Performance Optimization of the Regulation Loop of a Buck Chopper Driven by Duty Cycle Modulation

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Abstract- **The control of switching power supplies in general, and particularly type Buck is a topic much discussed and highly invested by sense current technological applications simultaneously require a high level of accuracy and performance, and DC-DC converters have a very important role to play in systems requiring conversion and adaptation of the energy level. We were interested in the course of this work to the optimal control of a Buck controlled by the duty cycle modulation diet, with the aim of optimizing the performance of the control loop of our Buck chopper DCM. An analysis of modeling approaches and control law synthesis to ensure stability and to guarantee a certain level of performance throughout the area have been studied in this work while takingin to account the problems of their application inan industrial environment. Our study based on the optimal control of the Buck chopper controlled by the DCM allowed us both in the design of the continuous state model and in the design of the discrete state model and through the LQR controller (Linear Quatratic Régulator) to discuss the responses of our system in closed loop and open loop. By exploring advanced control laws, we were able to control our system, namely the state variable that we want to control and to impose a very small overshoot, and consequently a very damped dynamic regime. We believe that we have been able to provide power electronics systems with systematic methods to model, analyze and control DC-DC converters using optimal control and the DCM technique.**

Keywords **: Duty Cycle Modulation (DCM), Linear Quatratic Regulator (LQR), Buck Chopper Driven, Pulse Width Modulation (PWM).**

I. INTRODUCTION

The most Buck-type switching power supply structures in power electronics are controlled by the classic pulse-width modulation (PWM) technique. DC-DC converters therefore have a very important role in energy conversion systems. For this type of function, several circuit topologies are proposed. We can broadly classify them by simple topologies, which are the second order DC-DC converters from a mathematical modeling perspective, and complex topologies, which are the higher order ones. The wide use of switching power supplies comes from the fact that they can provide galvanic isolation, regulate the output voltage according to variations in load and input, and have good conversion efficiency. Despite this, there are constraints inherent to the implementation of these transmission techniques, among others we have the complicated hardware interface, the complex programming logic, the reconstructed signal is not always very reliable due to the presence of harmonics. Faced with this problem posed by the PWM, the performances resulting from a Buck chopper controlled by the new duty cycle modulation (DCM) technique developed since 2005, are superior under the same operating conditions to that of a chopper PWM equivalent [1],[2],[3] et [4] [5]. The exploration of advanced control laws of DCM choppers, with a view to best benefit from the virtues of the DCM technique in power electronics, constitutes our object of study.

II. LITERATURE SURVEY

Since 2005, the duty cycle modulation technique has been used in several application areas, from instrumentation to signal processing to process control. DCM technology is increasingly used in industrial electronic applications, including instrumentation systems, interface drivers and signal transmission chains. However, in existing research related to new applications of the DCM technique, the linear approximation policy is used for reasons of structural simplicity and low implementation cost, at the expense of rigorous analysis and low approximation errors. . It is based on an overall least square cubic fit model of the original intractable nonlinear duty cycle structure. The resulting simple linear demodulation model consists of a linear low-pass filter, which is connected in series to a static amplifier with a cubic input-output characteristic. Virtual simulation results obtained under a variety of modulating inputs with arbitrary waveforms showed the efficiency of the proposed suboptimal DCM architecture[6].

Next, a new DCM control scheme for Buck power converters is studied. Compared to the pulse width modulation technique widely used in power electronics, the proposed control scheme offers a variety of advantages under the same operating conditions, including greater hardware simplicity, lower modulation frequency and better transient response. In addition, while offering new features, it offers disturbances under load of the same level of robustness as the standard pulse width modulation control technique. The characteristics of both control techniques are validated and compared on a well-tested prototyping buck converter, using a mix of analytical reasoning, virtual simulations and real tests on a laboratory bench.[7]

