Sum and Product of Strong Restrained Domination Number of Path and its Derived Graph

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***Abstract*: Let G = (V, E) be a simple graph with p vertices and q edges. Let S $\subseteq$ V(G). S is called a strong restrained dominating set of G if for every u $\in$ V – S, there exists v $\in$ S and w $\in$ V – S such that v and w strongly dominate u. The minimum cardinality of a strong restrained dominating set of G is called the strong restrained domination number of G and is denoted by $\gamma_{srd}(G)$. The existence of a strong restrained dominating set of G is guaranteed, since V(G) is a strong restrained dominating set of G. In this paper, the sum and product of strong restrained domination number of path and its derived graph are studied.**

***Keywords*: Domination, strong domination, restrained domination, strong restrained domination number**

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1. INTRODUCTION

Throughout this paper only path is considered. Let G = (V, E) be a simple graph with p vertices and q edges. The degree of any vertex u in G is the number of edges incident with u and is denoted by deg u. The minimum and maximum degree of a vertex is denoted by $\delta(G)$ and $\Delta(G)$ respectively. A vertex of degree zero in G is called an isolated vertex and a vertex of degree one in G is called a pendant vertex. A subset S of V(G) of a graph G is called a dominating set of G if every vertex in V(G) \ S is adjacent to a vertex in S [8]. The domination number $\gamma(G)$ is the minimum cardinality of a dominating set of G. The concept of strong domination in graphs was introduced by Sampathkumar and Pushpalatha [5] and the restrained domination was introduced by Domke [3] et al. A set S $\subseteq$ V(G) is said to be a strong dominating set of G if every vertex v $\in$ V – S is strongly dominated by some vertex u in S. A set S $\subseteq$ V(G) is a restrained dominating set of G, if every vertex not in S is adjacent to a vertex in S and to a vertex in V – S. The restrained domination number of a graph G, denoted by $\gamma_r(G)$, is the minimum cardinality of a restrained dominating set in G. The strong restrained domination was introduced by Selvaloganayaki and Namasivayam [6]. For all graph theoretic terminologies and notations, Harary [4] is referred to. In this paper, the sum and product of strong restrained domination number of path and its derived graph are studied.

Definition 1.1: Let G = (V, E) be a simple graph with p vertices and q edges. Let S $\subseteq$ V(G). S is called a strong restrained dominating set of G if for every u $\in$ V – S, there exists v $\in$ S and w $\in$ V – S such that v and w strongly dominate u. The minimum cardinality of a strong restrained dominating set of G is called the strong restrained domination number of G and is denoted by $\gamma_{srd}(G)$. The existence of a strong restrained dominating set of G is guaranteed, since V(G) is a strong restrained dominating set of G.

Result 1.2 [6]: Let G = (V, E) be a simple connected graph. If the degree of any support vertex is exactly two, then it belongs to any strong restrained dominating set of G.

Result 1.3 [6]: For the path P_m, $\gamma_{srd}(P_m) = \begin{cases} n+2 \text{ if } m = 3n \\ n+3 \text{ if } m = 3n+1 \\ n+4 \text{ if } m = 3n+2 \end{cases}$ where $n \geq 1$.

2. MAIN RESULTS

In this section, the authors studied the sum and product of strong restrained domination number of path and its derived graphs.

Definition 2.1: The complement of a graph G is a graph H on the same vertices such that two distinct vertices of H are adjacent if and only if they are not adjacent in G.

Theorem 2.2: When $n \geq 2$,

$$\gamma_{srd}(P_m) + \gamma_{srd}(\overline{P}_m) = \begin{cases} 4 \text{ if } m = 2 \\ 6 \text{ if } m = 3 \\ 8 \text{ if } m = 4 \text{ or } m = 5 \\ n + 4 \text{ if } m = 3n \\ n + 5 \text{ if } m = 3n + 1 \\ n + 6 \text{ if } m = 3n + 2 \end{cases} \text{ and}$$

$$\gamma_{srd}(P_m) \times \gamma_{srd}(\overline{P}_m) = \begin{cases} 4 \text{ if } m = 2 \\ 9 \text{ if } m = 3 \\ 16 \text{ if } m = 4 \\ 15 \text{ if } m = 5 \\ 2(n + 2) \text{ if } m = 3n \\ 2(n + 3) \text{ if } m = 3n + 1 \\ 2(n + 4) \text{ if } m = 3n + 2 \end{cases}$$

Proof: Case(i): Suppose m = 2. Let $V(\overline{P}_2) = \{v_1, v_2\}$. $\{v_1, v_2\}$ is the unique strong restrained dominating set of P_2 as well as $\overline{P}_2$. Therefore $\gamma_{srd}(P_2) + \gamma_{srd}(\overline{P}_2) = 4$ and $\gamma_{srd}(P_2) \times \gamma_{srd}(\overline{P}_2) = 4$.

