# NUMERICAL ANALYSIS OF BLOOD FLOW THROUGH AN CIRCULAR RIGID TUBE WITH MILD STENOSIS

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*Abstract-* **In this paper we study a mathematical model on unsteady blood flow through rigid tube in the presence of mild stenosis has been studied numerically by finite difference method. The effects on the pressure gradient, wall shear stress are obtained and the velocity profiles have been investigated are obtained and shown graphically.**

*Key Word:* **Unsteady blood flow, Rigid tube, Pressure gradient, Wall shear stress.**

## INTRODUCTION

The blood flow problem a stenosed artery are of increasing interests to the researcher due to its physiological and clinical importance. The investigation of pulsatile flow in circular tubes is understanding and predicting blood flow in large arteries are of considerable importance in many cardiovascular diseases particularly atherosclerosis. The normal flow of blood is disturbed due to some abnormal growth like stenosis in the lumen of the artery. The actual reasons for formation of stenosis is not known, but its effect over the flow characteristics has been studied by many research workers. Bhardwaj et.al. [1] the considerable effect of a magnetic field on blood flow through on indented tube in the presence of erythrocytes. Shukla et. al. [2] considered the effects of stenosis on blood flow through the artery with mild stenosis.

Halder and Dey [3] solved the problem Effects of erythrocytes on the flow characteristics of blood in on indented tube. The response of blood flow through an artery under stenotic conditions has been attempted by
Chakrabarty et.al. [4] We studied Blood flow through an artery under stenotic conditions. Halder and Ghosh [5] investigated the Effects of a magnetic field on blood flow through on indented tube in the presence of erythrocytes. Sanyal et. al. [6] considered the Unsteady arterial blood flow with mild stenosis. Kumar et. al. [7] studied Performance modeling and analysis of blood flow in elastic arteries. Haldar, K. [8] et. al. discussed the Oscillatory flow of blood in a stenosed artery.

The aim of the present investigation is to study the pulsatile flow characteristics of blood in a single constricted blood vessel. The analytical expression for velocity, volumetric flow rate, pressure gradient and the wall shear stress are obtained. The numerical solutions for pressure gradient and wall shear stress have also been obtained and discussed.

## MATHEMATICAL MODEL

It is obvious that stenosis has no well-defined geometrical configuration. In general complex three dimensional flow pattern have been developed near the stenosis which are virtually impossible analyses. In this paper ‘Collar like’ stenosis model i.e. axisymmetric constriction in a tube has been considered. It is assumed that flow is unsteady and laminar, the artery is a constant diameter ($2R_0$) proceeding and following the stenosis.

For mathematical convenience we take the artery to be a long cylindrical tube with the axis coinciding with z axis.

The basic equations of motion in the cylindrical co-ordinate system
($r$, $\theta$, $z$) are given by

$$\frac{\partial w}{\partial z} = 0 \tag{1}$$

$$\frac{\partial p}{\partial r} = 0 \tag{2}$$

$$\frac{\partial w}{\partial T} = -\frac{1}{\rho}\frac{\partial p}{\partial z} + \frac{1}{r}\frac{\partial}{\partial r}\left(vr\frac{\partial w}{\partial r}\right) \tag{3}$$

Where

| | | |
|---|---|---|
| $w$ | – | is the axial velocity. |
| $p$ | – | is the fluid pressure. |
| $\frac{\partial p}{\partial z}$ | – | is the pressure gradient. |
| $\rho$ | – | density of blood. |
| $\nu(=\mu/\rho)$ ) | – | the kinematic viscosity of blood. |

$\mu\{=\mu_0[1+\beta h(r)]\}$ – is the viscosity of blood as proposed by Einstein.
$\mu_0$ – the coefficient of viscosity of plasma.
$\beta$ – a constant.
$h(r)\{=h_m[1-(r/R)^n]\}$ – the hematocrit.
$h_m$ – is maximum hematocrit at the centre of the tube.
$R_0$ – is radius of the normal tube.
$n(\geq 2)$ – is a parameter determining the shape of the constriction.

