

A Low Power Fault Tolerant Reversible Decoder Using MOS Transistor

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Abstract— This paper demonstrates the reversible logic synthesis for the n -to- $2n$ decoder, where n is the number of data bits. The circuits are designed using only reversible fault tolerant Fredkin and Feynman double gates. Thus, the entire scheme inherently becomes fault tolerant. Algorithm for designing the generalized decoder has been presented. In addition, several lower bounds on the number of constant inputs, garbage outputs and quantum cost of the reversible fault tolerant decoder have been proposed. Transistor simulations of the proposed decoder are shown using standard p-MOS 901 and n-MOS 902 model with delay of 0.030 ns and 0.12 μm channel length, which proved the functional correctness of the proposed circuits. The comparative results show that the proposed design is much better in terms of quantum cost, delay, hardware complexity and has significantly better scalability than the existing approach.

Keywords— Decoder, Delay, Garbage Output, Low Power Design, Quantum Cost, Reversible & Fault Tolerant Computing.

I. INTRODUCTION

Logic plays an extensively important role in low power computing as it recovers from bit loss through unique mapping between input and output vectors [1]. No bit loss property of reversible circuitry results less power dissipation than the conventional one [2]. Moreover, it is viewed as a special case of quantum circuit as quantum evolution must be reversible [3]. Over the last two decades, reversible circuitry gained remarkable interests in the field of DNA-technology [4], nano-technology [5], optical computing [6], program debugging and testing [7], quantum dot cellular automata [8], discrete event simulation [9] and in the development of highly efficient algorithms [10].

On the other hand, parity checking is a popular mechanisms for detecting single level fault. If the parity of the input data is maintained throughout the computation, then intermediate checking wouldn't be required and an entire circuit can preserve parity if its individual gate is parity preserving [11].

Reversible fault tolerant circuit based on reversible fault tolerant gates allows to detect faulty signal in the primary outputs of the circuit through parity checking [12]. Hardware of digital communication systems relies heavily on decoders as it retrieve information from the coded output.

Decoders have also been used in the memory and I/O of microprocessors [13]. In [7], a reversible fault tolerant decoder was designed, but it was not generalized and compact. Therefore, this paper investigates the generalized design methodologies of reversible fault tolerant decoders.

II. BASIC DEFINATIONS AND LITERATURE REVIEW

This section formally defines reversible gate, garbage output, delay, hardware complexity and presents popular reversible fault tolerant gates along with their input-output specifications, transistor and quantum equivalent representations.

A. Reversible and Fault Tolerant

An $n \times n$ reversible gate is a data stripe block that uniquely maps between input vector $I_v = (I_0, I_1, \dots, I_{n-1})$ and output vector $O_v = (O_0, O_1, \dots, O_{n-1})$ denoted as $I_v \leftrightarrow O_v$. Two prime requirements for the reversible logic circuit are as follows [14]:

- There should be equal number of inputs and outputs.
- There should be one-to-one correspondence between inputs and outputs for all possible input-output sequences.

A *Fault tolerant gate* is a reversible gate that constantly preserves same parity between input and output vectors. More specifically, an $n \times n$ fault tolerant gate clarify the following property between the input and output vectors [12]:

$$I_0 \oplus I_1 \oplus \dots \oplus I_{n-1} = O_0 \oplus O_1 \oplus \dots \oplus O_{n-1} \quad (1)$$

Parity preserving property of Eq.1 allows to detect a faulty signal from the circuit's primary output. Researchers [11], [12], [15] have showed that the circuit consist of only reversible fault tolerant gates preserves parity and thus able to detect the faulty signal at its primary output.

B. Qubit and Quantum Cost

The main difference between the *qubits* and conventional bits is that, qubits can form linear combination of states $|0\rangle$ or $|1\rangle$ called superposition, while the basic states $|0\rangle$ or $|1\rangle$ are an orthogonal basis of two-dimensional complex vector [3]. A superposition can be denoted as, $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$, which means the probability of particle being measured in states 0 is $|\alpha|^2$, or results 1 with probability $|\beta|^2$, and ofcourse $|\alpha|^2 + |\beta|^2 = 1$ [16]. Thus, information stored by a qubit are different when given different α and β . Because of such properties, qubits can perform certain calculations exponentially faster than conventional bits. This is one of the main motivation behind the quantum computing. Quantum computer demands its underneath circuitry be reversible [1]~[6].