Case(ii): Suppose m = 3. $V(\overline{P}_3) = \{v_1, v_2, v_3\}$. $\overline{P}_3 = K_2 \cup K_1$. $\{v_1, v_2, v_3\}$ is the strong restrained dominating set of P_3 as well as $\overline{P}_3$. Hence $\gamma_{srd}(P_3) + \gamma_{srd}(\overline{P}_3) = 6$ and $\gamma_{srd}(P_3) \times \gamma_{srd}(\overline{P}_3) = 9$.

Case(iii): Suppose m = 4. $\overline{P}_4$ is again P_4. By result 1.3, $\gamma_{srd}(P_4) = 4$. Hence $\gamma_{srd}(P_4) + \gamma_{srd}(\overline{P}_4) = 8$ and $\gamma_{srd}(P_4) \times \gamma_{srd}(\overline{P}_4) = 16$.

Case(iv): Suppose m = 5. $V(\overline{P}_5) = \{v_1, v_2, v_3, v_4, v_5\}$. $\{v_1, v_3, v_5\}$ is the unique strong restrained dominating set of $\overline{P}_5$. Hence $\gamma_{srd}(\overline{P}_5) = 3$. By result 1.3, $\gamma_{srd}(P_5) = 5$. Therefore $\gamma_{srd}(P_5) + \gamma_{srd}(\overline{P}_5) = 8$ and $\gamma_{srd}(P_5) \times \gamma_{srd}(\overline{P}_5) = 15$.

Case (v): Let $G = \overline{P}_{3n}$. Let $n \geq 2$. $V(G) = \{v_1, v_2, ..., v_{3n-1}, v_{3n}\}$. deg v_1 = deg $v_{3n} = 3n–2 = \Delta(G)$ and deg $v_i = 3n–3$, $2 \leq i \leq 3n–1$. Let $S = \{v_1, v_{3n}\}$. Therefore $V–S = \{v_2, v_3, ..., v_{3n-2}, v_{3n-1}\}$. The vertices v_i, $3 \leq i \leq 3n–2$, are strongly dominated by both v_1 and v_{3n} in S and strongly dominated by all the vertices except v_{i+1}, v_{i-1} in V–S. The vertex v_2 is also strongly dominated by v_{3n} in S and strongly dominated by all the vertices except v_3 in V–S. Similarly the vertex v_{3n-1} is also strongly dominated by v_1 in S and strongly dominated by all the vertices except v_{3n-2} in V–S. Therefore S is a strong restrained dominating set of G. Hence $\gamma_{srd}(G) \leq 2$ ---(1).

Suppose let T be any strong restrained dominating set of G such that $|T| = 1$. Since v_1 and v_{3n} are the only maximum degree vertices, v_1 and v_{3n} are adjacent, either v_1 or v_{3n} belongs to T. Suppose v_1 belongs to T. The vertex v_{3n} is strongly dominated by v_1 in T but no vertex in V–T. The case is similar if v_{3n} belongs to T, a contradiction. Hence there is no strong restrained dominating set with only one element. Therefore $\gamma_{srd}(G) \geq 2$ ---(2). From (1) and (2) we get $\gamma_{srd}(G) = 2$. By result 1.3, $\gamma_{srd}(P_{3n}) = n + 2$. Hence $\gamma_{srd}(P_{3n}) + \gamma_{srd}(\overline{P}_{3n}) = n + 4$ and $\gamma_{srd}(P_{3n}) \times \gamma_{srd}(\overline{P}_{3n}) = 2(n + 2)$.