In another publication, a low-cost duty cycle modulation scheme is studied in depth and compared to the standard pulse width modulation technique. Using a mixture of analytical reasoning and electronic simulation tools, it is shown that under the same operating conditions, most of the features of the proposed duty cycle modulation scheme are better than those provided by a

standard pulse width modulation technique. Simulation results obtained from testing both modulation control policies on prototyping systems indicate that the proposed duty cycle modulation approach appears to be a high-quality control policy in a wide variety of application areas, including A/D and N/A conversion, signal transmission, and switching control in power electronics [8]. other article to show the interest of the DCM we have a design methodology and a virtual simulation framework, of the optimal feedback control scheme for DCM (Duty-Cycle Modulation) Buck choppers. The transfer function of an open loop DCM chopper, is built from the numerical analysis of a virtual step response, with Ref = 2 volts as step control input. Then, the optimal PID controller for DCM Buck Choppers is formulated and synthetized as a LQR (Linear Quadratic Regulator) equivalent control, and the associated Riccati equations is easily solvable using Matlab/LQR tool. As a relevant implication, the parameters of the corresponding optimal PID, are straightforwardly obtained from the LQR gains according to a LQR/PID equivalence principle, which is proven for a relevant class of second order dynamic systems. In addition, in order to show the feasibility of the designed optimal PID control scheme, a whole prototyping DCM Buck chopper with given operating conditions (main DC supplied (12 V), load resistance (3.3 Ω), basic DCM frequency (30 kHz)), is virtually implemented and well tested using Multisim. The virtual simulation results obtained from numerous operating conditions (e.g., open loop, closed loop, load variations), are presented and discussed. Furthermore, relevant results and findings are reported, e.g., transient characteristics (i.e. controllability, stability, overshoot (4%), and time response (1.5 ms) and static performance levels (e.g. static error (0 %), high robustness under 50 % load variations). These results are a great challenge for the first virtual DCM Buck chopper, operating under a well-tested optimal PID-based control policy[9].
Finally, in a recent publication, a new algorithmic scheme for building DDCM (duty-cycle modulation) drivers is presented. It is modeled in the analog domain as a continuous time-hop Markovian dynamic model, with a two-state deterministic Markov chain. An equivalent discrete jump dynamic model is calculated using a BOZ transformation. Second, the resulting numerical iterative algorithm consists of simple operators and numerical structures. The proposed DDCM algorithm is simulated under the Matlab framework, and implemented using Arduino IDE-C++ with download in an ESP32 system-on-chip (SoC) device. The monitoring device connected to the ESP32 via USB communication cable, Arduino/IDE virtual monitor. It is configured for communication at 230400 baud. The DAC (digital-to-analog converter) based on ESP32, is virtually implemented and well tested as a case study of the proposed new generation of DDCM drivers. Matlab numerical simulation results and ESP32 processing and virtual monitoring results are presented and discussed, in order to show the realistic nature and grand challenge of the proposed DDCM algorithmic scheme for SOC devices[10].

The control law is one of the most important and complex aspects in the design of static converters and the MRC technique in view of the studies carried out and the results obtained increasingly demonstrate its effectiveness in terms of efficiency, simplicity of implementation and constitutes a real alternative to the problems caused by PWM.

III. METHODOLOGY

The theory of optimal control makes it possible to determine the control of a system that minimizes (or maximizes) a performance criterion, possibly under constraints. This the implementation of algorithms and control laws requires models that reflect the behavior of the electronic circuit of a converter. The quality of a control law is above all related to its ability to reject disturbances and robustness parametric uncertainties or modeling before any consideration concerning these dynamic performances [11]. As a first step, the optimal control system in continuous time of the Buck chopper in DCM, then in a second step, the optimal discrete time control system of the Buck chopper in DCM will be studied, Finally, the simulation results will be presented to validate the performance of the control loop.

III.1 CONTINUOUS LINEAR QUADRATIC REGULATOR (CLQR)

The LQR controller we are going to use is based on the minimization of a quadratic criterion related to the energy of state variables and control signals.

$$J = \int_0^\infty (X^T QX + U^T RU)dt \qquad (1)$$

Q and R are the x and u weighting matrices of the cost function. The optimal control is therefore to systematically calculate the gain matrix of the state feedback control, because LQR is a consolidated control technic’s.