The geometry of the stenosis is shown in figure and described as Chakrabartry [1998]

$$\frac{R(z)}{R_0}=1-A\left[l_0^{s-1}(z-d)-(z-d)^s\right] \qquad d\leq z\leq d+l_0 \tag{4}$$

Here s $(\geq 2)$ is a parameter which determines the shape of the stenosis, $l_0$ is the length of stenosis, R(z) is the radius of the tube in the stenotic region, d is the location of the stenosis and $A=\frac{\in}{R_0 l_0^s}\frac{S^{s\ /s-1)}}{(s-1)}$, $\in$ being the maximum height of stenosis z with ,

z = d + $\frac{l_0}{S^{1/s-1}}$ and $\in\ <R_0$

The boundary condition are

$$\left.\begin{aligned} & w=0 \text{ at } r=R(z) \\ \text{and} \quad & \frac{\partial w}{\partial r}=0 \quad \text{at} \quad r=0 \end{aligned}\right\} \tag{5}$$

Solution of the problem we assumed the pulsatile sinusodal flow, we have
.Substituting the value of v, $\mu$, h(r) in the eq.(3) and then using the transformation.

$$y=\frac{r}{R_0},\ t=\frac{T}{t_0} \tag{6}$$

We get

$$\frac{\rho R^2{}_0}{t u_0}\frac{\partial w}{\partial t} = -\frac{R^2{}_0}{\mu_0}\frac{\partial P}{\partial z}+\frac{1}{y}\frac{\partial}{\partial y}\left[(a-ky^n)y\frac{\partial w}{\partial y}\right] \tag{7}$$

With $\beta h_m$= k , a =1+k (8)

Then we have form equation (7).

$$\frac{1}{y}\frac{\delta}{\delta y}\left[(a-ky^n)y\frac{\delta w}{\delta y}\right] - \alpha^2 w = -c \tag{10}$$

$$\text{with} \quad \alpha^2=\frac{\rho R^2{}_0}{t_0\mu_0}\,iw \tag{11}$$

$$W(y, r, t) = W(y, r)\ e^{i\omega t} \tag{12}$$

$$\frac{\partial p}{\partial z}=-p\ e^{i\omega t},\ (i=\sqrt{-1}) \quad \text{and} \tag{13}$$

$$-\frac{R^2{}_0}{\mu_0}\frac{\partial p}{\partial z} = c\ e^{iwt} \tag{14}$$

This means that the real part gives the velocity for pressure gradient P cos$\omega$t and the imaginary part gives the velocity for the pressure gradient P sin$\omega$t.

From equation (3), (12) and (13) we have

$$i\omega W\rho = P+\mu\left(\frac{d^2W}{dr^2}+\frac{1}{r}\frac{dW}{dr}\right) \tag{15}$$

$$\frac{d^2W}{dr^2}+\frac{1}{r}\frac{dW}{dr}-\frac{i\omega}{\mu}\rho W=-\frac{P}{\mu} \tag{16}$$

The corresponding boundary conditions (5) are transformed to

$$\left.\begin{aligned} & W=0 \text{ at } r=R(z) \\ \textit{and} \quad & \frac{dW}{dr}=0 \quad \textit{at} \quad r=0 \end{aligned}\right\} \tag{17}$$

**[IV] SOLUTION OF THE PROBLEM**

Now the general solution of the equation (15)

$$\frac{d^2y}{dx^2}+\frac{1}{x}\frac{dy}{dx}-K^2y=0 \tag{18}$$

is $y = AJ_0(ikx)+BY_0(ikx)$ (19)

where both $J_0(x)$ and $Y_0(x)$ are Bessel functions of zero order and are of the first and second kind respectively.

Thus the equation of (8) is

$$W = AJ_0\left[i^{3/2}\sqrt{\omega\rho/(\mu r)}\right]+BY_0\left[i^{3/2}\sqrt{\omega\rho/(\mu r)}\right]+\frac{p}{iw\rho} \tag{20}$$

Since $\omega$ and W have to be finite on the axis (i.e at r = 0) and $Y_0(0)$ is not finite, B has to be zero. Also, because of the no slip condition W(r)=0 when r = R, we have

$$AJ_0\left[i^{3/2}/\omega\rho/\mu R\right]+\frac{p}{\omega\rho i}=0, B=0 \tag{21}$$