Case (vi): Let $G = \overline{P}_{3n+1}$, $n \geq 2$ or $G = \overline{P}_{3n+2}$, $n \geq 2$. Proof is similar to the case (v). Hence $\gamma_{srd}(G) = 2$. Using result 1.3, $\gamma_{srd}(P_{3n+1}) + \gamma_{srd}(\overline{P}_{3n+1}) = n + 5$, $\gamma_{srd}(P_{3n+1}) \times \gamma_{srd}(\overline{P}_{3n+1}) = 2(n + 3)$ and $\gamma_{srd}(P_{3n+2}) + \gamma_{srd}(\overline{P}_{3n+2}) = n + 6$, $\gamma_{srd}(P_{3n+2}) \times \gamma_{srd}(\overline{P}_{3n+2}) = 2(n + 4)$. Hence the theorem.

Definition 2.3 [7]: The line graph L(G) of G is the graph whose vertex set is E(G) in which two vertices are adjacent if and only if they are adjacent in G. If e = uv is an edge of G then $d_{L(G)}(e) = d_G(u) + d_G(v) – 2$.

Theorem 2.4:

$$\gamma_{srd}(P_m) + \gamma_{srd}(L(P_m)) = \begin{cases} 3 \text{ if } m = 2 \\ 5 \text{ if } m = 3 \\ 2n + 5 \text{ if } m = 3n, n > 1 \\ 2n + 5 \text{ if } m = 3n + 1, n \geq 1 \\ 2n + 7 \text{ if } m = 3n + 2, n \geq 1 \end{cases} \text{ and}$$

$$\gamma_{srd}(P_m) \times \gamma_{srd}(L(P_m)) = \begin{cases} 2 \text{ if } m = 2 \\ 6 \text{ if } m = 3 \\ n^2 + 5n + 6 \text{ if } m = 3n, n > 1 \\ n^2 + 5n + 6 \text{ if } m = 3n + 1, n \geq 1 \\ n^2 + 7n + 12 \text{ if } m = 3n + 2, n \geq 1 \end{cases}$$

Proof: Since $L(P_2) = P_1$. Using result 1.3, $\gamma_{srd}(P_2) + \gamma_{srd}(L(P_2)) = 3$, $\gamma_{srd}(P_2) \times \gamma_{srd}(L(P_2)) = 2$, $\gamma_{srd}(P_3) + \gamma_{srd}(L(P_3)) = 5$, $\gamma_{srd}(P_3) \times \gamma_{srd}(L(P_3)) = 6$. Suppose m = 3n, $n \geq 2$, $\gamma_{srd}(P_{3n}) + \gamma_{srd}(L(P_{3n})) = \gamma_{srd}(P_{3n}) + \gamma_{srd}(P_{3(n-1)+2}) = 2n + 5$, $\gamma_{srd}(P_{3n}) \times \gamma_{srd}(L(P_{3n})) = n^2 + 5n + 6$. Similarly m = 3n + 1, $n \geq 1$, $\gamma_{srd}(P_{3n+1}) + \gamma_{srd}(L(P_{3n+1})) = \gamma_{srd}(P_{3n+1}) + \gamma_{srd}(P_{3n}) = 2n + 5$, $\gamma_{srd}(P_{3n+1}) \times \gamma_{srd}(L(P_{3n+1})) = n^2 + 5n + 6$ and m = 3n + 2, $n \geq 1$, $\gamma_{srd}(P_{3n+2}) + \gamma_{srd}(L(P_{3n+2})) = \gamma_{srd}(P_{3n+2}) + \gamma_{srd}(P_{3n+1}) = 2n + 7$, $\gamma_{srd}(P_{3n+2}) \times \gamma_{srd}(L(P_{3n+2})) = n^2 + 7n + 12$. Hence the theorem.

Definition 2.5 [2]: The jump graph J(G) of G is the graph whose vertex set is E(G) in which two vertices are adjacent if and only if they are non-adjacent in G.

Theorem 2.6:

$$\gamma_{srd}(P_m)+\gamma_{srd}(J(P_m))=\begin{cases}3 & \text{if } m=2\\ 5 & \text{if } m=3\\ 7 & \text{if } m=4 \text{ or } m=6\\ 9 & \text{if } m=5\\ n+4 & \text{if } m=3n, n>2\\ n+5 & \text{if } m=3n+1, n\geq 2\\ n+6 & \text{if } m=3n+2, n\geq 2\end{cases} \quad \text{and}$$

$$\gamma_{srd}(P_m)\times\gamma_{srd}(J(P_m))=\begin{cases}2 & \text{if } m=2\\ 6 & \text{if } m=3\\ 12 & \text{if } m=4 \text{ or } m=6\\ 20 & \text{if } m=5\\ 2(n+2) & \text{if } m=3n, n>2\\ 2(n+3) & \text{if } m=3n+1, n\geq 2\\ 2(n+4) & \text{if } m=3n+2, n\geq 2\end{cases}$$

Proof: Obviously $\gamma_{srd}(J(P_2)) = 1$ and $J(P_m) = \overline{P}_{m-1}$, as discussed in the theorem 2.2. We get the theorem.