III.1.1 FORMULATION OF THE LQR PROBLEM CONTINUOUS TIME

To use the LQR method, whether in continuous time or discrete time it will be necessary to find the following 2 parameters:

- Performance Index Matrix R
- State cost matrix Q

Q and R are positive definite symmetric matrices. To obtain the numerical values of Q and R, we proceed by numerical simulation by varying them until satisfactory answers are obtained.

III.1.2 STRUCTURE OF THE THEORETICAL SOLUTION

our system is described by a state representation of the form (Ac,Bc,Cc) and controlled by a state return control law: $u(t) = -Kx(t) + \mathrm{Re}\,f$ and K is the gain of the quadratic linear regulator. To find K we apply the following formula:

$K = -R^{-1} B_c^T P$ P is the solution of the RICCATI equation

$$Q + A_c^T K + K A_c - K B_c R^{-1} B_c^T K = 0$$

The closed-loop system is then given by:

$$\overset{*}{x}(t) = (A_c - B_c K)x(t) + B_c \operatorname{Re} f \qquad (2)$$

Ac and Bc are determined by the physical relationship between the components.

III.1.3 BLOCK DIAGRAM OF THE CONTINUOUS-TIME SYSTEM

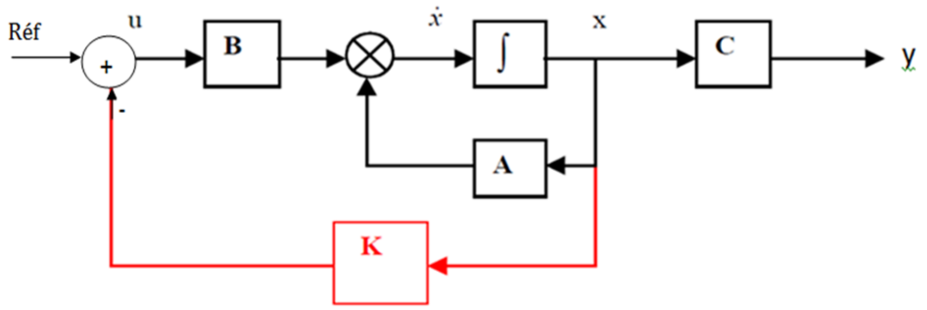

Figure 1: Continuous Time State Return Loop Diagram

III.1.4 THE MATLAB PROGRAM STRUCTURE OF OUR BUCK CHOPPER IN DCM

```
Clear
%III.1.4.1) Identifying Simulated Buck Chopper Parameters in Matlab
R=3.3;   %Buck chopper load resistance
C=220*10^(-6);  %Vc filter capacitor
Ref=2; %In Volt
%III.1.4.2) Measured data on the response at the open loop step
% Vc(t) obtained in Matlab
Yinf=5.5; %Yfinal (static or permanent)
r=1;                    % or r=1%
Ts=0.006;               % Response time Ts to r%
YD=7.12;                %Ymax on 1st overtake
%III.1.4.3) Data estimated from measured data
ks=yinf/ref(1); %Static gain
D=YD-Yinf; %Amplitude of 1st exceedance relative to yinf
d=D/Yinf; %Relative amplitude of 1st exceedance
%III.1.4.4) Transfer function calculation: Vc(t)/U(p)
z=log(d)/sqrt(pi^2+(log(d)^2));
wn=log(100/r)/(z*Ts);
SysVcBo=tf(Ks*wn^2,[1 2*z*wn wn^2]); %Vc(p)/U'(p)
%III.1.4.5) Transfer function calculation: IL(p)/U(p)
SysRC=tf([R*C 1],[R]);
SysILBo=tf(series(SysRC, SysVcBo)); % IL(p)/U(p)
