Dynamic mechanical analysis of composites via the linear hereditary theory

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***Abstract*- In this paper the authors use the hereditary theory of Boltzmann to describe the elastoviscous behavior of composites and introduce them to the dynamic mechanical analysis (DMA). In order to well describe the experimental data concerning the creep and stress relaxation in large time interval we used a sum of singular kernels in the integral hereditary s. In the linear case we have obtain the storage and loss modules as well as the loss factor as a function of the hereditary functions and kernels. One has obtained the loss factor by cycling of composites as a function of the strain amplitude and frequency. Experimental results for epoxy fiber composite with glass fibers (GFC) and natural fiber composite, namely Hemp 2.5 x 2.5 composite (HFC) [1] illustrate the applicability of the proposed approach.**

***Key words*: cycling, composites, viscoelasticity, hereditary theory, DMA, loss factor.**

I. INTRODUCTION

Composites are increasingly used in modern industry [1,2]. Epoxy fiber composites are elastoviscous solids. At small deformations they are linear, but at elevated deformations they possess non-linear behaviors. For glass fiber composites their creep and relaxation are linear according to the applied stresses for small deformations [3,4]. Thus, glass fiber and natural fiber composites require identification and description in the linear domain.
The more important parameters to describe the DMA are the complex modules. In the linear case one can use the constitutive hereditary s with singular kernels. Vibration attenuation capability of composites is the so-called loss factor. In the general viscoelastic case this factor can be defined as the ratio of the dissipated and the stored energy. These energies are related to the constitutive mechanical stress-strain s. It is well known that the Boltzmann hereditary theory using integral s of Volterra [3,4,5,6] can well describe the creep and stress relaxation of different viscoelastic solids. In this study we propose an analytical approach to describe the complex modules and the loss factor of such a composites as a function of the imposed strain (stress) amplitude and frequency.

II. COMPLEX MODULES IN THE LINEAR CASE

In the case of small deformations one can use the linear hereditary theory to describe the viscoelastic behavior of composites. First of all, we need to obtain the complex modules of such viscoelastic solids from relaxation (creep) functions and respective kernels.
The hereditary theory of Boltzmann, involving integral s of Volterra, is often used to describe the viscoelastic behavior of polymers, elastomers and composite materials. This theory is sometime expressed with the help of creep and relaxation functions, but sometimes one introduces the respective integral kernels [2,3,4]. From this point of view, we have obtained some useful relations concerning the complex modules and the loss factor of viscoelastic solids expressed as a function of the creep and relaxation functions and on the other hand as a function of the respective kernels in the linear case.
The constitutive viscoelastic relation in the linear hereditary Boltzmann's theory can be expressed as [3,4]

$$\sigma(t) = E\varepsilon(t) - E\int_0^t R(t-\tau)\varepsilon(\tau)d\tau \quad . \qquad (1)$$

Here $\sigma(t)$ is the stress as a function of the time, E is the Young modulus, $\varepsilon(t)$ is the imposed strain, $R(t-\tau)$ is the relaxation kernel which can be found from stress relaxation tests, t - the current time and $0 \le \tau \le t$. Equation (1) represents an integral equation of Volterra.
The solution of this integral equation looks like [3,4]

$$\varepsilon(t) = \frac{1}{E}\sigma(t) + \frac{1}{E}\int_0^t K(t-\tau)\sigma(\tau)d\tau \quad . \qquad (2)$$

Concerning the creep kernel $K(t-\tau)$, we can say the following. It is the resolving kernel of the relaxation one. As kernels in the integral equations of Volterra like equations (1, 2) it is recommended to take singular kernels which better describe the enhanced creep and relaxation rate at the beginning. In this work we assume the singular relaxation kernel of Koltunov [4]

$$R(t) = A\frac{e^{-\beta t}}{t^{\alpha}}, \qquad (3a)$$

which resolving kernel (the creep kernel) looks like [4]

$$K(t) = \frac{e^{-\beta t}}{t}\sum_{n=1}^{\infty} A\Gamma(\alpha)^n\, t^{\alpha n}/\Gamma(\alpha n). \qquad (3b)$$

