Performance Comparison of various Observers for a Nonlinear System

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***Abstract*- This paper compares the performance of various observers for a nonlinear system. The objective is to determine better performance of observer to fast estimation of states of the system. The inverted pendulum problem is the benchmark of modern control theory. Three observers are presented viz. Extended Kalman Filter (EKF), Extended Luenberger Observer (ELO) and Sliding Mode Observer (SMO) are used to estimate the states of the systems for generating state feedback control signal to control the angle and position of cart. Simulation study has been done in Matlab/simulink environment shows that observers are capable to estimate states of multi output inverted pendulum system successfully. The result shows that SMO produced better response compared to EKF and ELO techniques presented in time domain.**

***Keywords*: Extended Luenberger Observer, Sliding Mode Observer, Inverted Pendulum System (IPS).**

I.INTRODUCTION

Designing of an observer for nonlinear system that provides the desired performance to the closed loop system in the presence of the disturbance/uncertainties is one of the most challenging and difficult subject of modern control theory[1]-[3].

In State variable design, we assume that all the state variables are measurable and utilize them in a full state feedback control law. Full state feedback control is usually not practical because it is not possible to measure all the states of the system. In practice, only certain states (or linear combinations thereof) are measured and provided as systems output [4].

Aim of the observer is to estimate the states that are not directly sensed and available for as output and connect to control input $(u(t))$ as given in (1)

$u(t) = -\boldsymbol{Kx(t)}$ (1)

Determining the gain matrix **K** is the objective of the state feedback design procedure [1]-[4].

Number of states variable in higher order nonlinear systems are not available for measurement in real world application.

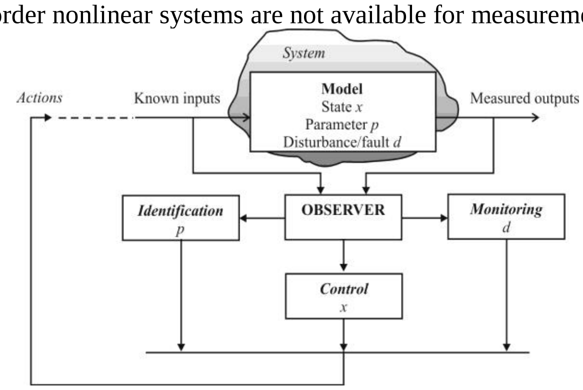

Fig.1. Observer as the heart of control

The unmeasurable states are generally estimated based on available measurements and the knowledge of the physical system by an observer which is the heart of control system [3] as shown in the figure.1.

II.OBSERVER METHODS

This section provides a very brief description of each of the observer design method developed for state estimation of nonlinear system [3].

$$\left.\begin{aligned} \dot{x} &= f(x) + gu \\ y &= h(x) \end{aligned}\right\} \quad (2)$$

where $x \in R^n$ is the state vector and $y \in R^p$ is the output vector. It is assumed that $f: R^n \to R^n$ and $h: R^n \to R^p$ are mappings and that for some $x^* \in R^n$ the (3) hold:

$f(x^*) = 0 \; and \; h(\,x^*) = 0$ (3)

The solutions x^* of the equation $f(x) = 0$ are called the equilibrium points of the plant dynamics.

$$\dot{x} = f(x) \qquad (4)$$

For the formulation of sufficient condition of observability of system (2), consider the linearization of (2) at the equilibrium $x = x^*$ given as

$$\left.\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx \end{aligned}\right\} \qquad (5)$$

Where $\boldsymbol{A} = \left[\frac{df}{dx}\right]_{x=x^*}$ and $\boldsymbol{C} = \left[\frac{dh}{dx}\right]_{x=x^*}$ (6)

A). The Extended Kalman Filter(EKF)

The Kalman Filter (EKF) is a non linear filter that minimizes the mean square estimation error [5]-[7]. This method of estimation assumes a linear dynamic system

$$\left.\begin{aligned} \dot{x} &= Ax(t) + Bu(t) + w(t) \\ y &= Cx(t) + v(t) \end{aligned}\right\} \qquad (7)$$

Where w and v are uncorrelated Gausian noise of zero mean and of intensities Q and R respectively. The observer scheme is of the form

$$\left.\begin{aligned} \dot{\hat{x}}(t) &= A\hat{x}(t) + Bu(t) - H(\hat{y}(t) - y) \\ \hat{y}(t) &= C\,\hat{x}(t) \end{aligned}\right\} \qquad (8)$$

The kalman filter gain H is

$$H = SC^T R^{-1}$$

Where S is symmetric and positive definite matrix, satisfying the Riccati equation:

$$SA^T + AS - SC^T R^{-1} CS + Q = 0 \qquad (9)$$

B). Extended Luenberger Observer (ELO).