%III.1.4.6) Simulation of models estimated by transfer functions
t=0:0.0001:0.008; u=Ref*ones(1,length(t));
Vc=lsim(SysVcBo,u,t); IL=lsim(SysILBo,u,t); %Simulation
%III.1.4.7) Continuous State Model by Posing: x1=Vc(t), x2=Ic(t)=Cdvc/dt
%III.1.4.8) Calculation of terms: Ac,Bc,Cc,Dc
A11=0; A12=1/C;
A21=-C*wn^2;
A22=-2*z*wn;
Ac=[A11 A12;A21 A22]; Bc=[0;Ks*wn^2*C]; Cc=[1 0]; Dc=0;
%III.1.4.9) Simulation
SyscABCD=ss(Ac,Bc,Cc,Dc);
[Y,T,X]=lsim(SyscABCD,u,t);          % Continuous state model level response

IR=Vc/R; Ic=X(:,2);                  % Continuous-time state vector data
subplot(211); plot(t,Vc,'k',t,IL,'k'); grid
title('Estimated Step Responses of DCM Buck Converter');
subplot(212); plot(t,X,'k',t,Vc/R,'k',t,Ic+(Vc/R),'k'); grid
title('Step Responses for the state space model');
%III.1.4.10) Calculation of the LQR in continuous time over infinite horizon
```

```
%III.1.4.10.1 Cost Weighting Matrix
Qc=[0.07 0; 0.5 0]; Rc=0.6;
%III.1.4.10.2 LQR calculation
[K,S,e]=lqr(SyscABCD,Qc,Rc) %K=optimal gain, S= LQR solution Riccati Equation
        af=ac-bc*k; Bf=Bc; cf=cc; Df=Dc;
        SyscfABCD=ss(Af,Bf,Cf,Df);
%III.1.4.10.3 Closed-loop simulation of the MRC Hasher Controlled by LQR
 [Yf,T,Xf]=lsim(SyscfABCD,u,t);  %Continuous state model level response
 figure(2); plot(t,Xf(:,1),'r',t,Xf(:,2),'r',t,X(:,1),'k',t,X(:,2),'k'); Grid
 title('Step Responses for the State space model');
```

III.2 The Quadratic Linear Regulator in discrete time

The discrete-time method is very rigorous in nature and is best suited to converters operating in Modulation in Duty Cyclique or variable frequency.

III.2.1 Formulation of the LQR Problem in discrete time

The application of the principle of dynamic programming to LQR consists in decomposing the optimal control problem of into a sequence of optimal control subproblems, admitting any point ( ) of the optimal trajectory for initial condition and is the required value function, this results in the discrete H-J-B (Hamilton-Jacobi-Bellman) equation

$$Min\ J=\sum_{k=0}^{N-1}\left(\frac{1}{2}x_k^T Q\, x_k+\frac{1}{2}u_k^T R\, u_k + V^*(Ax_k+Bu_k)\right) \quad (3)$$