Such a presentation of the elastoviscous linearity is used in many practical problems [4,5]. To better describe the viscoelastic behaviors of different materials we can use a sum of such kernels (3a) in equation (1) [5]. In this case the resolving kernel in equation (2) can be expressed as a sum of the respective resolving kernels (3b) [5].
The constitutive relation (1) can be obtained from the following more general linear relation

$$\sigma(t) = E\int_{-\infty}^{t} r(t-\tau)\dot{\varepsilon}(\tau)d\tau, \qquad (4)$$

where $r(t-\tau)$ is the relaxation function and $\dot{\varepsilon}(t)$ is the strain rate. This can be made integrating by parts equation (4) reduced to

$$\sigma(t) = E\int_{0}^{t} r(t-\tau)\dot{\varepsilon}(\tau)d\tau. \qquad (4a)$$

This reduction is possible in the case of no aging materials which kernels are time difference ones [3,4]. After integration by parts equation (4a) becomes

$$\sigma(t) = Er(0)\varepsilon(t) + E\int_{0}^{t} r'(t-\tau)\varepsilon(\tau)d\tau, \qquad (5a) \qquad \text{where } r'(t) = dr(t)/dt. \qquad (5b)$$

If we need to obtain the Hooke's low for $t \to 0$, we should impose $r(0) = 1$ (6a). Imposing $R(t) = -dr(t)/dt$ (6b), from equations (5) and (6) we arrive to equation (1).

II.1 COMPLEX MODULES AS A FUNCTION OF THE RELAXATION AND CREEP KERNELS

Suppose we have the following imposed strain low

$$\varepsilon_{imp}(t) = \varepsilon_o\, exp(i\omega t). \qquad (7)$$

Then the stress response has the form

$$\sigma(t) = \sigma_o\, exp(i(\omega t + \phi)). \qquad (8)$$

Here ϕ is the phase shift angle. Making the substitution

$$z = t - \tau, \quad \text{we have } \tau = t - z, d\tau = -dz;\ \tau \to t,\ z \to 0\,;\ \tau \to -\infty,\ z \to \infty. \qquad (9)$$

Replacing equation (7) into equation (1) and using equation (9) after some transformations we have

$$\sigma(t) = E\varepsilon_o[exp(i\omega t) - \int_0^t R(t-\tau)\, exp(i\omega t)]d\tau = E\varepsilon_o\, exp(i\omega t)[1 - R_c + iR_s], \qquad (10)$$

where

$$R_c = \int_0^{\infty} R(x)\, cos(\omega x)dx, \qquad R_s = \int_0^{\infty} R(x)\, sin(\omega x)dx. \qquad (11)$$

Here we need to take the kernels from equation (1) and equation (3a).

The complex module E^* can be defined as [3,5]

$$E^* = E' + iE''. \tag{12}$$

where E', E'' are the storage and the loss modules, $i = \sqrt{-1}$.
Using the correspondence principle [4] we can write

$$\sigma(t) = E^* \varepsilon(t) \quad . \tag{13}$$

Compare equation (10) and equations (12, 13). Thus, to the loss and storage modules as a function of the relaxation kernels, we have

$$E' = E(1 - R_c), \qquad E'' = E R_s \quad . \tag{14}$$

Employing the same manner with the help of equation (8) and equation (2) we express the storage and loss modules as a function of the creep kernels

$$E' = \frac{1+K_c}{(1+K_c)^2 + K_s^2}, \qquad E'' = \frac{K_s}{(1+K_c)^2 + K_s^2} \quad , \tag{15}$$

where

$$K_c = \int_0^\infty K(x)\cos(\omega x)dx, \qquad K_s = \int_0^\infty K(x)\sin(\omega x)dx. \tag{16}$$

II.2 COMPLEX MODULES AS A FUNCTION OF THE RELAXATION AND CREEP FUNCTIONS

Often, researchers do not dispose with the creep and relaxation kernels. Thei possesses only the creep or relaxation curves. Using the more general integral description - equation (4), we can express the complex modules as a function of the relaxation and creep functions.
Equation (7) concerning the imposed strains can be represented as follows

$$\varepsilon(\tau) = \varepsilon_o \cos(\omega\tau) + \varepsilon_o i \sin(\omega\tau). \tag{17a}$$

To the strain rate we have

$$\dot{\varepsilon}(\tau) = -\varepsilon_o \omega \sin(\omega\tau) + \varepsilon_o \omega i \cos(\omega\tau) \quad , \tag{17b}$$

