Nonlinear dynamic mechanical analysis of composites as a function of strain amplitude and frequency

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Abstract- **In this paper the authors use the nonlinear hereditary theory of Boltzmann to describe the elastoviscous behavior of glass fiber composites and introduce them to the dynamic mechanical analysis (DMA). In order to well describe the experimental data concerning the creep and stress relaxation in large time interval and large strains, they used a sum of singular kernels and introduce the nonlinear Ogden Equation in the integral hereditary theory. One has obtained the loss factor by cycling of composites as a function of the strain amplitude and frequency for two imposed regimes – sinusoidal and pulsation loads. Experimental results for epoxy fiber composite with 30% glass fibers (GFC) [1] illustrate the applicability of the proposed approach.**

Key words: **cycling, glass fiber composites, nonlinear viscoelasticity, hereditary theory, DMA, loss factor**

I. INTRODUCTION

Glass fiber composites are increasingly used in modern industry [1-4]. Glass fiber composites are elastoviscous solids. In the case of enhanced strains, they possess nonlinear behaviors. Their creep is also nonlinear according to the applied stresses [5,6]. The last non-linearity can be observed excluding the time from the creep curves (the so-called isochrones). Thus, glass fiber composites require identification and description of these nonlinearities. Here was used epoxy matrix composite with 30% glass fibers volume fraction [3].The more important parameter to describe the vibration behavior of glass fiber composites is the so-called loss factor [1,2]. In the general nonlinear viscoelastic case this factor can be defined as the ratio of the dissipated and the stored energy. These energies are related to the constitutive mechanical stress-strain equation. It is well known that the Boltzmann hereditary theory using integral equations of Volterra [6-10] can well describe the creep and stress relaxation of different viscoelastic solids. In this study we propose an analytical approach to describe the loss factor of glass fiber composites as a function of the imposed strain amplitude and frequency using as constitutive relations nonlinear integral equations.

II. GENERAL FRAMEWORK

In the case of Assuming similarity of the isochrones stress relaxation curves, let introduce the following nonlinear integral Equation to describe the mechanical behavior of such a solid [6-9]

$$\sigma(t) = \phi(\varepsilon(t)) - \int_0^t R(t,\tau)\phi(\varepsilon(\tau))d\tau \quad . \quad (1)$$

Here $\sigma(t)$ is the stress as a function of time t, $\varepsilon(t)$ is the imposed strain, $R(t,\tau)$ is the relaxation kernel, which can be found from stress relaxation tests, $\phi(\varepsilon(t))$ is the instantaneous stress-strain curve. To well describe this curve one can apply the Ogden relation [11]

$$\phi(\varepsilon) = \sum_{i=1}^{3} \mu_i(\lambda(\varepsilon)^{\kappa_i - 1} - \lambda(\varepsilon)^{-\frac{\kappa_i}{2} - 1}), \quad (2)$$

here $\mu_1, \mu_2, \mu_3, \kappa_1, \kappa_2, \kappa_3$ are parameters obtained from instantaneous stress-strain test and the stretch λ is related with the engineering strain as follows $\lambda(\varepsilon) = 1 + \varepsilon$.

The solution of Equation (1) can be represented as follows [6-9]

$$\phi(\varepsilon(t)) = \sigma(t) \quad + \int_0^t K(t,\tau)\sigma(\tau))d\tau. \quad (3)$$

To obtain the strain creep curve (nonlinear creep) one should use the inverse function $\psi(\varepsilon(t)) = \phi^{-1}(\varepsilon(t))$. The above-mentioned integral equations of Volterra have been longtime employed to describe the viscoelastic behavior of polymers, glass fiber composites and other materials [6-8]. Due to the extremely high strain (stress) rate at the beginning in creep (relaxation) conditions one needs to introduce singular kernels. In order to increase the creep or stress relaxation time interval and thus well describe the experimental data from the beginning to the end in the case of large time interval, we have proposed in our previous work [9] to involve in the hereditary theory a sum of singular kernels as follows

$$R(t) = \sum_{n=1}^{N} R_i(t), \quad \text{with} \quad R_i(t) = A_i \frac{e^{-\beta_i t}}{t^{\alpha_i}}. \quad (5)$$