III.2.2 BLOCK diagram of the discrete time system

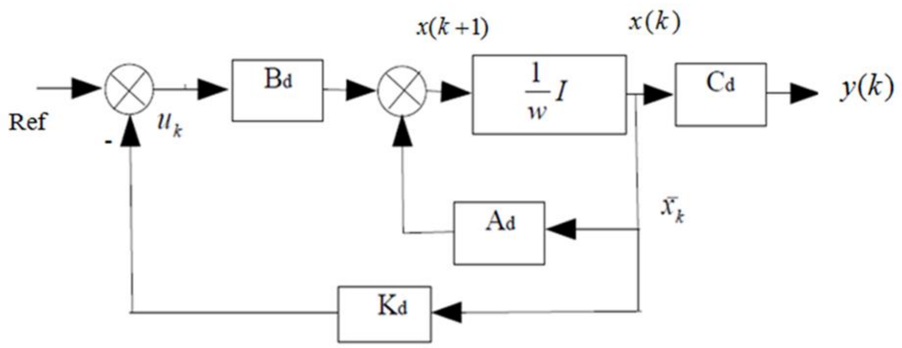

Figure 2 : Discrete-time state feedback loop diagram

III.2.3 Discretization Program

The structure of the discretization program from the continuous state model to a discrete state model is presented.

```
%III.2.3.1) Discretization and simulation of the discrete process
tech=0.0001; %Sampling period
 Sysd=c2d(SyscABCD,tech,'zoh')
 [A,B,C,D]=ssdata(Sysd) %Retrieving A,B,C,D matrices from the discrete model
 [Yd,Xd]=dlsim(A,B,C,D,u); %Response at the BO continuous state model level
 figure(3); subplot(211); plot(t,Xd(:,1),'.',t,Xd(:,2),'.',t,X(:,1),'k',t,X(:,2),'k')
 axis([0 0.006 -1 8]);
title('LQR-Step responses of discrete and losed loop models')
xlabel('Temps(s)'); grid; title('Open loop Step responses of discrete and continuous models')
% III.2.3.2) Discrete-time LQR (DLQR) calculation over infinite horizon
 Qd=[0.6 0; 0.5 0]; Rd=0.5;
 [Kd,Sd,ed]=dlqr(A,B,Qd,Rd)
 Afd=A-B*Kd; Bfd=B; Cfd=C; Dfd=D;
 [Yfd,Xfd]=dlsim(Afd,Bfd,Cfd,Dfd,u);
 subplot(212); plot(t,Xfd(:,1),'.',t,Xfd(:,2),'.',t,Xf(:,1),'k',t,Xf(:,2),'k');
 title('LQR-Step responses of discrete and closed loop models')
 axis([0 0.006 -0.5 3]); xlabel('Time(s)'); Grid
```

IV.RESULTS AND DISCUSSION

For a better understanding of the results presented it is important the choice of variables

$X_1 = V_C$ et $X_2 = i_C = C\dfrac{dV_C}{dt}$, $donc$ $i_L = i_C + \dfrac{V_C}{R} = X_2 + \dfrac{X_1}{R}$ From the above we have the following state model

$$\begin{cases} \dot{x} = \begin{bmatrix} 0 & \dfrac{1}{c} \\ -C_s\omega_n^2 & -2\xi\omega_n \end{bmatrix} x + \begin{bmatrix} 0 \\ C_s K_s \omega_n^2 \end{bmatrix} u \\ y = \begin{bmatrix} 1 & 0 \end{bmatrix} x + 0u \end{cases} \tag{4}$$

IV.1: Presentation of simulation results obtained in Matlab

IV.1.1 solution of the continuous time state space model in Matlab

$$A_c = \begin{bmatrix} 0 & 4.5455 \\ -0.9857 & -1.5240 \end{bmatrix} B_c = \begin{bmatrix} 0 \\ 2710.8 \end{bmatrix} C_c = \begin{bmatrix} 1 & 0 \end{bmatrix} \text{Et enfin } D_c = 0 \tag{5}$$

$$\textbf{Sc=}\begin{bmatrix} -0.2423 & 0.2994 \\ 0.2994 & 0.6942 \end{bmatrix} \qquad \textbf{K=}-[0.1353 \quad 0.3137] \tag{6}$$

Comment: Our system is controllable and observable with regard to the results that result from the simulation in Matlab. Our system is also stable because the poles are non-zero negative real parts; the return gain of state K since this is what interests us more is:

$$K = -[K_1 \text{ pour } X_1 \text{ et } K_2 \text{ pour } X_2]$$

IV.1.2 simulation results of the continuous time state space model in Matlab

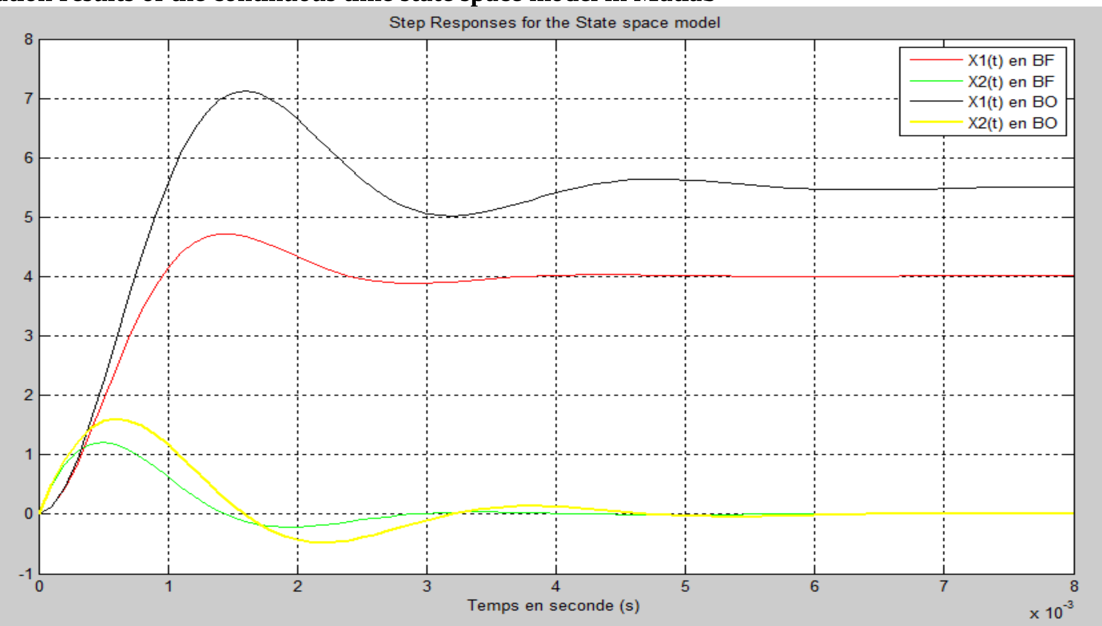

Figure 3: Space of LQR-controlled states from Buck chopper state model to DCM

Comment: Figure 3 shows us the simulation of the state model of the Buck chopper at DCM in continuous time so we see that in open loop the damping of X1(t) and X2(t) is very important, the response time and the overshoot of X1(t) and X2(t) are almost identical. We can also notice that for low values of the damping coefficient as is the case in our system (< 0.7), the response time increases when decreases because the amplitude of the oscillations increases and the transient regime is longer and longer. With regard to the overshoot when < 1, X1(t) reaches its final value after one or more exceedances. In closed loop and thanks to the quadratic linear regulator our has very good performance and stabilizes a little quickly (4ms).

IV.2 Simulation of the discrete state model obtained

IV.2.1 Discrete results of the continuous state model in Matlab

$$x(k+1)=\begin{bmatrix}0.9424 & 0.3563\\ -0.2462 & 0.5727\end{bmatrix}x(k)+\begin{bmatrix}0.0584\\ 0.2494\end{bmatrix}u(k) \qquad (7)$$
$$y(k)=\begin{bmatrix}0 & 1\end{bmatrix}x(k)$$

IV2.2 Simulation the open-loop discrete state model

The simulation of the discrete state model obtained in an open loop makes it possible to verify that the discretization of the continuous model has not caused a loss of information or characteristic properties of the dynamical system. This, after the simulation, it will be necessary to compare the responses at the level of the discrete system, with those of the continuous system obtained under the same simulation conditions (same signal input step).