The luenberger observer [8]-[11] is given for the non-linear system (4) given as

Where

$$\left.\begin{aligned} \dot{x} &= Ax(t) + Bu(t) \\ y &= Cx(t) + Du(t) \end{aligned}\right\} \qquad (10)$$

According to Luenberger the full state observer for the system (10) is given by

$$\dot{\hat{x}} = A\hat{x}(t) + Bu(t) + L(y - C\hat{x}) \qquad (11)$$

Where $\hat{x}(t)$ is the estimate value of state $x(t)$. The matrix L is the observer gain matrix.

The goal of the observer is to provide an estimate of $\hat{x}(t)$ so that $\hat{x} \rightarrow x(t)$ as $t \rightarrow \infty$ with $\hat{x}(t_0)$ and $e(t) \rightarrow 0)$ as $t \rightarrow \infty$

C). The Sliding Mode observer (SMO).

This technique is based on theory of variable structure systems.

(1) Selection of a hyper surface or a manifold (i.e., the sliding surface) such that the system trajectory exhibits desirable behavior when confined to this manifold.

(2)Finding feedback gains so that the system trajectory intersects and stays on the manifold [12]-[16].

A nonlinear system is described by dynamical equations in state space form as:

$$\ddot{x} = f(x, u, t) \qquad (12)$$

where $x \in R^n$ is the state vector, $u \in R$ is control input, $y \in R^m$ is measurement output. $f: R^n \rightarrow R^n$,

The sliding-mode designer picks a switching function σ that represents a kind of "distance" that the states X are away from a sliding surface.

- A state X that is outside of this sliding surface has σ (x) ≠0.
- A state that is on this sliding surface has σ (X)=0

$$\left.\begin{aligned}\dot{\hat{x}}_1 &= -\alpha_1\hat{x}_1 + \hat{x}_2 - k_1 sgn(e_1)\\ \dot{\hat{x}}_2 &= -\alpha_2\hat{x}_1 + \hat{f}(\hat{x},u,t) - k_2 sgn(e_1)\end{aligned}\right\} \quad (13)$$

Where $e_1 = \hat{x}_1 - x_1$ and $\hat{f}(\hat{x},u,t)$ is the estimated value of $f(x,u,t)$. The design parameters are α_i and k_i, and they are chosen so that

$$\left.\begin{aligned}\hat{x}_2 &\leq \ \ k_1 + \alpha_1 e_1, if\ x_1 > 0\\ \hat{x}_2 &\geq -k_1 + \alpha_1 e_1, if\ x_1 < 0\\ k_2 &\geq |\Delta f|\end{aligned}\right\} \quad (14)$$

$$\Delta f = \hat{f}(\hat{x},u,t) - f(x,u,t) \quad (15)$$

To force the system states to satisfy $\sigma(x) = 0$ one must:
(1) Ensure that the system is capable of reaching $\sigma(x) = 0$ from any initial condition.
(2) Having reached $\sigma(x) = 0$ the control action is capable of maintaining the system at $\sigma(x) = 0$.
Condition for existence of sliding mode

$$V(\sigma(x) = \frac{1}{2} * \ \sigma^T(x) * \sigma(x) = \frac{1}{2} * \left|\left|\sigma(x)\right|\right|_2^2 \quad (16)$$

Where $\|.\|$ is a Euclidean norm (i.e. $\left|\left|\sigma(x)\right|\right|_2$ is the distance away from the main fold where $(x) = 0$.