The *quantum cost* for all 1×1 and 2×2 reversible gates are considered as 0 and 1, respectively [6]~[14]. Hence, quantum cost of a reversible gate or circuit is the total number of 2×2 quantum gate used in that reversible gate or circuit.

C. Delay, Garbage Output and Hardware Complexity

The delay of a circuit is the delay of the critical path. The path with maximum number of gates from any input to any output is the critical path [1]. There may be more than one critical path in a circuit and it is an NP-complete problem to find all the critical paths [17]. So, researchers pick the path which is the most likely candidates for the critical paths [18]. Unused output of a reversible gate (or circuit) is known as *garbage output*, i.e., the output which are needed only to maintain the reversibility are the garbage output. The number of basic operations (Ex-OR, AND, NOT etc.) needed to realize the circuit is referred to as the *hardware complexity* of the circuit. Actually, a constant complexity is assumed for each basic operation of the circuit, such as, α for Ex-OR, β for AND, γ for NOT etc. Then, total number of operations are calculated in terms of $\alpha, \beta,$ and γ .

D. Popular Reversible Fault Tolerant Gates

1) **Feynman Double Gate:** Input vector (I_v) and output vector (O_v) for 3×3 reversible Feynman double gate (F2G) is defined as follows [19]: $I_v = (a, b, c)$ and $O_v = (a, a \oplus b, a \oplus c)$. Block diagram of F2G is shown in Fig. 1(a). Fig. 1(b) represent the quantum equivalent realization of F2G. From Fig. 1(b) we find that it is realized with two 2×2 Ex-OR gate, thus its quantum cost is two (Sec. II-B). According to our design procedure, twelve transistors are required to realize F2G reversibly as shown in Fig. 1(c). Fig. 3(a) represents the corresponding timing diagram of F2G.

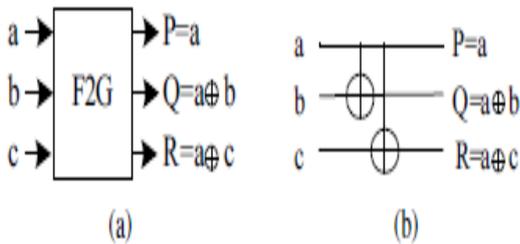
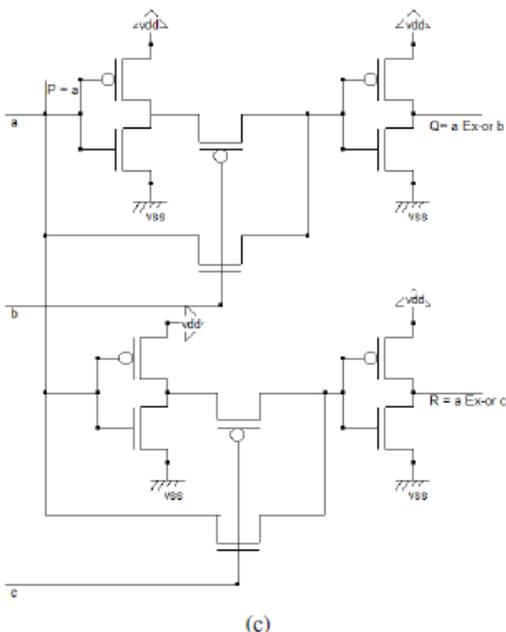


Fig. 1: Reversible Feynman double gate (a) Block diagram (b) Quantum equivalent realization (c) Transistor realization



2) **Fredkin Gate:** The input and output vectors for 3×3 Fredkin gate (FRG) are defined as follows [20]: $I_v = (a, b, c)$ and $O_v = (a, a \oplus b \oplus ac, a \oplus c \oplus ab)$. Block diagram of FRG is shown in Fig. 2(a). Fig. 2(b) represents the quantum realization of FRG. In Fig. 2(b), each rectangle is equivalent to a 2×2 quantum primitives, therefore its quantum cost is considered as one [13]. Thus total quantum cost of FRG is five. To realize the FRG, four transistors are needed as shown in Fig. 2(c) and its corresponding timing diagram is shown in Fig. 3(b).