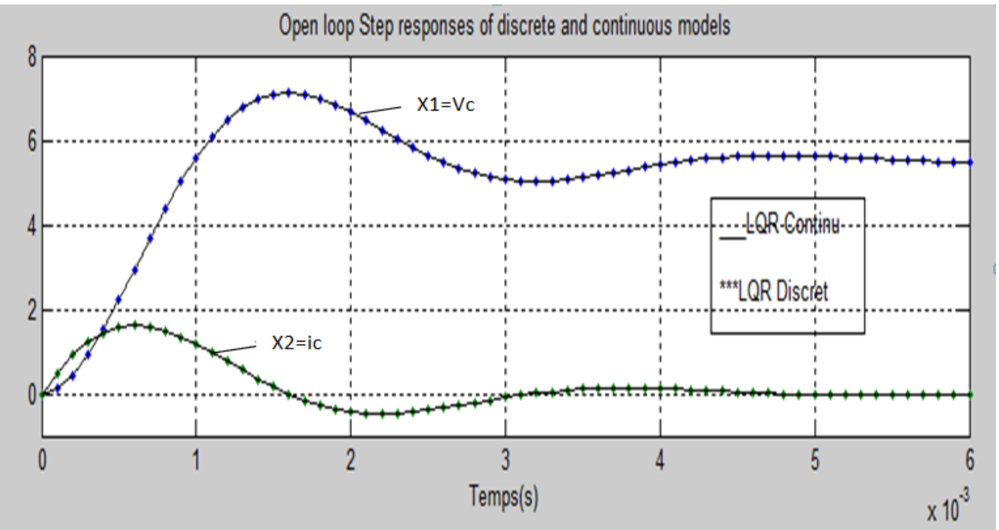

Figure 4: Results of comparison of responses of continuous and discrete open-loop dynamic models

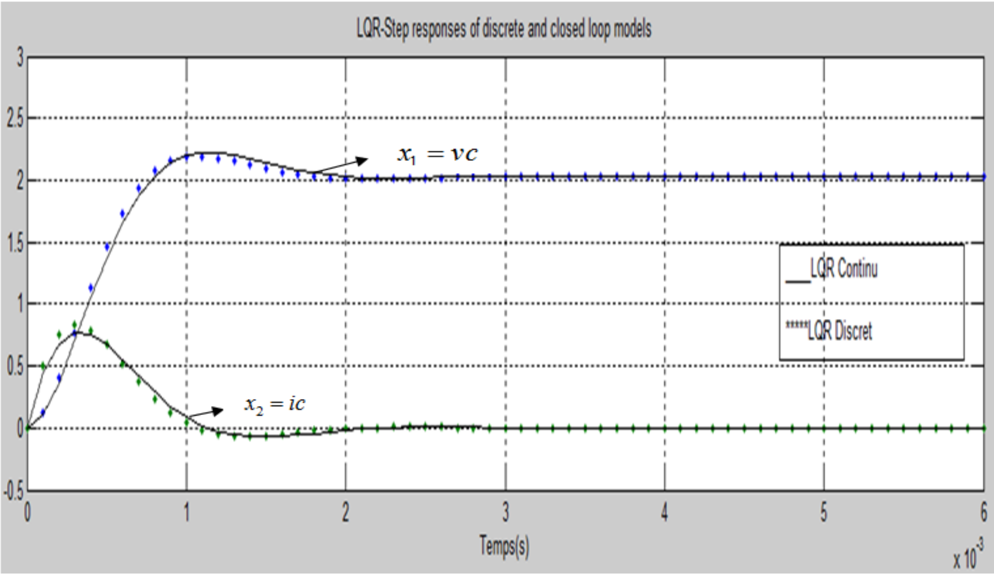

Figure 5: Closed-loop continuous and discrete dynamic model response comparison results

Comment: We generally know in essence that discretization when it is necessary always leads to a loss of information, but it is clear here that the discrete and continuous state model have retained the same parameters and evolved symmetrically. The level of damping of our continuous time and discrete time LQR is comparable and satisfactory. The LQR control of our closed-loop system as can be seen has the advantages in terms of robustness compared to disturbances and uncertainty on the parameters; And sensitivity reduction. These figures show a small exceedance just before the establishment of the steady state.

V.CONCLUSION

The objective assigned to this work was to optimize the performance of the control loop of a Buck chopper controlled by DCM. the LQR method highlighted in this work is well known in control theory. It uses the theory of optimal control which lies in the minimization of the criterion in order to find a gain of the state return. Thus in this approach of synthesis of the laws of control, we used the Euler-Lagrange and Hamiltonian energy models well adapted to the formulation and the energy "vision" of our Buck chopper in DCM. Through the results presented above, we believe that we have been able to bring to the industrial environment systematic methods to model, analyze and control DC-DC converters using optimal control and DCM technics. This showed that it is necessary to take into account the control law envisaged from the modelling phase. In addition, we were able to give a vision on the applicability or exploitation of several principles of modeling and nonlinear control for DC-DC converters keeping in mind to achieve a balance between reasonable theoretical / practical compromises in order to be able to experimentally implement the control laws finally obtained.

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