Definition 2.7 [4]: The subdivision graph S(G) of a graph G is obtained from G by inserting a new vertex into every edge of G.

Theorem 2.8: When $n \geq 1$,

$$\gamma_{srd}(P_m)+\gamma_{srd}(S(P_m))=\begin{cases}5 \text{ if } m=2\\ 3n+5 \text{ if } m=3n\\ 3n+6 \text{ if } m=3n+1\\ 3n+7 \text{ if } m=3n+2\end{cases} \quad \text{and}$$

$$\gamma_{srd}(P_m)\times\gamma_{srd}(S(P_m))=\begin{cases}6 \text{ if } m=2\\ 2n^2+7n+6 \text{ if } m=3n\\ 2n^2+9n+9 \text{ if } m=3n+1\\ 2n^2+11n+12 \text{ if } m=3n+2\end{cases}$$

Proof: Case(i): Suppose m = 2. $S(P_2) = P_3$. Hence $\gamma_{srd}(P_2) + \gamma_{srd}(S(P_2)) = 5$ and $\gamma_{srd}(P_2) \times \gamma_{srd}(S(P_2)) = 6$. Suppose m = 3n, $n \geq 1$. Using result 1.3, $\gamma_{srd}(P_{3n}) + \gamma_{srd}(S(P_{3n})) = \gamma_{srd}(P_{3n}) + \gamma_{srd}(P_{3(2n-1)+2}) = 3n + 5$ and $\gamma_{srd}(P_{3n}) \times \gamma_{srd}(S(P_{3n})) = 2n^2 + 7n + 6$. Similarly m = 3n + 1, $n \geq 1$, $\gamma_{srd}(P_{3n+1}) + \gamma_{srd}(S(P_{3n+1})) = \gamma_{srd}(P_{3n+1}) + \gamma_{srd}(P_{3(2n)+1}) = 3n + 6$, $\gamma_{srd}(P_{3n+1}) \times \gamma_{srd}(S(P_{3n+1})) = 2n^2 + 9n + 9$ and m = 3n + 2, $n \geq 1$, $\gamma_{srd}(P_{3n+2}) + \gamma_{srd}(S(P_{3n+2})) = \gamma_{srd}(P_{3n+2}) + \gamma_{srd}(P_{3(2n+1)}) = 3n + 7$, $\gamma_{srd}(P_{3n+2}) \times \gamma_{srd}(S(P_{3n+2})) = 2n^2 + 11n + 12$. Hence the theorem.

Definition 2.9: The paraline graph P(L(G)) is a line graph of subdivision graph of G.

Theorem 2.10: When $n \geq 1$,

$$\gamma_{srd}(P_m)+\gamma_{srd}(P(L(P_m)))=\begin{cases}4 \text{ if } m=2\\ 3n+4 \text{ if } m=3n\\ 3n+5 \text{ if } m=3n+1\\ 3n+8 \text{ if } m=3n+2\end{cases} \quad \text{and}$$

$$\gamma_{srd}(P_m)\times\gamma_{srd}(P(L(P_m)))=\begin{cases}4 \text{ if } m=2\\ 2n^2+6n+4 \text{ if } m=3n\\ 2n^2+8n+6 \text{ if } m=3n+1\\ 2n^2+14n+20 \text{ if } m=3n+2\end{cases}$$