In this case the resolving kernel is [6-8]

$$K_i(t) = \frac{e^{-\beta_i t}}{t} \sum_{n=1}^{\infty} A_i \Gamma(\alpha_i)^n \, t^{\alpha_i n} / \Gamma(\alpha_i n) \ . \quad (5a)$$

Here $\Gamma(\alpha)$ is the gamma function.

In the case of sinusoidal loading, we introduce the imposed strain as follows

$$\varepsilon_{imp}(t) = \varepsilon_o \, exp(\, i\omega t) \quad . \quad (6a)$$

In this case the stress response has the form

$$\sigma(t) = \sigma_o \, exp(\, i(\omega t + \phi)) \ . \quad (6b)$$

In the case of imposed positive sinusoidal strains (pulsations), we have

$$\varepsilon_{imp}(t) = \varepsilon_o + \varepsilon_o \sin(\, \omega t - \pi/2) \ . \ (6c)$$

In this case the stress response can be obtained using Equations (1, 2) and (6c).

In Equations 6 ϕ is the phase angle shift, ε_o , σ_o are the imposed strain and stress amplitude and ω is the angular frequency related with the imposed period T as follows

$$T = 2\pi/\omega. \quad (7)$$

On the other hand, to the stored and the dissipated energy per cycle in the more general case (nonlinearity) as a function of the strain amplitude ε_o and the angular frequency ω in the case of pulsations, we can respectively write [10]

$$U(\varepsilon_o) = \int \sigma d\varepsilon = \int_{(n-1)T}^{(n-1)T+T/2} \sigma(t,\varepsilon_o)\dot{\varepsilon}(t,\varepsilon_o)dt, \qquad D(\varepsilon_o) = \oint \sigma d\varepsilon = \int_{(n-1)T}^{nT} \sigma(t,\varepsilon_o)\dot{\varepsilon}(t,\varepsilon_o)dt. \quad (8)$$

$$U(\omega) = \int \sigma d\varepsilon = \int_{2(n-1)\pi/\omega}^{(2n-1)\pi/\omega} \sigma(t,\omega)\dot{\varepsilon}(t,\omega)dt, \qquad D(\omega) = \oint \sigma d\varepsilon = \int_{2(n-1)\pi/\omega}^{2n\pi/\omega} \sigma(t,\omega)\dot{\varepsilon}(t,\omega)dt. \quad (9)$$

Here n = 1, 2…N, where N is the final cycle number.

It is evident that these energies are cycle number dependent. But this dependence is not strong. To derive the lower and upper limits in Equation 9 we have used Equation 7.

In the case of imposed sinusoidal load, we need to change the upper limits for the stored energy as follows: $(n-1)T + T/4$ and $(2n-1.5)\pi/\omega$ respectively.

Damping is generally characterized by the amount of energy dissipated under steady harmonic motion. The most common measure of this dissipation is the loss factor, which can be defined as the ratio of the average energy dissipated per radian to the peak potential energy during a cycle. Using this way, we can obtain the loss factor as a function of the strain amplitude ε_o and the angular frequency ω. Using Equations 8 and Eq.9 we respectively have [10]

$$\eta(\varepsilon_o) = \frac{D(\varepsilon_o)}{2\pi U(\varepsilon_o)} \ , \qquad \eta(\omega) = \frac{D(\omega,)}{2\pi U(\omega)} \quad . \quad (10)$$

This approach is valid in the case of imposed strains. If we need to obtain analogical results imposing the stresses, we should use Equation 3 with the resolving (creep) kernel - Equation 5a and using the inversion function of Equation 2. In this case the imposed sinusoidal stress should be presented in similar way as Equation 6 with imposed stress amplitude.