Let $\alpha^2=\frac{\omega\rho}{\mu}R^2=\frac{\omega R^2}{\nu}$ (22)

so that $A=\frac{Pi}{\omega\rho}\left[\frac{1}{J_0(i^{3/2}\alpha)}\right]$ (23)

$$W(r)=\frac{-P_i}{\omega\rho}\left[1-\frac{J_0(i^{3/2}\alpha s)}{J_0(i^{3/2}\alpha)}\right] \tag{24}$$

Where s = r/R (25)

Finally, we get the velocity of the hematocrit

$$W(r,t)=\frac{-PR^2}{\mu\alpha^2}i\left[1-\frac{J_0(i^{3/2}\alpha s)}{J_0(i^{3/2}\alpha)}\right]e^{i\omega t} \tag{26}$$

Eq. (10) can be written as

$$A_1\frac{d^2W}{dy^2}+A_2\frac{dW}{dy}+A_3w+ C y = 0 \tag{27}$$

Where $A_1$=y (a - k$y^n$) , $A_2$= a - k$y^n$- n k $y^n$,$A_3$= -$\alpha^2$y

Applying finite difference scheme for (27), We get

$B_1$ [i]w [i+1] + $B_2$[i]w[i] + $B_3$[i]w[i-1]= $B_4$[i] (28)

Where $B_1$ [i] = $A_1$[i]+h$A_2$[i] , $B_2$ [i] =$-2A_1$[i]-h$A_2$[i]+ $h^2A_3$ [i]

$A_3$ [i] = $A_1$[i], $B_4$ [i]= -ci$h^2$ and h is mesh size along space direction.In eq.( 27 ) taking i=1(1)m and using the eq.(17) boundary conditions ,we get the following tridiagonal system of equation.

EW=D (29)

Where E is tridiagonal matrix of order m and these element are defined by $E_{i,i}$ [i] = $B_2$[i],i=1(1)m, $E_{i-1,i}$ [i] = $B_1$[i],i=2(1) m, $E_{i,i-1}$ [i] = $B_3$[i],i=2(1)m and W,D are column matrices having m components, they are w[i] and $B_4$[i],i=1(1)m , respectively.

The eq. (29) is solved by Gauss-Seidel iteration method. To prove convergence of finite difference scheme , the computation is carried out for slightly changed value was observed in the value of w and also after each cycle of iteration the convergence check is performed the tolerance is set at $10^8$ is satisfied at all point .Thus if is concluded that the finite difference scheme is convergent and stable.

Let Q be the volumetric flow rate of the fluid in stenotic region, then

$$Q=2\pi\int_0^{R/R_0} wy\, dy \tag{30}$$

The pressure gradient in stenotic region from eq. (10 ) and (12 ) as :

$$\frac{\partial p}{\partial z}=e^{iwt}\frac{\mu_0}{R^2{}_0}\left[\frac{1}{y}\frac{d}{dy}\{(a-ky^n)y\frac{dW}{dY}\}-\alpha^2W\right] \tag{31}$$

The shear stress at the surface of stenosis is defined by:

$$\tau_s= -\left[\mu(r)\frac{dW}{dr}\right]_{r=R} \tag{32}$$

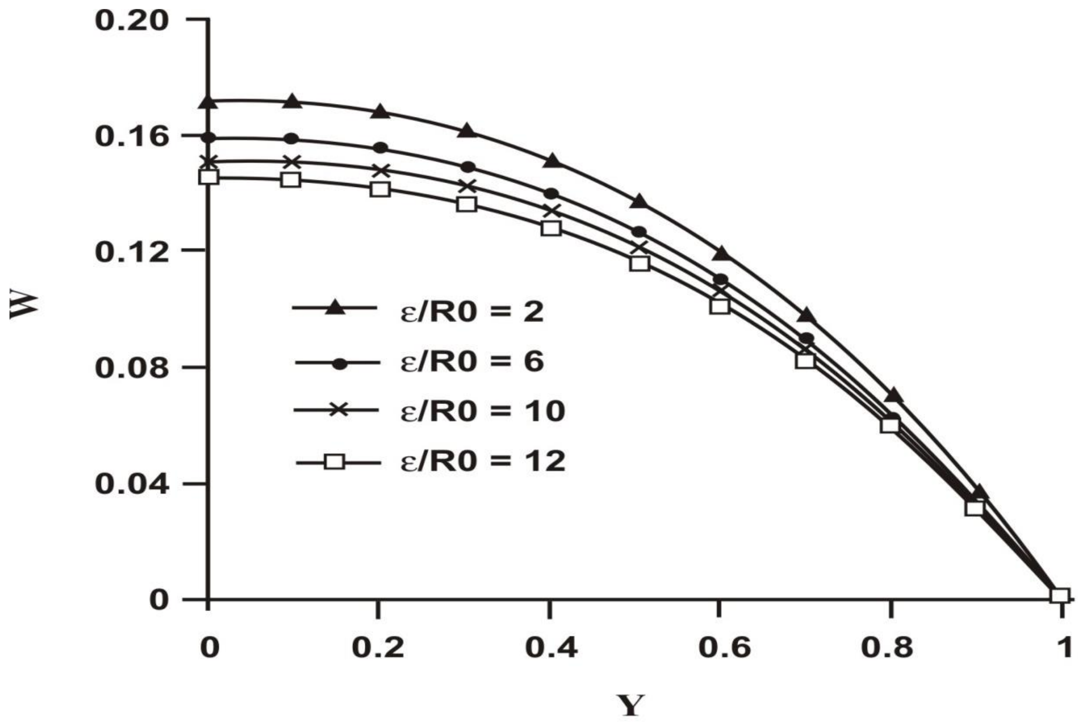


**Fig. 1 : Axial velocity profile for different value of $h_m$.**

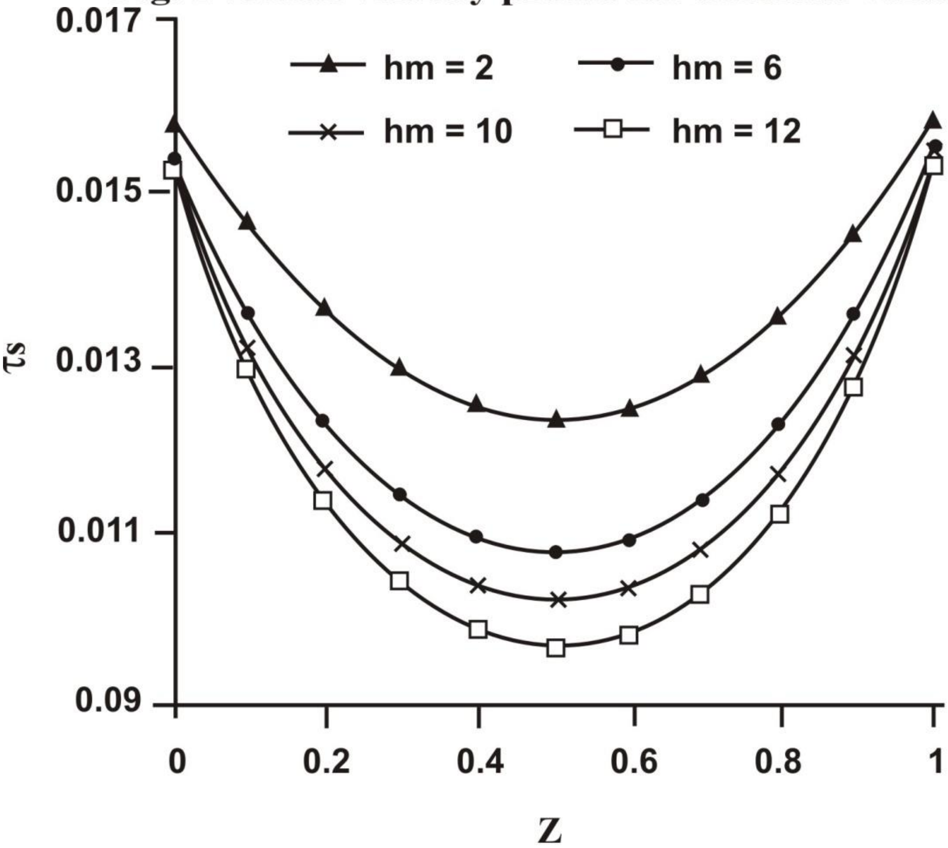


**Fig. 2 : Shear stress at the surface of stenosis for different values of $h_m$ where $\alpha = 1.0$ and $\varepsilon/R_0 = 0.1$**