Proof: Case(i): Suppose m = 2. $P(L(P_2)) = P_2$. Therefore $\gamma_{srd}(P_2) + \gamma_{srd}(P(L(P_2))) = 4$ and $\gamma_{srd}(P_2) \times \gamma_{srd}(P(L(P_2))) = 4$. Suppose m = 3n, $n \geq 1$. Using result 1.3, $\gamma_{srd}(P_{3n}) + \gamma_{srd}(P(L(P_{3n}))) = \gamma_{srd}(P_{3n}) + \gamma_{srd}(P_{3(2n-1)+1}) = 3n + 4$ and $\gamma_{srd}(P_{3n}) \times \gamma_{srd}(P(L(P_{3n}))) = 2n^2 + 6n + 4$. Similarly m = 3n + 1, $n \geq 1$, $\gamma_{srd}(P_{3n+1}) + \gamma_{srd}(P(L(P_{3n+1}))) = \gamma_{srd}(P_{3n+1}) + \gamma_{srd}(P_{3(2n)}) = 3n + 5$, $\gamma_{srd}(P_{3n+1}) \times \gamma_{srd}(P(L(P_{3n+1}))) = 2n^2 + 8n + 6$ and m = 3n + 2, $n \geq 1$, $\gamma_{srd}(P_{3n+2}) + \gamma_{srd}(P(L(P_{3n+2}))) = \gamma_{srd}(P_{3n+2}) + \gamma_{srd}(P_{3(2n)+2}) = 3n + 8$, $\gamma_{srd}(P_{3n+2}) \times \gamma_{srd}(P(L(P_{3n+2}))) = 2n^2 + 14n + 20$. Hence the theorem.

Definition 2.11 [1]: The corona $G_1 \odot G_2$ of two graphs G_1 and G_2 is defined as the graph by taking one copy of G_1 (which has p_1 points) and p_1 copies of G_2 and then joining the i^{th} point of G_1 to every point in the i^{th} copy of G_2.

Theorem 2.12: When $n \geq 1$,

$$\gamma_{srd}(P_m)+\gamma_{srd}(P_m\odot K_1)=\begin{cases}6 \; if \; m=2\\ 5n+4 \; if \; m=3n\\ 5n+7 \; if \; m=3n+1\\ 5n+10 \; if \; m=3n+2\end{cases}$$

$$\gamma_{srd}(P_m) \times \gamma_{srd}(P_m \odot K_1) = \begin{cases} 8 \text{ if } m = 2 \\ 4n^2 + 10n + 4 \text{ if } m = 3n \\ 4n^2 + 16n + 12 \text{ if } m = 3n + 1 \\ 4n^2 + 22n + 24 \text{ if } m = 3n + 2 \end{cases}$$

Proof: When m = 2, $P_2 \odot K_1 = P_4$. Using result 1.3, $\gamma_{srd}(P_2) + \gamma_{srd}(P_2 \odot K_1) = 6$, $\gamma_{srd}(P_2) \times \gamma_{srd}(P_2 \odot K_1) = 8$. Suppose m ≥ 3. Let G be the graph $P_m \odot K_1$. Let V(G) = {v_i, u_i / 1 ≤ i ≤ n} and E(G) = {v_iv_{i+1} / 1 ≤ i ≤ n – 1} ∪ {v_iu_i / 1 ≤ i ≤ n}. Then deg v_1 = 2 = deg v_n, deg v_i = 3, 2 ≤ i ≤ n – 1 and deg u_i = 1, 1 ≤ i ≤ n. Let S be a strong restrained dominating set of G. Since any strong restrained dominating set contains all the end vertices, therefore all u_i, 1 ≤ i ≤ n belong to S. The subgraph H induced by the remaining vertices is the path Pm. The pendent vertices do not strongly dominate the vertices of the path Pm. Using result 1.3, $\gamma_{srd}(H) = \gamma_{srd}(P_m)$. Therefore │S│= $\gamma_{srd}(P_m)$ + nK1. Hence $\gamma_{srd}(P_{3n}) + \gamma_{srd}(P_{3n} \odot K_1)$ = 5n + 4, $\gamma_{srd}(P_{3n}) \times \gamma_{srd}(P_{3n} \odot K_1)$ = 4n^2 + 10n + 4, n ≥ 1. Similarly, $\gamma_{srd}(P_{3n+1}) + \gamma_{srd}(P_{3n+1} \odot K_1)$ = 5n + 7, $\gamma_{srd}(P_{3n+1}) \times \gamma_{srd}(P_{3n+1} \odot K_1)$ = 4n^2 + 16n + 12 and $\gamma_{srd}(P_{3n+2}) + \gamma_{srd}(P_{3n+2} \odot K_1)$ = 5n + 10, $\gamma_{srd}(P_{3n+2}) \times \gamma_{srd}(P_{3n+2} \odot K_1)$ = 4n^2 + 22n + 24, n ≥ 1. Hence the theorem.