To obtain the damage $d\ (n)$ as a function of the cycle number n we need to define the damage for the *n-th* cycle as in [11]

$d = 1 - \frac{U(n)}{U_{in}}$, (11)

where $U(n)$ and U_{in} are the stored energies for the *n-th* cycle and the initial one. Then the relative damage accumulation per cycle is $d(n) = [d(n+1)-d(n)]/din$. After summation of the relative damage accumulation per cycle we obtain the damage as a function of the cycle numbers

$$D(N) = \sum_{n=1}^{N} d(n) . \quad (12)$$

When we have strain-controlled test, these energies can be expressed as the energy stored upon loading from zero to maximum strain [11]

$$U = \int_0^{\varepsilon_{max}} \sigma d\varepsilon \quad . \quad (13)$$

In the case of pulsations for the *n-th* cycle we can respectively write (see Equations (8))

$$U(\varepsilon_o) = \int \sigma d\varepsilon = \int_{(n-1)T}^{(n-0.5)T} \sigma(t,\varepsilon_o)\dot{\varepsilon}(t,\varepsilon_o)dt = \int_{(n-1)2\pi/\omega}^{(2n-1)\pi/\omega} \sigma(t,\varepsilon_o)\dot{\varepsilon}(t,\varepsilon_o)dt. \quad (13a)$$

In the case of sinusoidal load we should change the upper integral limit as follows $(n\text{-}0.75)T$ or $(2n\text{-}1.5)\pi/\omega$. Here $n = 1,2\ldots N$. In the case of imposed stresses (stress-controlled test) we should use another definition concerning the stored energy [11], namely the energy stored upon loading from zero to maximum stress with the same integral limits as in Equation 13a

$$U = \int_0^{\sigma_{max}} \sigma d\varepsilon \ . \quad (13b)$$

Note that in the case of small strains (linear stress-strain relation) these energies coincide [10].

III. EXPERIMENTAL RESULTS AND COMPARISONS

For glass fiber composite (GFC) with 30% fiber volume fraction we have obtain the following Ogden parameters [12,13]

$$\mu_1 = 9.93, \mu_2 = -39.9, \mu_3 = 1.2, \kappa_1 = 29.95, \kappa_2 = -59.8, \kappa_3 = 5.4 \ .$$

To the hereditary kernel parameters, we have:

$$\alpha_1 = 0.58, \alpha_2 = 0.94, \alpha_3 = 0.31, \beta_1 = 0.02, \beta_2 = 0.1, \beta_3 = 0.005, A_1 = 0.00141, A_2 = 0.015, A_3 = 0.018.$$

The respective relative stress relaxation curve - Equation 1 is shown in **Fig.1**. In **Figures 2,3** we have plotted the imposed strain in the cases of sinusoidal and pulsations regime and the respective stress response according to Equations 1,2 and Eq. 6a,6c.