Using the substitution (9) and replacing equation (17b) into equation (4) we arrive to

$$\sigma(t) = E\int_{-\infty}^{t} r(t-\tau)\,\dot{\varepsilon}(\tau)d\tau = -E\int_{\infty}^{0} r(z)\,\dot{\varepsilon}(t-z)dz, \tag{18}$$

Knowing that $\dot{\varepsilon} = -\varepsilon_0 \omega \sin \omega \tau$ and $\tau = t - z$, we can wright

$$\sigma(t) = E\int_0^\infty r(z)\varepsilon_0 \omega i \exp i\,\omega(t-z)dz \quad . \tag{18a}$$

From equation (18a) after some continuous mathematical transformations to the stress response we obtain

$$\sigma(t) = E\varepsilon_0 \omega \exp i\,\omega t[r_s + i r_c] \quad , \tag{19}$$

where

$$r_c = \int_0^\infty r(z)\cos(\omega z)dz, \qquad r_s = \int_0^\infty r(z)\sin(\omega z)dz \quad . \tag{20}$$

Now from equations (12) and (19) to the components of the complex module we have

$$E' = E\omega r_s, \qquad E'' = E\omega r_c. \tag{21}$$

To compare with equations (14).
Now in order to express the components of the complex module via the creep functions, we suppose an imposed cyclic stress

$$\sigma_{imp}(t) = \sigma_o\, exp(\,i\omega t) \quad , \text{ or } \quad \sigma(\tau) = \sigma_o\, cos(\,\omega\tau) + \sigma_o i\, sin(\,\omega\tau) \quad . \tag{22}$$

We know that $\varepsilon(t) = \frac{1}{E}\int_{-\infty}^{t} k(\tau)\sigma_0\omega i\, exp\, i\, \omega\tau d\tau$. Using the substitution (9) we obtain

$$\varepsilon(t) = \frac{1}{E}\int_0^{\infty} k(z)\sigma_0\omega i\, exp\, i\, \omega(t-z)dz \quad . \tag{23}$$

After some transformations over the integral in equation (23) we have

$$\varepsilon(t)E = \sigma_0\omega i\, exp\, i\, \omega t \int_0^{\infty} k(z)\, exp - i\omega z dz \quad , \tag{24}$$

We can simplify equation (24) using the well-known relations $exp\, i\, \omega t = cos\, \omega\, t + i\, sin\, \omega\, t$ and $i\, exp\, i\, \omega t = i\, cos\, i\, \omega t - sin\, \omega\, t$

$$\varepsilon(t)E = \sigma_0\omega i\, exp\, i\, \omega t \int_0^{\infty} k(cos\, \omega\, z - i\, sin\, \omega\, z)dz \quad , \tag{24a}$$

One can reduce the integral in equation (24a) to

$$\int_0^{\infty} k(cos\, \omega\, z - i\, sin\, \omega\, z)dz = \left(\int_0^{\infty} k(z)\, cos\, \omega\, zdz - i\int_0^{\infty} k(z)\, sin\, \omega\, zdz\right).$$

Introducing this in equation (24a) we have

$$\varepsilon(t)E = \sigma_0\omega i\, exp\, i\, \omega t\left[\int_0^{\infty} k(z)\, cos\, \omega\, zdz - i\int_0^{\infty} k(z)\, sin\, \omega\, zdz\right] \quad . \tag{24b}$$

Putting

$$k_c = \int_0^{\infty} k(z)\, cos(\,\omega z)dz, \qquad k_s = \int_0^{\infty} k(z)\, sin(\,\omega z)dz \quad . \tag{25}$$

in equation (24b), we have

$$\varepsilon(t)E = \sigma_0\omega i\, exp\, i\, \omega t[k_c - ik_s] \quad . \tag{26}$$

After simplification of equation (26) and using equations (27) and (12, 13) we have

$$\varepsilon(t)E = \sigma_0\omega\, exp\, i\, \omega t[k_s + ik_c] \; , \quad (27a) \qquad \varepsilon = (E' + iE'')^{-1}\sigma_0\, exp\, i\, \omega t \quad . \quad (27b)$$

Thus, we can write $E' + iE'' = E\omega[k_s + ik_c]$.