Sufficient conditions for the existence of sliding mode are:

$$\frac{dV}{dt} < 0\ where \frac{dV}{dt} = \frac{dV}{d\sigma} * \frac{d\sigma}{dt} \quad (17)$$

III.INVERTED PENDULUM SYSTEM

This section provides a brief description on the modeling of the inverted pendulum system, as a basis of a simulation environment for development and assessment of above said observer techniques. It consists of a control link and a pendulum link connected by a low-friction pivot. Two measurements are taken: the angular displacement of the control and pendulum link via encoders and counters. The control torque, supplied by a DC motor, is such that the pendulum and control link are held vertically at 90 degrees. The objective of the observer techniques is to ***es***timate the angular velocities of the control and pendulum link [17]-[18].

The system consists of an inverted pole hinged on a cart which is free to move in the *x* direction as shown in Figure 2.

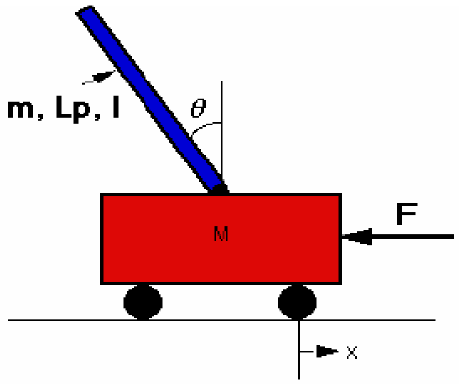

Fig.2. Inverted Pendulum System

In order to obtain the dynamic model of the system, the following assumptions have been made:

I. The system starts in a state of equilibrium meaning that the initial conditions are therefore assumed to be zero.

II. The pendulum does not move more than a few degrees away from the vertical to satisfy a linear model.

Free body diagram of the system is shown in the fig.3. From the free body diagram, the following dynamic equations in horizontal direction in (1) and vertical direction in (2) are determined.

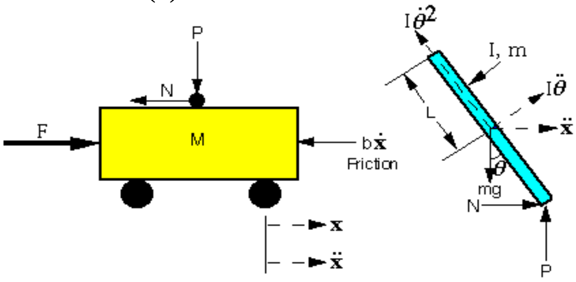

Fig.3.Free body diagram of the System

The equations describing the model of inverted pendulum system[] are given as
$(M+m)\ddot{x}+b\dot{x}+ml\ddot{\theta}\cos\theta-ml\dot{\theta}^2\sin\theta=F$ (17)
$(I+ml^2)\ddot{\theta}-mgl\sin\theta=ml\ddot{x}\cos\theta$ (18)

Our goal is to keep inverted pendulum vertical, we can assume that $\theta(t)$ and $\dot{\theta}(t)$ are small quantities such that $\sin\theta(t)\sim\theta(t)$, $\cos\theta(t)\sim 1$, and $\theta(t)\dot{\theta}^2=0$.
After linearization the dynamic equations in (13) and (14) are:
$(M+m)\ddot{x}+b\dot{x}+ml\ddot{\theta}=F$ (19)
$(I+ml^2)\ddot{\theta}-mgl\theta(t)=ml\ddot{x}$ (20)

By manipulating the dynamics equations in (19) and (20), and substituting the parameter values of the cart and pendulum, the state space model of the system is given as

$$\begin{bmatrix}\dot{x_1}\\ \dot{x_2}\\ \dot{x_3}\\ \dot{x_4}\end{bmatrix}=\begin{bmatrix}0 & 1 & 0 & 0\\ 0 & -\beta b & mlg\alpha & 0\\ 0 & 0 & 0 & 1\\ 0 & -\alpha b & g(M+m)\alpha & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\end{bmatrix}+\begin{bmatrix}0\\ \beta\\ 0\\ \alpha\end{bmatrix}\vec{F}(t) \quad (21)$$

$$y(t)=\begin{bmatrix}1000\\ 0010\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\end{bmatrix}+[0]\vec{F}(t) \quad (22)$$

Where $\alpha=\frac{ml}{I(M+m)+Mml^2}$; $\beta=\frac{I+ml^2}{I(M+m)+Mml^2}$.

The parameters of the system are shown in Table 1.