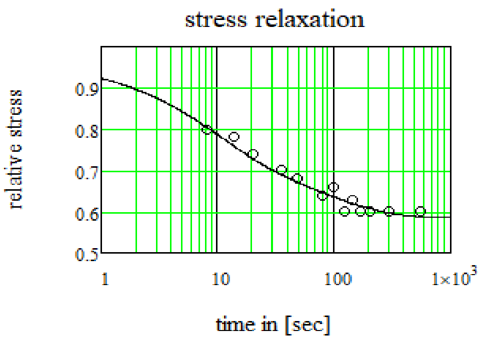


Fig.1 Stress relaxation curve

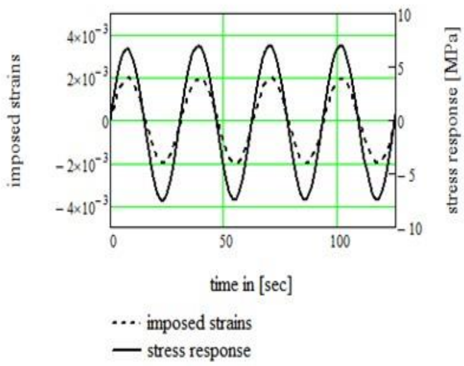
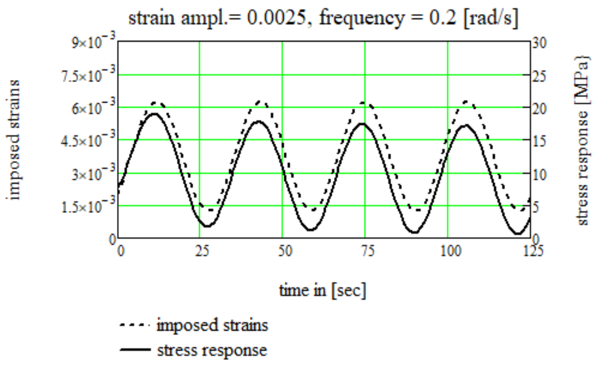


Fig.2 Imposed sinusoidal strains and stress response

On **Fig.4** we have illustrated the strain-controlled hysteresis loop for our GFC according to Equations 1 and Eq. 6a in the case of sinusoidal imposed strains.



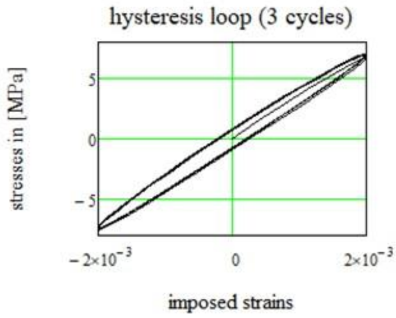
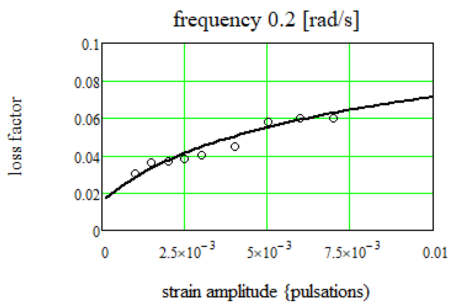


Fig.3 Imposed strain pulsations and stress response **Fig.4 Hysteresis loop – sinusoidal imposed load**

In the next **Figures 5** and **6** one can see the loss factor as a function of the imposed angular strain amplitude corresponding to the first Equation 10. In these figures the angular frequency was 0.2 [rad/s]. The loss factor increases continuously with strain amplitude. More intensively in the case of sinusoidal load. The imposed strain lows follow Equations 6c and 6a respectively.



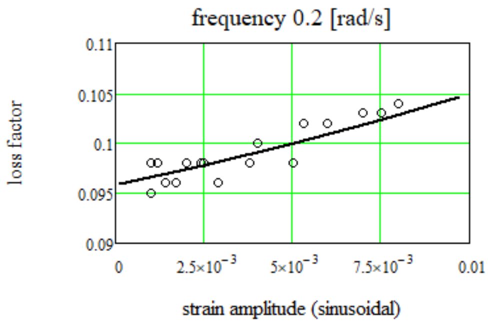
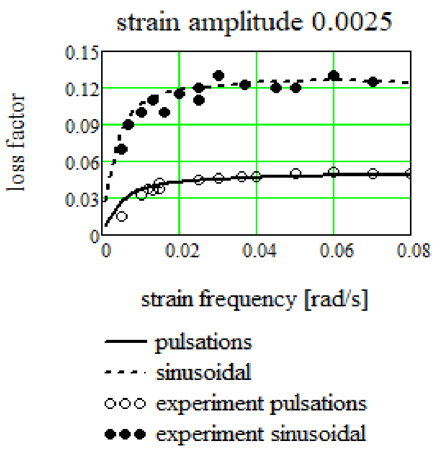


Fig.5. Loss factor as a function of the strain amplitude (pulsations)
Fig.6. Loss factor as a function of the strain amplitude (sinusoidal load)

The loss factor as a function of the strain frequency from the second Equation 10 is illustrated in **Fig.7** in the case of pulsations and sinusoidal load respectively. As on can see on **Fig.7**, the loss factor increases with increasing frequency, but after 0.04 [rad/s] remains constant.