Theorem 2.13: When k ≥ 1,

$$\gamma_{srd}(P_m) + \gamma_{srd}(P_m \odot \bar{K}_n) = \begin{cases} 2(n+2) \text{ if } m = 2 \\ 2k + nm + 4 \text{ if } m = 3k \\ 2k + nm + 6 \text{ if } m = 3k + 1 \\ 2k + nm + 8 \text{ if } m = 3k + 2 \end{cases}$$

$$\gamma_{srd}(P_m) \times \gamma_{srd}(P_m \odot \bar{K}_n) = \begin{cases} 4(n+1) \text{ if } m = 2 \\ (k+2)(k+nm+2) \text{ if } m = 3k \\ (k+3)(k+nm+3) \text{ if } m = 3k + 1 \\ (k+4)(k+nm+4) \text{ if } m = 3k + 2 \end{cases}$$

Proof: When m = 2, $P_2 \odot \bar{K}_n = P_4 \cup 2(n-1) K_1$. Therefore $\gamma_{srd}(P_2) + \gamma_{srd}(P_2 \odot \bar{K}_n)$ = 2(n + 2), $\gamma_{srd}(P_2) \times \gamma_{srd}(P_2 \odot \bar{K}_n)$ = 4(n + 1). Suppose m ≥ 3. Let G be the graph $P_m \odot \bar{K}_n$. Let V(G) = {v_i, v_{ij} / 1 ≤ i ≤ m, 1 ≤ j ≤ n} and E(G) = {v_iv_{i+1}, v_iv_{ij} / 1 ≤ i ≤ m – 1, 1 ≤ j ≤ n} ∪ {v_mv_{mj} / 1 ≤ j ≤ n}. Let S be a strong restrained dominating set of G. Since any strong restrained dominating set contains all the end vertices, therefore all v_{ij}, 1 ≤ i ≤ m, 1 ≤ j ≤ n belong to S. The subgraph H induced by the remaining vertices is the path Pm. The pendent vertices do not strongly dominate the vertices of the path P_m. Using result 1.3, $\gamma_{srd}(H) = \gamma_{srd}(P_m)$. Therefore │S│= $\gamma_{srd}(P_m)$ + nm. Hence $\gamma_{srd}(P_{3k}) + \gamma_{srd}(P_{3k} \odot \bar{K}_n)$ = 2k + nm + 4, $\gamma_{srd}(P_{3k}) \times \gamma_{srd}(P_{3k} \odot \bar{K}_n)$ = (k + 2) (k + nm + 2), n ≥ 1. Similarly, $\gamma_{srd}(P_{3k+1}) + \gamma_{srd}(P_{3k+1} \odot \bar{K}_n)$ = 2k + nm + 6, $\gamma_{srd}(P_{3k+1}) \times \gamma_{srd}(P_{3k+1} \odot \bar{K}_n)$ = (k + 3) (k + nm + 3) and $\gamma_{srd}(P_{3k+2}) + \gamma_{srd}(P_{3k+2} \odot \bar{K}_n)$ = 2k + nm + 8, $\gamma_{srd}(P_{3k+2}) \times \gamma_{srd}(P_{3k+2} \odot \bar{K}_n)$ = (k + 4) (k + nm + 4), n ≥ 1. Hence the theorem.

Definition 2.14: Let G be a graph with a fixed vertex v and let (P_m : G) be the graph obtained from m copies of G and the path P_m: u_1, u_2, …., u_m by joining u_i with the vertex of the jth copy of G by means of an edge for 1 ≤ i ≤ n.

