Study of Finite Element Analysis of Heterogeneous Object Using Material Based Graded Elements

Shashank Shekhar[1], Rohit Sharma[2]

[1]M. Tech Scholar, [2]Assistant Professor
Mechanical Engineering Department
BM Group of Institutions, Gurgaon

***Abstract*: A heterogeneous object is referred to a solid object made of different constituent materials. The object is made up of finite collection of regions having set of prescribed material classes of continuously or discontinuously varying material properties. This paper we study the heterogeneous object using material based graded element and continuously and discontinuously distribution of material property within the domain as well try to developing a new technique for the defining the each element property with material variation (linearly, Non-linearly) within the element so that our result comes near to the exact. Also study on different meshing type so that we able to select the right meshing size in our problem.**

I. INTRODUCTION

A heterogeneous object (HO) is referred to as a solid component consisting of two or more material primitives distributed continuously or discontinuously within the object. The continuously varying material composition produces gradation in material properties, as they are often known as functionally gradient materials (FGM), shown in Figure 1. A part contains two compositions, metal and heat resistance material (such as ceramic), and the material distribution. Metal increases its fraction gradually from one side to another (the blue line), while the heat resistance material linearly reduces its fraction (the red line), which can avoid the stress concentration like composite materials for the thermal stress relaxation in transition of two materials. A discontinuous change in material composition generates distinct regions of material in the solid, which is usually called multi-material object (MMO) such as composite materials. MMO has been extensively used in industry for a long time, while FGM has shown tremendous potential in many fields, such as aeronautics and astronautics, biomedical engineering, and nano-technology.

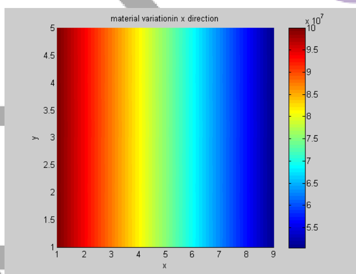


Fig.1 Model of Functionally Gradient Materials

II. HETEROGENEOUS OBJECT MODELING AND ANALYSIS

a. Heterogeneous object Modeling

In general, heterogeneous object modeling embraces two fundamental processes: geometric modeling and material modeling. Geometric modeling is concerned with the geometric representations of the objects and material modeling is targeted at modeling material distributions defined over the geometric domain. Most of the traditional CAD systems are based on the premise of material homogeneities. The modeling space is three-dimensional Euclidean space E3 (or a subspace), and the major focus in geometric modeling is the shape and spatial relations.

b. FE analysis of heterogeneous object

When subject specific FE model are used for the analysis of the heterogeneous object for near to the exact mechanical behavior of object. The method of assignment of the material property strongly influences the predicted mechanical behavior of object. In most of the FE studies of heterogeneous object elastic modulus is constant within the element and computed by power law relation with averaged apparent density within the element such model can be treated as composite model .In the present study the material property is taken according to the variation of the material within the object so that it is used to improve the performance of the analysis of this model.

III. GEOMETRY REPRESENTATION

To represent and manipulate heterogeneous objects within the computer and to develop the ability of describing how material composition varies within a heterogeneous objects. The reason for this is because the interior of heterogeneous objects is decomposed into a set of sub-regions. These sub- regions can be considered as the finite elements in this paper. It is also necessary to build up a data structure to represent the model which is created by FEM. This model can be considered as a 2D finite element model with material information, including some geometry elements like a node, edge, face and finite element, the topology relationship.

IV. MATERIAL-BASED GRADED ELEMENT FORMULATION

There are two preconditions to design material distribution of Heterogeneous Objects. The first condition is the material composition of each node. The second condition is the material composition of discretional point inside each finite element can also be achieved if the first condition is satisfied. On the basis of these two conditions material property of each element is defined.

Solution done with help of this 4 technique to get near to accurate results by varying material property within the element

1. Average Material property for whole domain
2. Average material property for element
3. Material linear variation for element
4. Material non linear variation for element

Finite element analysis is a numerical technique in this method all the complexities of the problem like varying shape ,boundary condition and loads are maintained but the solution obtained are approximate because of its diversity and flexibility.