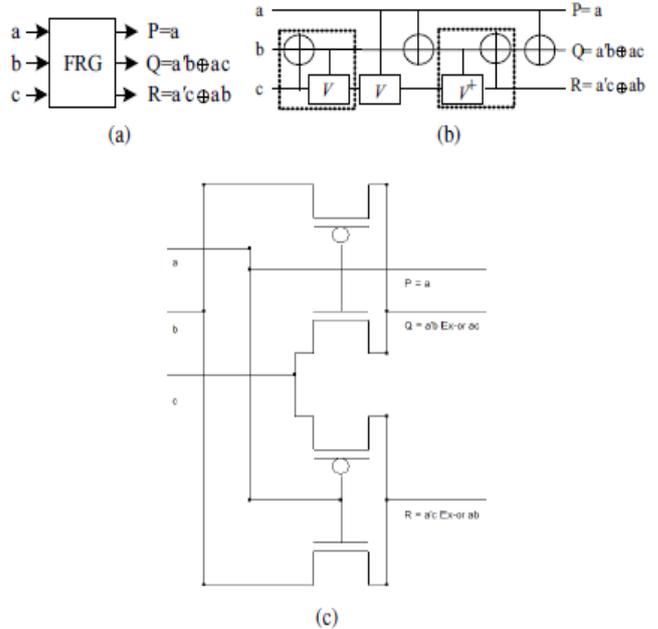


Fig. 2: Reversible Fredkin gate (a) Block diagram (b) Quantum equivalent realization (c) Transistor realization

Reversible Fredkin and Feynman double gate obey the rule of Eq.1. The fault tolerant (parity preserving) property of Fredkin and Feynman double is shown in Table. I.

TABLE I: Truth table for F2G and FRG

Input			Output of F2G			Output of FRG			Parity
A	B	C	P	Q	R	P	Q	R	
0	0	0	0	0	0	0	0	0	Even
0	0	1	0	0	1	0	0	1	Odd
0	1	0	0	1	0	0	1	0	Odd
0	1	1	0	1	1	0	1	1	Even
1	0	0	1	1	1	1	0	0	Odd
1	0	1	1	1	0	1	1	0	Even
1	1	0	1	0	1	1	0	1	Even
1	1	1	1	0	0	1	1	1	Odd

E. Decoder

Decoders are the collection of logic gates fixed up in a specific way such that, for an input combination, all outputs terms are low except one. These terms are the *minterms*. Thus, when an input combination changes, two outputs will change. Let, there are n inputs, so number of outputs will be $2n$. There are several designs of reversible decoders in the literature. To the best of our knowledge, the designs from [7] is the only reversible design that preserve parity too.

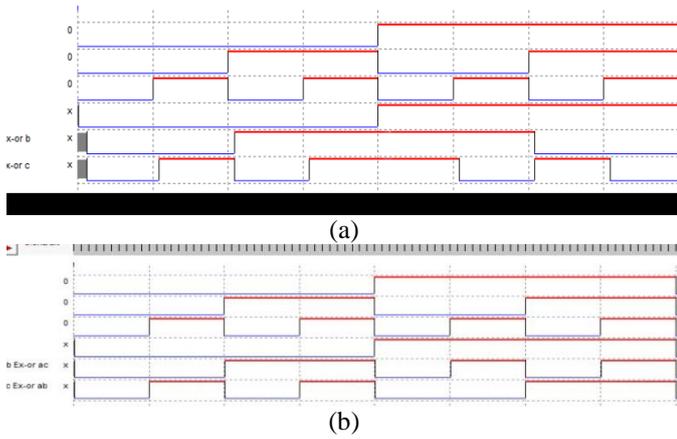


Fig. 3: Simulation with DSCH-2.7 [21] of the transistor realization of (a) Feynman double gate (b) Fredkin gate.