Theorem 2.15: When n ≥ 2,

$$\gamma_{srd}(P_m) + \gamma_{srd}(P_m : S_n) = \begin{cases} 2(n+2) \text{ if } m = 2 \\ 3(n+3) \text{ if } m = 3 \\ k(3n+4) + 2 \text{ if } m = 3k,\ k \geq 2 \\ 4(k+1) + n(3k+1) \text{ if } m = 3k+1, k \geq 1 \\ 2(2k+3) + n(3k+2) \text{ if } m = 3k+2, k \geq 1 \end{cases}$$

$$\gamma_{srd}(P_m) \times \gamma_{srd}(P_m : S_n) = \begin{cases} 4(n+1) \text{ if } m = 2 \\ 9(n+2) \text{ if } m = 3 \\ m(k+2)(n+1) \text{ if } m = 3\text{k},\ k \geq 2 \\ m(k+3)(n+1) \text{ if } m = 3k+1, k \geq 1 \\ m(k+4)(n+1) \text{ if } m = 3k+2, k \geq 1 \end{cases}$$

Proof: Let G be a graph (P_m: S_n), n ≥ 2, where P_m be the path having m vertices and $S_m = K_{1,m}$. Let u_1, u_2, …., u_m be the vertices of path P_m. Then V((P_m : S_n)) = {u_i, v_i, v_{ij}: 1 ≤ i ≤ m, 1 ≤ j ≤ n} and E((P_m : S_n)) = {u_iu_{i+1}, u_jv_j, v_jv_{jk} : 1 ≤ i ≤ m – 1, 1 ≤ j ≤ m, 1 ≤ k ≤ n}. Suppose m = 2, P_2 : S_n = $P_6 \cup 2(n-1)$K1. Therefore $\gamma_{srd}(P_2 : S_n) = \gamma_{srd}(P_6)$ + 2(n – 1). Hence $\gamma_{srd}(P_2) + \gamma_{srd}(P_2 : S_n)$ = 2(n + 2) and $\gamma_{srd}(P_2) \times \gamma_{srd}(P_2 : S_n)$ = 4(n + 1). Suppose m = 3, V(P_3 : S_n) is the unique strong restrained dominating set of P_3 : S_n. Therefore $\gamma_{srd}(P_3) + \gamma_{srd}(P_3 : S_n)$ = 3(n + 3) and $\gamma_{srd}(P_3) \times \gamma_{srd}(P_3 : S_n)$ = 9(n + 2). Suppose m ≠ 3, S = {v_i, v_{ij} / 1 ≤ i ≤ n, 1 ≤ j ≤ m} is the unique strong restrained dominating set of G and │S│ = m(n + 1). Therefore $\gamma_{srd}(P_{3k}) + \gamma_{srd}(P_{3k} : S_n)$ = k(3n + 4) + 2 and $\gamma_{srd}(P_{3k}) \times \gamma_{srd}(P_{3k} : S_n)$ = m(k + 2)(n + 1), k ≥ 2. Similarly $\gamma_{srd}(P_{3k+1}) + \gamma_{srd}(P_{3k+1} : S_n)$ = 4(k + 1) + n(3k + 1) and $\gamma_{srd}(P_{3k+1}) \times \gamma_{srd}(P_{3k+1} : S_n)$ = m(k + 3)(n + 1) and $\gamma_{srd}(P_{3k+2}) + \gamma_{srd}(P_{3k+2} : S_n)$ = 2(2k + 3) + n(3k + 2) and $\gamma_{srd}(P_{3k+2}) \times \gamma_{srd}(P_{3k+2} : S_n)$ = m(k + 4)(n + 1), k ≥ 1. Hence the theorem.

Definition 2.16: The coconut tree graph T(n, m) is obtained by joining the central vertex of the star $K_{1,m}$ and a pendent vertex of a path P_n by an edge.

Theorem 2.17: When $k \geq 1$,

$$\gamma_{srd}(P_n) + \gamma_{srd}(T(n,m)) = \begin{cases} m+5 \text{ if } n=2 \\ 2k+m+5 \text{ if } n=3k, 3k+1 \\ 2k+m+7 \text{ if } n=3k+2 \end{cases}$$

$$\gamma_{srd}(P_n) \times \gamma_{srd}(T(n,m)) = \begin{cases} 2(m+3) \text{ if } n=2 \\ (k+2)(k+m+3) \text{ if } n=3k \\ (k+3)(k+m+2) \text{ if } n=3k+1 \\ (k+4)(k+m+3) \text{ if } n=3k+2 \end{cases}$$