Finally, we obtain the components of the complex module via the creep function (see equations (25))

$$E' = E\omega k_s \quad , \qquad E'' = E\omega k_c \quad . \tag{28}$$

III. LOSS FACTOR

Damping is generally characterized by the amount of energy dissipated under steady harmonic motion. The most common measure of this dissipation is the loss factor, which can be defined as the ratio of the average energy dissipated per radian to the peak potential (stored) energy during a cycle and in the case of sinusoidal loading can be expressed as [6]

$$\eta = \frac{D}{2\pi U} = tg\phi = E''/E'. \tag{29}$$

Using equations (14 and 15) to the loss factor in this case we obtain

$$\eta = E^{"}/E' = \frac{R_s}{1-R_c} = \frac{K_s}{1+K_c}. \tag{30}$$

To the loss factor in the case, we dispose with the relaxation function, using equation (21) we have

$$\eta = E^{"}/E' = \frac{E\omega r_c}{E\omega r_s} = \frac{r_c}{r_s} \quad . \tag{31}$$

To the loss factor in the case, we dispose with the creep function, using equation (28) we arrive to

$$\eta = E^{"}/E' = \frac{E\omega k_c}{E\omega k_s} = \frac{k_c}{k_s} \quad . \tag{32}$$

Finally, we arrive to express the loss factor as a function of the creep and relaxation functions and kernels

$$\eta = r_c/r_s = k_c/k_s \quad . \tag{33}$$

IV. EXPERIMENTAL RESULTS AND COMPARISONS

The relative stress relaxation using the hereditary integral equation (1) with sum of three kernels (equation (3a)) for glass fiber composite and hemp composite are illustrated on **Fig. 1.**

The imposed sinusoidal strains and the stress responses are shown in **Fig. 2**. Here the angular frequency was 0.1 [rad/s].

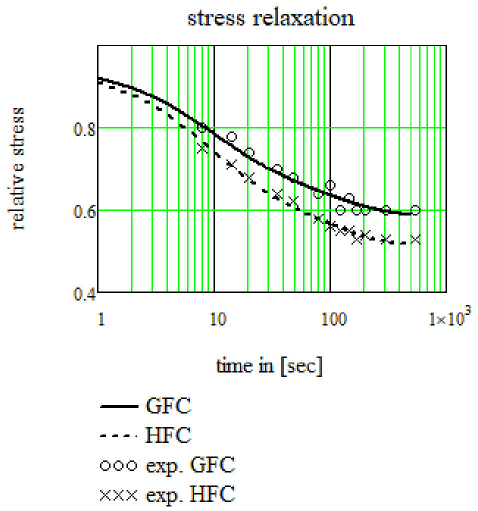


Fig.1 Normalized stress relaxation

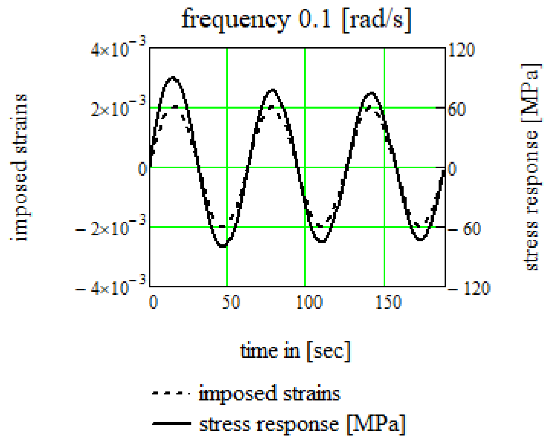


Fig.2 Sinusoidal imposed strains and stress response

Equations (14) on the basis of equations (11) concerning the storage and loss modules are illustrated in **Fig. 3** and **Fig.4**. On the next **Fig.5** and **Fig.6** we have illustrated these dependances for our two composite materials. As on can see the hemp composite possesses higher loss factor.