In all the figures the experimental data are plotted with circles.



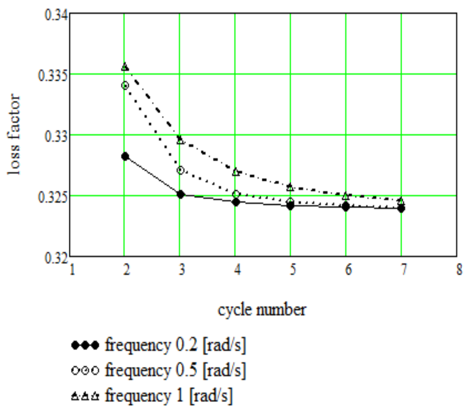
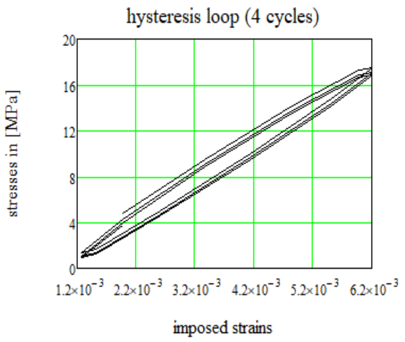


Fig.7 Loss factor as a function of the frequency for two regimes
Fig.8 Loss factor as a function of the strain cycle number *n* for three frequencies (sinusoidal) load

At the beginning the loss factor decrease with cycle numbers, but after the seventh cycle remains constant – see **Fig.8**. This example concerns the sinusoidal load.

In **Fig.9** one can see the hysteresis loops for 4 cycles.

On the next **Fig.10** one can see the curve of the damage evolution in the pulsation and sinusoidal loading cases with frequency 0.2 [rad/s] and strain amplitude 0.0025, according to Equations 12 and taking into account Equations 11, 6a and Eq. 6c.



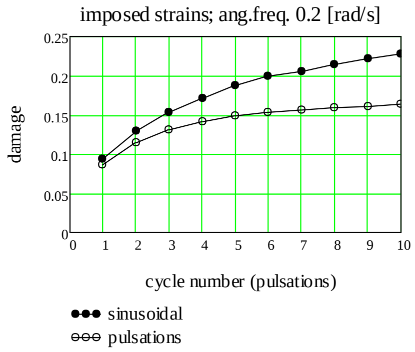


Fig.9 Hysteresis loops for 4 cycles – imposed pulsations
Fig.10 Damage evolution in the case of two regimes

From these figures one can made the conclusion that enhancement in the strain amplitude considerably increase the losses, whereas enhancement in frequency slowly increases the losses. On the other hand the sinusoidal regime is harder as the pulsation's one. Thus, the losses in the first mentioned regime are greater as in the second one.

In order to impose different strain and stress regimes, we have used here a special dispositive to impose sinusoidal imposed strains. The description of this dispositive can be found in [14,15].

Note that the averaged experimental curves are obtained by smoothing data using the Mathcad software averaging process in the case of data scattered along a band whose width fluctuates considerably.

In this work we have used a glass-fiber composite [3] produced in the URCA of Reims-France.

IV. CONCLUSIONS

Using nonlinear integral Equations with three singular kernels, we have obtained the stress responses, the hysteresis loops, the stored and dissipated energies, the loss factors and the damage evolution by cycling in the case of imposed strains and stresses.

The experimental hysteresis curves agree with the theoretical ones obtained from the stress (strain) responses by imposing sinusoidal and pulsation loads to the strain (stress) laws.

The proposed approach to obtain the loss factor of glass fiber composites in the non-linear case using a sum of relaxation kernels in the hereditary theory well describe the mechanical behavior by cycling loading.

The experimental data confirm this approach. The nonlinear hereditary theory works well due to the great number of parameters and the singularity of the kernels.

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