III. PROPOSED REVERSIBLE FAULT TOLERANT DECODER

Considering the simplest case, $n=1$, we have a 1-to-2 decoder. Only a $F2G$ can work as 1-to-2 Reversible Fault Tolerant Decoder (RFD) as shown in Fig. 4(a) and its corresponding timing diagram is shown in Fig. 4(b). From now on, we denote a reversible fault tolerant decoder as RFD

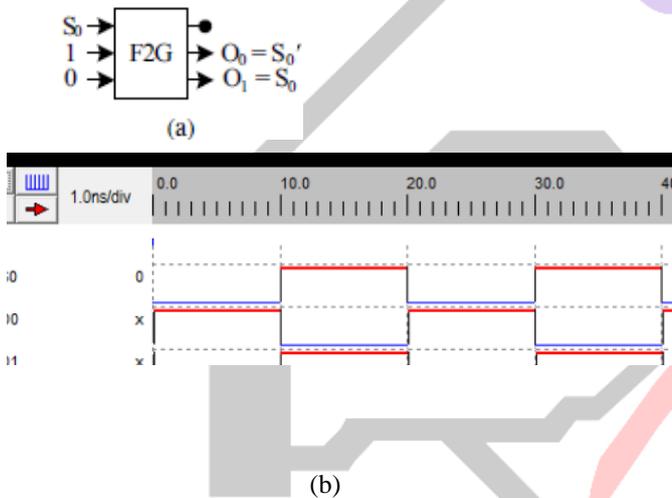


Fig. 4: Proposed 1-to-2 RFD (a) Architecture (b) Simulation with DSCH-2.7 [21].

The Fig. 5(a) and Fig. 5(d) represent the architecture of 2-to-4 and 3-to-8 RFD, respectively. Timing diagram of Fig. 5(a) is shown in Fig. 5(c). From Fig. 5(d), we find that 3-to-8 RFD is designed using 2-to-4 RFD, thus a schema of Fig. 5(a) is created which is shown in Fig. 5(b). Algorithm 1 presents the design procedure of the proposed n -to- $2n$ RFD. Primary input to the algorithm are n control bits. Line 6 of the proposed algorithm assigns the input to the Feynman double gate for the first control bit (S_0), whereas line 9 assigns first two inputs to the Fredkin gates for all the remaining control bits. Line 10-12 assign third input to the Fredkin gate for $n = 2$.