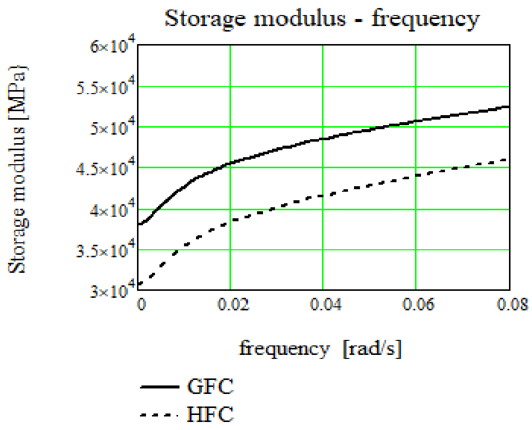


Fig.3 Storage modulus via frequency

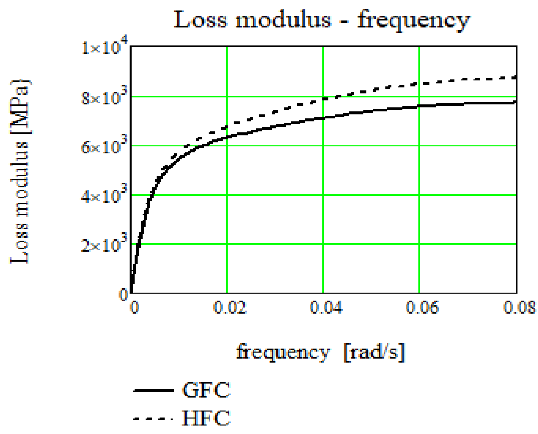


Fig.4 Loss modulus via frequency

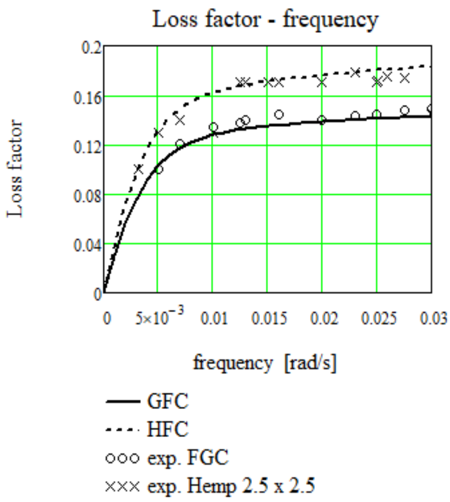


Fig.5 Loss factor via frequency

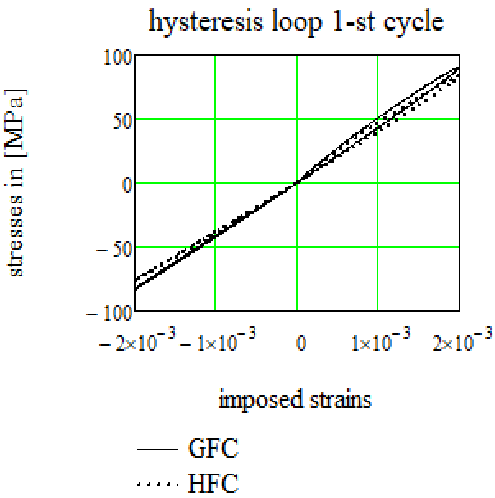
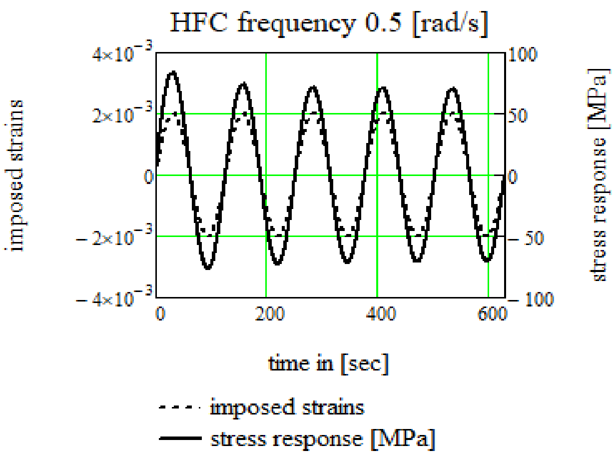


Fig.6 Hysteresis loops-first cycle (sinusoidal imposed strains)

Figures 7 and **8** illustrate the sinusoidal imposed strains and the respective stress responses for HFC and GFC respectively. In order to make the respective comparisons we have imposed the same sinusoidal strains lows for both composite materials.