Proof: Let G be a coconut tree T(n, m). Let $v_1, v_2, \ldots v_n$ be the vertices of path P_n, u be the central vertex of $K_{1,m}$ and $u_1, u_2, \ldots, u_m$ be the pendent vertices of $K_{1,m}$. The vertices u and v_n is joined by an edge. The subgraph H induced by the vertices $v_1, v_2, \ldots v_n$, u, u_1 is the path P_{n+2}. Let S be the strong restrained dominating set of P_{n+2}. The vertices of S together with the pendent vertices $u_2, u_3, \ldots. u_m$ form a strong restrained dominating set T of G and $|T| = |S| + m - 1$. Therefore $\gamma_{srd}(P_2) + \gamma_{srd}(T(2,m)) = m + 5$, $\gamma_{srd}(P_2) \times \gamma_{srd}(T(2,m)) = 2(m+3)$. If $n = 3k$, $k \geq 1$, then $\gamma_{srd}(P_n) + \gamma_{srd}(T(n,m)) = \gamma_{srd}(P_{3k+2}) + \gamma_{srd}(T(3k,m)) = 2k + m + 5$, $\gamma_{srd}(P_n) \times \gamma_{srd}(T(n,m)) = (k+2)(k+m+3)$. If $n = 3k+1$, $k \geq 1$ then $\gamma_{srd}(P_n) + \gamma_{srd}(T(n,m)) = \gamma_{srd}(P_{3(k+1)}) + \gamma_{srd}(T(3k+1,m)) = 2k + m + 5$, $\gamma_{srd}(P_n) \times \gamma_{srd}(T(n,m)) = (k+3)(k+m+2)$ and if $n = 3k+2$, $k \geq 1$ then $\gamma_{srd}(P_n) + \gamma_{srd}(T(n,m)) = \gamma_{srd}(P_{3(k+1)+1}) + \gamma_{srd}(T(3k+2,m)) = 2k + m + 7$, $\gamma_{srd}(P_n) \times \gamma_{srd}(T(n,m)) = (k+4)(k+m+3)$. Hence the theorem.

Definition 2.18: The twig graph G obtained from the path P_n by attaching exactly two pendent edges to each internal vertex of the path.

Theorem 2.19: Let G be a twig graph. When $k \geq 1$,

$$\gamma_{srd}(P_m) + \gamma_{srd}(G) = \begin{cases} 4 \text{ if } m=2 \\ 8k \text{ if } m=3k \\ 4(2k+1) \text{ if } m=3k+1 \\ 8(k+1) \text{ if } m=3k+2 \end{cases}$$

$$\gamma_{srd}(P_m) \times \gamma_{srd}(G) = \begin{cases} 4 \text{ if } m=2 \\ 7k^2+12k-4 \text{ if } m=3k \\ 7k^2+22k+3 \text{ if } m=3k+1 \\ 7k^2+32k+16 \text{ if } m=3k+2 \end{cases}$$

Proof: Let G be a twig graph. Let $V(G) = \{v_i, u_j, w_j / 1 \leq i \leq n, 1 \leq j \leq n-2\}$ and $E(G) = \{v_iv_{i+1}, u_jv_{j+1}, w_jv_{j+1} / 1 \leq i \leq n-1, 1 \leq j \leq n-2\}$. Suppose $m = 2$, $G = P_2$. Therefore $\gamma_{srd}(P_2) + \gamma_{srd}(G) = 4$ and $\gamma_{srd}(P_2) \times \gamma_{srd}(G) = 4$. Suppose $m \geq 3$, let S be a strong restrained dominating set of path P_m. The vertices of S together with pendent vertices $\{u_j, w_j / 1 \leq j \leq m-2\}$ form a strong restrained dominating set T of G and $|T| = |S| + 2(m-2)$. Therefore $\gamma_{srd}(P_{3k}) + \gamma_{srd}(G) = 8k$, $\gamma_{srd}(P_{3k}) \times \gamma_{srd}(G) = 7k^2 + 12k - 4$, $k \geq 1$. Similarly $\gamma_{srd}(P_{3k+1}) + \gamma_{srd}(G) = 4(2k+1)$, $\gamma_{srd}(P_{3k+1}) \times \gamma_{srd}(G) = 7k^2 + 22k + 3$ and $\gamma_{srd}(P_{3k+2}) + \gamma_{srd}(G) = 8(k+1)$, $\gamma_{srd}(P_{3k+2}) \times \gamma_{srd}(G) = 7k^2 + 32k + 16$, $k \geq 1$. Hence the theorem.

3. CONCLUSION

In this paper, the authors studied sum and product of strong restrained domination number of path and its derived graphs. Similar studies can be made on this type for various derived graphs.

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