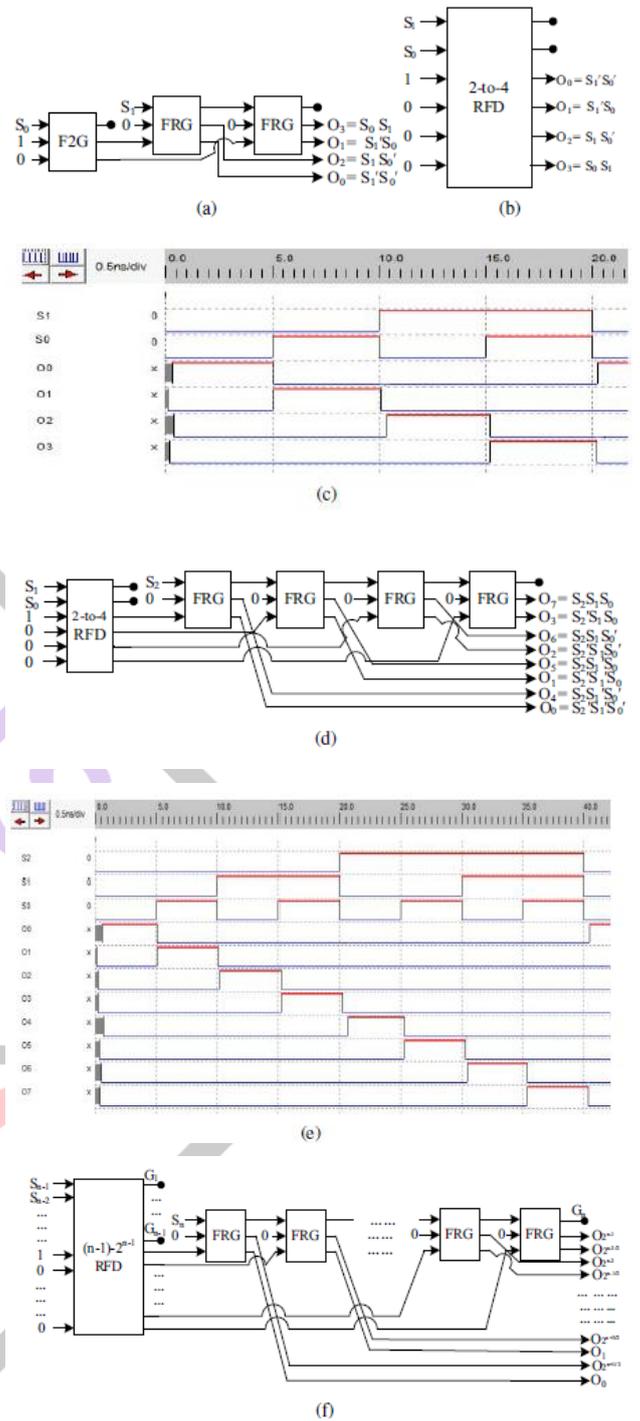


Fig. 5: (a) Block diagram of the proposed 2-to-4 RFD. (b) Schematic diagram of 2-to-4 RFD. (c) Simulation with DSCH-2.7 [21] of 2-to-4 RFD. (d) Block diagram of the proposed 3-to-8 RFD. (e) Simulation with DSCH-2.7 [21] of 3-to-8 RFD. (f) Block diagram of the proposed n -to- $2n$ RFD.

Algorithm 1: Algorithm for the proposed reversible fault tolerant n -to- 2^n decoder, $RFD(S, F2G, FRG)$

Input : Data input set $S(S_0, S_1, \dots, S_{n-1})$
Feynman double gate ($F2G$) and Fredkin gate (FRG)

Output: n -to- 2^n reversible fault tolerant decoder circuit

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1 begin
2   i = input
3   o = output
4   for j ← 0 to n - 1 do
5     if j = 0 then
6        $S_j \rightarrow first.i.F2G, 1 \rightarrow second.i.F2G, 0 \rightarrow$ 
7          $third.i.F2G$ 
8     end if
9     else
10       $S_j \rightarrow first.i.FRG, 0 \rightarrow second.i.FRG$ 
11      if j=2 then
12         $third.o.F2G \rightarrow third.i.FRG$ 
13      end if
14      else
15        call RFD(j-1),  $RFD.o.j \rightarrow third.i.FRG_j$ 
16      end if
17    end for
18    return  $FRG.o.3 \& FRG.o.2 \rightarrow desired\ output$ 
19           $remaining\ F2G.o \& FRG.o \rightarrow garbage$ 
20          outputs.
21 end

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While line 13-15 assigns third input to the Fredkin gate through a recursive call to previous RFD for $n > 2$. Line 18-19 returns outputs. The complexity of this algorithm is $O(n)$. According to the proposed algorithm architecture on n -to- 2^n RFD is shown in Fig. 5(e). In Sec. II-D, we present the transistor representations of FRG and $F2G$ using MOS transistors. These representations are finally used to get the MOS circuit of the proposed decoder. Each of the proposed circuit are simulated with DSCH-2.7 [21]. These simulations also show the functional correctness of the proposed decoders. Table. II shows a comparative study of the proposed fault tolerant decoders with existing fault tolerant one.

TABLE II: Comparison of reversible fault tolerant decoders

	GT	GO	QC	HC	UD
2-to-4 Existing Circuit [7]	3	2	15	$6\alpha + 12\beta + 6\gamma$	3
2-to-4 Proposed Circuit	3	2	12	$6\alpha + 8\beta + 4\gamma$	2
3-to-8 Existing Circuit [7]*	≥ 7	≥ 3	≥ 35	$\geq 14\alpha + 28\beta + 14\gamma$	≥ 7
3-to-8 Proposed Circuit	7	3	32	$14\alpha + 24\beta + 12\gamma$	4

GT = No of Gate, GO = Garbage Output, QC = Quantum cost, HC = Hardware Complexity, UD = Unit Delay.
* The design is not generalized one, i.e., it is not an n -to- 2^n decoder.

Theorem 1: An n -to- 2^n reversible fault tolerant decoder can be realized with at least n garbage outputs and $2n$ constant inputs, where n is the number of data bits.