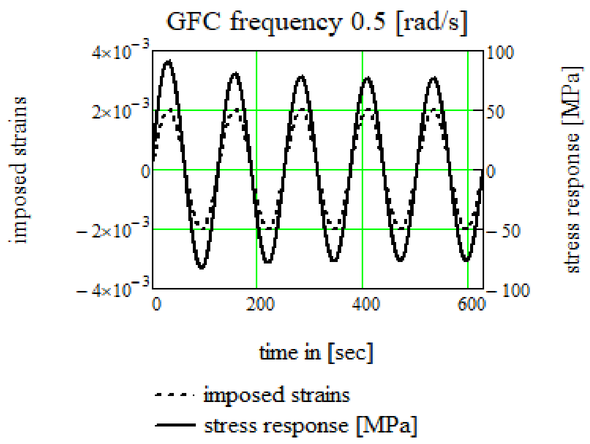


Fig.7 Sinusoidal imposed strains and stress response (HFC) **Fig.8 Sinusoidal imposed strains and stress response (GFC)**

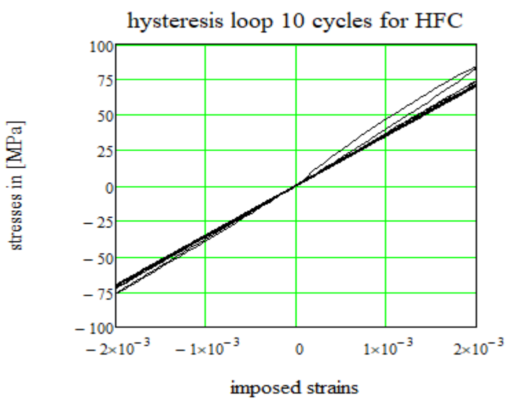
On the next **Fig.9** we have illustrated the hysteresis loops for HFC for 10 cycles with sinusoidal imposed strains with angular frequency 0.5 [rad/s] and strain amplitude 0.002. Similar result is observed for the GFC. In the positive strain-stress zone the hysteresis loop has greater surface in the first cycle. This can be due to the more intensive damage in the positive zone. Increasing cycle numbers do not influence the loss factor after the first cycle.

On **Fig. 10** one can see the influence of the strain amplitude on the hysteresis loop area in the positive zone. After the first cycle the hysteresis loop area remains relatively constant.

The module of elasticity and the kernel parameters for the FGC and the HFC were respectively:

E = 65000 [MPa], $A_1 = 0.001, A_2 = 0.015, A_3 = 0.018, \alpha_1 = 0.58, \alpha_2 = 0.94, \alpha_3 = 0.3, \beta_1 = 0.02, \beta_2 = 0.1, \beta_3 = 0.005$ for the FGC.

E = 60000 [MPa], $A_1 = 0.02, A_2 = 0.02, A_3 = 0.018, \alpha_1 = 0.58, \alpha_2 = 0.94, \alpha_3 = 0.3, \beta_1 = 0.02, \beta_2 = 0.1, \beta_3 = 0.005$ for the HFC.



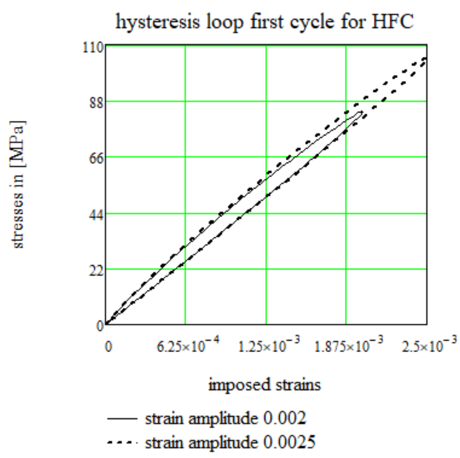


Fig.9. Hysteresis loops for 10 cycles (HFC) **Fig.10. Hysteresis loops first cycles (GFC-two frequencies)**

The experimental results are showed with circles for the FGC and with sterns for the HFC. As one can see the storage and loss modules increase with increasing strain amplitude and frequency. Our materials are hard and very weakly deformable; therefore the applied strains have very small amplitude.

We have used here a special dispositive to impose sinusoidal imposed strains. The description of this dispositive can be found in [8,9]. The data in **Fig.3** and **Fig.4** are obtained from the hysteresis curve measuring the hysteresis loop area and averaging.
The experimental samples had the following dimensions: cross section 15 x 6 [mm] and distance between the grips of the device 70 [mm].

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