the stability of the reduced 1st order system is decreases but the percentage overshoot is decreases (advantage). Also, for reduced system the rise time increases hence the transient response is not improved by use of this method. The peak value remained same.

**4.2 λ-Tuning Method for FOPDT systems:**

In this section same examples are taken and the demonstration of λ-tuning Methodfor tuning of FOPDT model is done. Firstly, the original higher order system is modeled into a FOPDT system by Ziegler-Nichols's technique (that is already discussed in previous section) and then tuning is done by λ-tuning method. Here three values of λ is taken for tuning that are, λ1=T, λ2=2T and λ3=3T (i.e., the integer multiple of time constant T). Same examples are being taken here for the comparative analysis .

**Example 1.4:** Let us consider a non-oscillatory system,

$$G(s) = \frac{1}{(s+1)^2(s+2)}e^{-0.3s}$$

DC gain k for modeled FOPDT process, k =G(0)=0.5

From, Fig. (1.1) It is clear that, the time delay, $t_0$=0.276, The residential time, $T_1$=4.56, hence the time constant, T=$T_1$-$t_0$=4.284.

The modeled or reduced FOPDT system is given by Eq. (1.13),

$$G_1(s) = k\frac{e^{-st_0}}{(1+Ts)} = 0.5\frac{e^{-0.276s}}{(1+4.284s)} \qquad (1.13)$$

The value of $K_p$ is calculated by Eq. (1.10), for three different values of λ,

$K_p = 2$, 1.015, and 0.68 for λ1=T, λ2=2T and λ3=3T respectively.

The value of $K_I$ is calculated by Eq. (1.11), $K_I$=4.422

The value of $K_D$ is calculated by Eq. (1.12), $K_D$=0.133

The controller gain transfer function is given by Eq. (1.19), Eq.(1.20) and Eq.(1.21) respectively,

$$K(s) = 2+\frac{4.422}{s} + 0.133s \text{ for } \lambda 1=4.284 \qquad (1.19)$$

$$K(s) = 1.015+\frac{4.422}{s} + 0.133s \text{ for } \lambda 2=8.568 \qquad (1.20)$$

$$K(s) = 0.68+\frac{4.422}{s} + 0.133s \text{ for } \lambda 3=12.852 \qquad (1.21)$$

Fig. (1.8), shows the overall closed loop step response (for unity feedback system) of reduced or modeled FOPDT process for different values of λ.

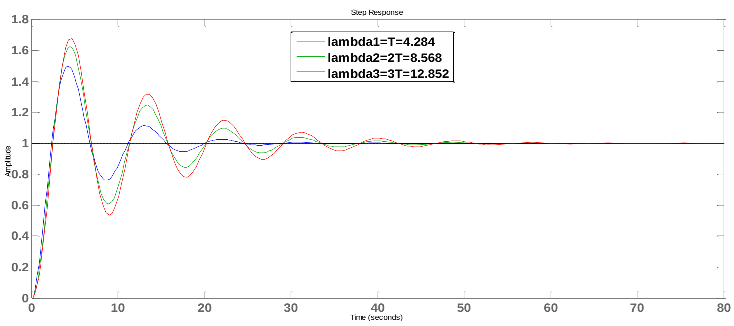


Fig. (1.8), Overall closed loop step response of $G_1(s) = 0.5\frac{e^{-0.276s}}{(1+4.284s)}$

**Example 1.5:** Consider a non-oscillatory system,

$$G(s) = \frac{1}{(s+2)^2(s+1)}e^{-0.5s}$$

DC gain k for modeled FOPDT process, k =G(0)=0.25

From, Fig. (1.4) It is clear that, the time delay, $t_0$=0.322, the residential time, $T_1$=4.56, hence the time constant, T=$T_1$-$t_0$=3.688.

The modeled or reduced FOPDT system is given by Eq. (1.22),

$$G_1(s) = k\frac{e^{-st_0}}{(1+Ts)} = 0.25\frac{e^{-0.322s}}{(1+3.688s)} \qquad (1.22)$$

The value of $K_p$ is calculated by Eq. (3.10), for three different values of λ,

$K_p = 4, 2.042$, and 1.371 for λ1=T, λ2=2T and λ3=3T respectively.

The value of $K_I$ is calculated by Eq. (1.11), $K_I$=3.849

The value of $K_D$ is calculated by Eq. (1.12), $K_D$=0.154

The controller gain transfer function is given by Eq. (1.23), Eq. (1.24) and Eq.(1.25) respectively,

$$K(s) = 4 + \frac{3.849}{s} + 0.154s \text{ for } \lambda1 = 3.688 \quad (1.23)$$

$$K(s) = 2.042 + \frac{3.849}{s} + 0.154s \text{ for } \lambda2 = 7.376 \quad (1.24)$$

$$K(s) = 1.371 + \frac{3.849}{s} + 0.154s \text{ for } \lambda3 = 11.064 \quad (1.25)$$

Fig. (1.9), shows the overall closed loop response (for unity feedback system) of reduced or modeled FOPDT process for different values of λ.