Proof: An n -to- 2^n decoder has n inputs and 2^n outputs. Thus, to maintain the property of reversibility, there should be at least $(2^n - n)$ constant inputs. However, this $(2^n - n)$ constant inputs don't preserve the parity. To preserve the parity, at least n more constant inputs are needed. So, there should be at least n garbage outputs. _

Example 1: Let the value of n be 1. Then, we have the 1-to-2 reversible fault tolerant decoder. As shown in Sec. II, for a

reversible circuit it is necessary to maintain the one-to-one correspondence between input and output vectors and thus, any reversible circuit should have equal number of inputs and outputs. In the 1-to-2 decoder, there are 2 primary outputs (O_0, O_1) but 1 input (S_0), hence according to the property of reversibility, 1-to-2 reversible decoder should have at least 1 constant input. The value of this constant input can be either 0 or 1. Table. III shows that whatever the value of this constant input, it will never be able to preserve the parity between input and output vectors, which is the prime requirement of the reversible fault tolerant logic circuit. Therefore, to preserve the parity for the 1-to-2 reversible fault tolerant decoder we need at least one more constant input, i.e., at least 2 constant inputs are required for the 1-to-2 reversible fault tolerant decoder.

TABLE III: 1-to-2 decoder with 1 constant input

Inputs		Outputs		Parity
CI	S_0	O_0	O_1	
0	0	1	0	$I_p=E, O_p=O$
0	1	0	1	$I_p=O_p$
1	0	1	0	$I_p=O_p$
1	1	0	1	$I_p=E, O_p=O$

Next, we must prove the existence of combinational circuit which can realize the reversible fault tolerant 1-to-2 decoder by 2 constant inputs. This can easily be accomplished by the circuit shown in Fig. 4(a). It can be verified that Fig. 4(a) is reversible and fault tolerant with the help of its corresponding truth table, there is no need to give more detail.

Now, in 1-to-2 reversible fault tolerant decoder there are at least 2 constant inputs and 1 primary input, i.e., total of 3 inputs. Thus, 1-to-2 reversible fault tolerant decoder should have at least 3 outputs, otherwise it will never comply with the properties of reversible parity preserving circuit. Among these 3 outputs, only 2 are primary outputs. So, remaining 1 output is the garbage output, which holds Theorem 1 for $n=1$.

Theorem 2: A 2-to-4 reversible fault tolerant decoder can be realized with at least 12 quantum cost.

Proof: A 2-to-4 decoder has 4 different 2×2 logical AND operations. A reversible fault tolerant AND2 operation requires at least 3 quantum cost. So, 2-to-4 reversible fault tolerant decoder is realized with at least 12 quantum cost.

Example 2: Fig. 5(a) is the proof for the existence of 2-to-4 reversible decoder with 12 quantum cost. Next, we want to prove that it is not possible to realize a reversible fault tolerant 2-to-4 decoder fewer than 12 quantum cost. In the 2-to-4 decoder, there are 4 different 2×2 logical AND operations, e.g., $S_1S_0, S_1\bar{S}_0, \bar{S}_1S_0, \bar{S}_1\bar{S}_0$. It will be enough if we prove that it is not possible to realize a reversible fault tolerant 2×2 logical AND with fewer than three quantum cost. Consider,

- i. If we make use of one quantum cost to design the AND, that of course is not possible according to our discussion in Sec. II.
- ii. If we make use of two quantum cost to design AND, then we must make use of two 1×1 or 2×2 gates. Apparently two 1×1 gates can't generate the AND. Aiming at two 2×2 gates, we have two combinations, which are shown in Fig. 6(a) and Fig. 6(b). In Fig. 6(a), the output must be (a, ab) if the inputs are (a, b) . The corresponding truth table is shown in Table. IV.

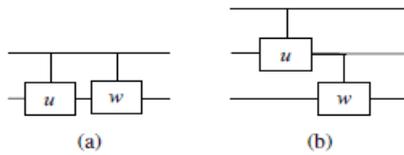


Fig. 6: Combinations of the two 2×2 quantum primitive gates

TABLE IV: Truth table of Fig. 6(a)

a	b	a	ab
0	0	0	0
0	1	0	0
1	0	1	0
1	1	1	1

From Table. IV, we find that, outputs are not at all unique to its corresponding input combinations (1st and 2nd rows have the identical outputs for different input combinations). So it can't achieve the reversible AND. For Fig. 6(b) if inputs are (a, b, c) then, the outputs of the lower level will be offered to the next level as a controlled input, this means that second output of Fig. 6(b) have to be ab , otherwise it will never be able to get output ab since third output of Fig. 6(b) is controlled by the second output, thereby according to Table. V, we can assert that the second combination is impossible to realize the AND no matter how we set the third output of Fig. 6(b) (third column of Table. V), the input vectors will never be one-to-one correspondent with the output vectors. Therefore, we can conclude that, a combinational circuit for reversible fault tolerant 2×2 logical AND operation can't be realized with less than three quantum cost. The above example clarifies the lower bound in terms of quantum cost of 2-to-4 RFD. Similarly, it can be proved that the n -to- $2n$ RFD can be realized with $5(2n - 85)$ quantum cost, when $n \geq 1$, and by assigning different values to n , the validity of this equation can be proved.