TABLE 1. Parameter of the system

Symbol	Parameter	Value	Unit
M	Mass of the cart	0.75	kg
m	Mass of the pendulum	0.25	kg
b	Friction of the cart	0.05	N/m/s
l	Length of the pendulum	0.4	m
I	Inertia of the pendulum	0.005	kgm^2
g	Gravity	9.8	m/s^2
u	Torque applied (N-m)		
x	Cart position(m)		
□	Pendulum Angle rad		

Open loop poles of the (21) are 0, -0.0500, -5.2987 and 5.2844.
As can be seen, one of the four poles, 5.2844 lies on right hand side of the s-plane which stated that the open system is unstable. Therefore, a controller has to be designed in order to stabilize the inverted pendulum system. The rank of controllability and observability is 4. That mean system is controllable and observable.
LQR is a method in modern control theory that uses state-space approach to compute control law to stabilize the system in the vertical position. The system can be stabilized using full state feedback. The schematic of this type of control system is shown in Figure 4.

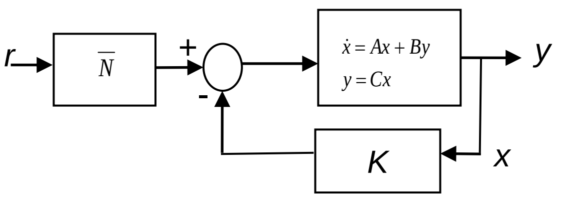

Fig.4.Block diagram with reference input

The control law obtained is computed as

$u = [0.3162\ 1.0288\ 23.0797\ 4.4625]x$ (23)

In order to reduce the steady state error of the system output, a value of constant gain, *Nbar* should be added after the reference. With a full-state feedback controller all the states are feedback. The steady-state value of the states should be computed, multiply that by the chosen gain *K*, and use a new value as the reference for computing the input. Figure 5 shows the estimation of states by ELO.