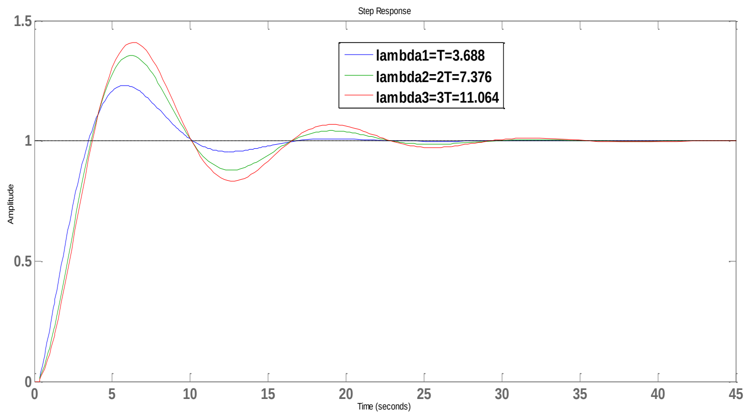


Fig. (1.9), Overall closed loop step response of $G_1(s)= 0.25\,\frac{e^{-0.322s}}{(1+3.688s)}$

**Example 1.6:** Let us consider a non-oscillatory system,

$$G(s) = \frac{1}{(s^2+s+1)(s+1)}e^{-0.3s}$$

DC gain k for modeled FOPDT process, k =G(0)=1

From, Fig. (1.6), it is clear that, the time delay**,** $t_0$=1.01, the residential time, $T_1$=4.61, hence the time constant, T=$T_1$-$t_0$=3.60

The modeled or reduced FOPDT system is given by Eq. (4.15),

$$G_1(s) = k\,\frac{e^{-st_0}}{(1+Ts)} = \frac{e^{1.01s}}{(1+3.6s)} \quad (1.26)$$

The value of $K_p$ is calculated by Eq. (1.10), for three different values of λ,

$K_p = 1, 0.532$, and 0.363 for λ1=T, λ2=2T and λ3=3T respectively.

The value of $K_I$ is calculated by Eq. (1.11), $K_I$=4.105

The value of $K_D$ is calculated by Eq. (1.12), $K_D$=0.235

The controller gain transfer function is given by Eq. (1.27), Eq.(1.28) and Eq.(1.29) respectively,

$$K(s)=1+\frac{4.105}{s}+0.235s \text{ for } \lambda1=T=3.60 \quad (1.27)$$

$$K(s)=0.532+\frac{4.105}{s}+0.235s \text{ for } \lambda2=2T=7.20 \quad (1.28)$$

$$K(s)=0.363+\frac{4.105}{s}+0.235s \text{ for } \lambda3=3T=11.8 \quad (1.29)$$

Fig. (1.10) shows the overall closed loop response (for unity feedback system) of reduced or modeled FOPDT process for different values of λ.

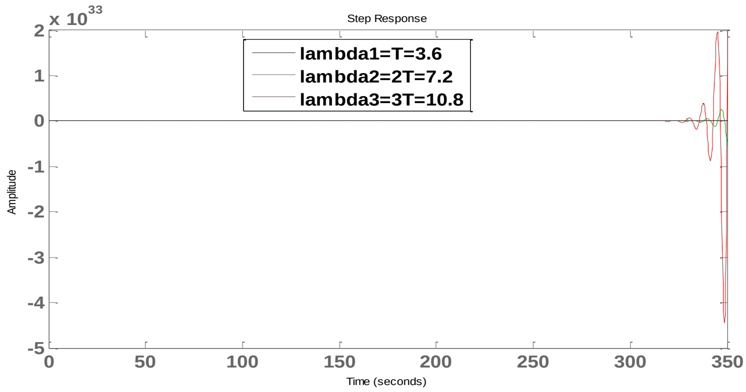


Fig. (1.10), Overall closed loop response of $G_1(s)=\frac{e^{-1.01s}}{(1+3.6s)}$

## VII ACKNOWLEDGEMENT:

In this research work, the focus has been given towards the applications of tuning of PID controllers for the higher order system to reduce the system order. In today's process control industries, the major attention is given to the development of those controllers which are most suitable and the best for application-based processes. There are several methods of tuning of PID controllers for FOPDT (First order plus dead time) systems, but there are some drawbacks of FOPDT tunning is that it is unable to generate peaks for monotonic systems.

In this work the first part is carried out for a FOPDT process using Ziegler-Nichols Technique. In this method the higher order system is reduced into a FOPDT system and then tunning of PID controller is done. Then a comparative analysis is also being done for higher order system and reduced FOPDT system. In the second part tunning is done by Lambda tuning method. The performance comparison (in terms of step response and overall response) for both the methods has been also carried out.

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