TABLE V: Truth table of Fig. 6(b)

a	b	c	a	ab
0	0	0	0	0
0	0	1	0	0
0	1	0	0	0
0	1	1	0	0
1	0	0	1	0
1	0	1	1	0
1	1	0	1	1
1	1	1	1	1

Lemma 1: An n -to- $2n$ RFD can be realized with $(2n-1)$ reversible fault tolerant gates, where n is the number of data bits.

Proof: According to our design procedure, an n -to- $2n$ RFD requires an $(n - 1)$ -to- $2n-1$ RFD plus n number of Fredkin gates, which requires an $(n - 2)$ -to- $2n-2$ RFD plus $(n - 1)$ Fredkin gates and so on till we reach to 1-to-2 RFD. 1-to-2 RFD requires a reversible fault tolerant Feynman double gate only. Thus total number of gates required for an n -to- $2n$ RFD is,

$$1 + 2 + 4 + \dots + n^{th} \text{ term} = \frac{2^0(2^n - 1)}{(2 - 1)} = 2^n - 1$$

Example 3: From Fig. 5(d) we find that the proposed 3-to-8 RFD requires total number of 7 reversible fault tolerant gates. If we replace n with 3 in Lemma 1, we get the value 7 as well.

Lemma 2: Let, α, β, γ be the hardware complexity for a two-input Ex-OR, AND and NOT operation, respectively. Then an n -to- $2n$ RFD can be realized with $(2n+1 - 2)\alpha + (2n+2 - 8)\beta + (2n+1 - 4)\gamma$ hardware complexity, where n is the number of data bits.

Proof: In Lemma 1, we proved that an n -to- $2n$ RFD is realized with a $F2G$ and $(2n - 2)$ FRG . Hardware complexity of a FRG and a $F2G$ are $2\alpha+4\beta + 2\gamma$ and 2α , respectively. Hence, hardware complexity for n -to- $2n$ RFD is

$$(2^n - 2)(2\alpha + 4\beta + 2\gamma) + 2\alpha = (2^{n+1} - 2)\alpha + (2^{n+2} - 8)\beta + (2^{n+1} - 4)\gamma$$

Example 4: Fig. 5(d) shows that the proposed 3-to-8 reversible fault tolerant decoder requires six Fredkin gates and one Feynman double gate. According to our previous discussion in Sec. II, hardware complexity of a Feynman double gate is 2α , whereas, hardware complexity of a Fredkin gate is $2\alpha + 4\beta + 2\gamma$. Thus, the hardware complexity of Fig. 5(d) is $6(2\alpha + 4\beta + 2\gamma) + 2\alpha = 14\alpha + 24\beta + 12\gamma$. In Lemma 2, if we put $n = 3$, we get exactly $14\alpha + 24\beta + 12\gamma$ as well.

IV. CONCLUSIONS AND FUTURE WORK

The In this paper, we presented the design methodologies of an n -to- $2n$ reversible fault tolerant decoder, where n is the number of data bits. We proposed several lower bounds on the numbers of garbage outputs, constant inputs and quantum cost and proved that the proposed circuit has constructed with the optimum garbage outputs, constant inputs and quantum cost. In addition, we presented the designs of the individual gates of the decoder using MOS transistors in order to implement the circuit of the decoder with transistors. Simulations of the transistor implementation of the decoder showed that the proposed fault tolerant decoder works correctly. The comparative results proved that the proposed designs perform better than its counterpart. We also proved the efficiency and supremacy of the proposed scheme with several theoretical explanations. Proposed reversible fault tolerant decoders can be used in parallel circuits [22], multiple-symbol differential detection [23], [24], network components [25] and in digital signal processing [26] etc.

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