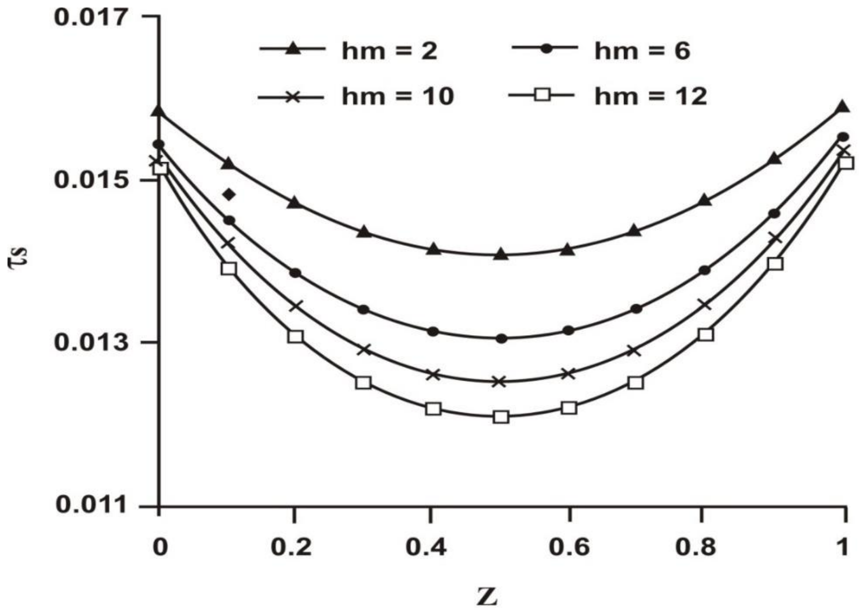


**Fig. 3 : Shear stress at the surface of stenosis for different values of $h_m$ where $\alpha = 1.0$ and $\varepsilon/R_0 = 0.2$**

## NUMERICAL RESULTS AND DISCUSSION:

In order to get physical insight into the problem , the axial velocity field ,Volumetric flow rate ,Pressure gradient and wall Shear stress are discussed taking different numerical value of the parameters encountered into the problem under consideration $\beta = 1, c = 1, n = 2, s = 2$ ,6,10,12 $l_0$=1,2,5,6 and d=0.

In Fig.1 ,the axial velocity field w has been plotted versus y for different values of maximum hematocrit[$h_m$],frequency parameter [$\alpha$],ratio of the maximum height of stenosis and radius of the normal tube [$\frac{\varepsilon}{R_0}$]and parameter determine the shape of the constriction [n], respectively .From these figures ,we observed commonly that an increase in $h_m$ ,$\alpha$, $\frac{\varepsilon}{R_0}$, and n decreases the axial velocity field.

In Fig. 2 and Fig. 3 , the shear stress at the surface of stenosis $\tau_s$ has been plotted versus z for different values of maximum hematocrit [$h_m$] at $\frac{\varepsilon}{R_0}$ =0.05, 0.10,0.15,and 0.20 respectively .From these figures ,we observed that the shear stress is maximum at z =0 ,z=1.0 and minimum at z =0.5 .The shear stress gradually decreases within the region [0$< z < 0.5$] and increases within the region [0.5$< z < 1.0$] for different values of $h_m$ . It is also observed that an increase in maximum hematocrit and maximum height of the stenosis decreases the shear stress in stenotic region. These figures are exactly symmetrical about z=0.5 .

## CONCLUSIONS

In this paper we have studied the unsteady blood flow through rigid tube in the presence of mild stenosis has been numerically .The blood flow in an artery through a symmetrical stenosis by considering the blood as Newtonian fluid. From the above discussion, it is clear that the pressure gradient increase with the increase of hematocrit value, indicating that there is higher value in systolic and lower value in diastolic pressure. In high systolic and low diastolic pressure, peripheral blood flow will increase, but coronary arterial blood flow will decrease.

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