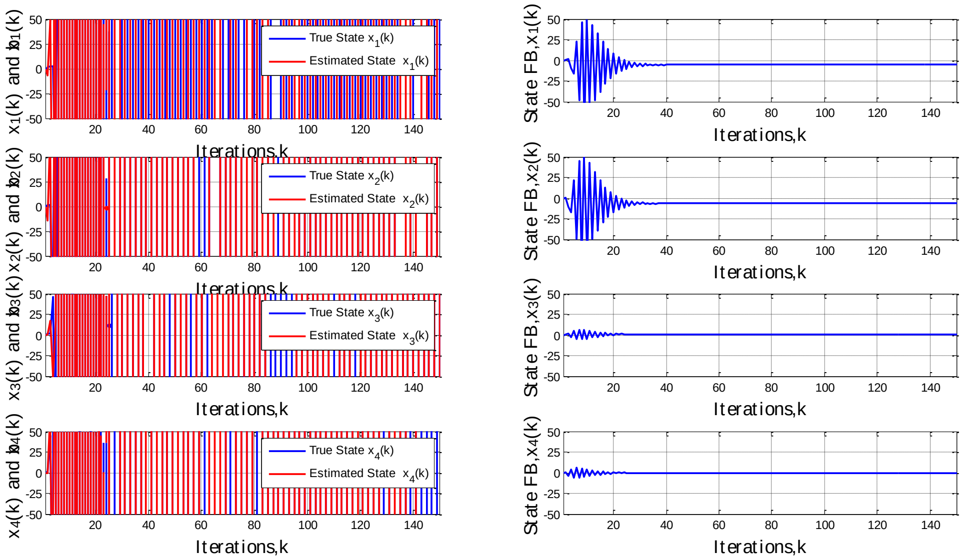

Fig.5.State Estimation by Luenberger Observer

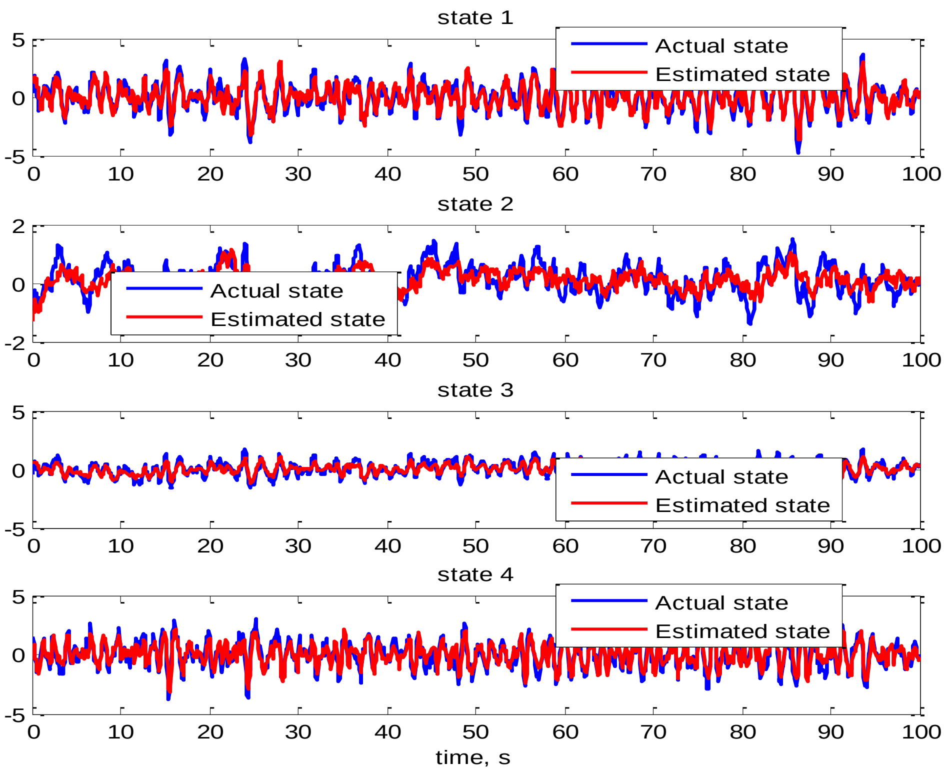

Fig.6.State Estimation by EKF

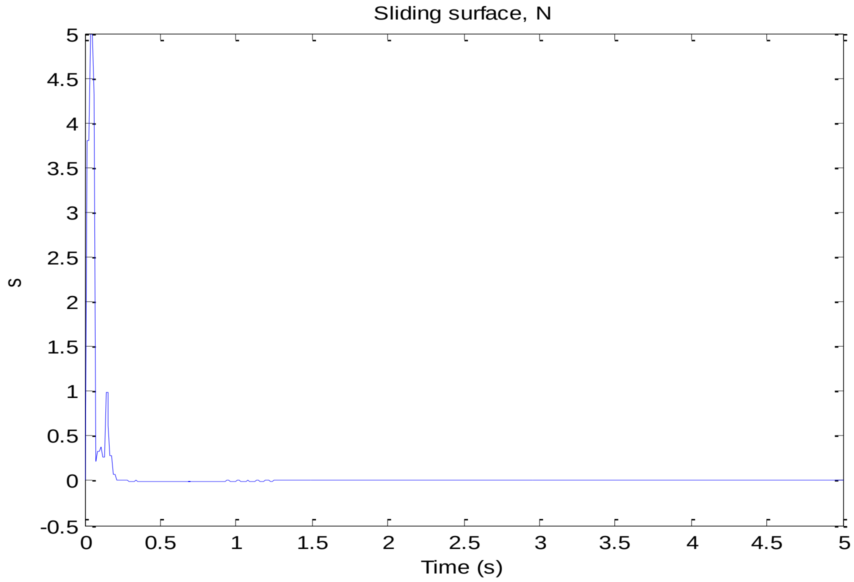

Fig.6.State Estimation by SMO

V. CONCLUSION

The rapid and continuing growth in the area of observer designed for nonlinear systems both from practical and theoretical point of view is presented. In this paper, three observer techniques EKF,Extended Luenverger and SMO are successfully designed and implemented in matlab/simulink for state estimation of inverted pendulum systems. Based on the results and the analysis, a conclusion has been made that both of All the successfully designed observers were compared. Simulation results show that SMO has better performance compared to other observer in estimation of states of the inverted pendulum system for state feedback law. Further improvement need to be done for SMO should be improved so that the chattering problem is reduce due to which overshoot for the pendulum's angle does not have very high range as required by the design criteria.

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