In engineering problem there are some basic unknown / field variable if they are found, the behavior of structure predicted like displacement in solid mechanics. For 1 D only U is consider and for 2D both U, V

$$U = \sum_{i=1}^{n} Niui$$

$$V = \sum_{i=1}^{n} Nivi$$

In a continuum there unknowns are infinite. The finite element procedure reduce such unknowns to a finite number by dividing the solution into small parts called element and shape function are defined in terms of field variable at specific points called nodes.

Shape function for 1Dimensional for linear element in global coordinates

N1=(X2 - X)/L,

N2=(X-X1)/L

X2, X1 is end points of the element, L length of the element; X is any point in domain

Shape function for 2 Dimensional for linear rectangular element in global coordinates

N1= (1-X/L)*(1-Y/W)

N2=(X*Y)*(1-Y/W)

N3=(X*Y)/ (L*W)

N4=(Y/W)*(1-X/L)

L, W is the length and breadth of rectangle, X, Y is any point in domain

After selecting element and nodes unknown next steps in Finite element analysis is to assembles the element property we have to find the force displacement matrix i.e. stiffness characteristics of each element

$$[k][\delta] = [f]$$

Where k for 1 Dimensional

$$k = \int_{x_1}^{x_2} [Bt]\,[d][B]\delta x$$

B is strain displacement matrix

Strain displacement matrix for 1 Dimensional

B=1/L*[-1 1]

B^t is B transpose

D is Material property matrix

D Material Matrix for 1 Dimensional

d= E for average young modulus of domain

d= (E2+E1)/2 for average young modulus of element

d=E1+ (E2-E1)*X/L linear variation of young modulus along X direction

d=E1+ (E2-E1)*X^2/L^2 non linear variation of young modulus along X direction

Stiffness matrix for two-dimensional elements is given as

Where k is for 2 Dimensional

$$k = \int_{-\frac{l}{2}}^{\frac{l}{2}} \int_{-\frac{w}{2}}^{\frac{w}{2}} t \times [Bt]\,[D][B]\partial x \partial y$$

B is strain displacement matrix

$$B=\begin{bmatrix}\frac{\partial N_1}{\partial x} & 0 & \frac{\partial N_2}{\partial x} & 0 & \frac{\partial N_3}{\partial x} & 0 & \frac{\partial N_4}{\partial x} & 0\\ 0 & \frac{\partial N_1}{\partial y} & 0 & \frac{\partial N_2}{\partial y} & 0 & \frac{\partial N_3}{\partial y} & 0 & \frac{\partial N_4}{\partial y}\\ \frac{\partial N_1}{\partial y} & \frac{\partial N_1}{\partial x} & \frac{\partial N_2}{\partial y} & \frac{\partial N_2}{\partial x} & \frac{\partial N_3}{\partial y} & \frac{\partial N_3}{\partial x} & \frac{\partial N_4}{\partial y} & \frac{\partial N_4}{\partial x}\end{bmatrix}$$

$N_1 N_2 N_3 N_4$ are shape functions

$$N_3 = (X * Y)/(L * W), N_4 = (Y/W) * (1 - X/L)$$

Bt= B transpose
t=thickness and
D= Material matrix
The material matrix [D] for above mentioned condition are given as

1. Average young modulus of domain

$$D=\frac{E}{(1+v)(1-2v)}\begin{bmatrix}1-v & v & 0\\ v & 1-v & 0\\ 0 & 0 & \frac{1-v}{2}\end{bmatrix}$$

υ = Poison ratio
E =Young's modulus

2. Average young modulus of element

$$D=\frac{\frac{(E_1+E_2)}{2}}{(1+v)(1-2v)}\begin{bmatrix}1-v & v & 0\\ v & 1-v & 0\\ 0 & 0 & \frac{1-v}{2}\end{bmatrix}$$

3. Bi-linear variation in element

$$D=\frac{E(x)}{(1+v)(1-2v)}\begin{bmatrix}1-v & v & 0\\ v & 1-v & 0\\ 0 & 0 & \frac{1-v}{2}\end{bmatrix}$$

Where $E(x) = E_1 + (E_2 - E_1) \times \frac{x}{l} + (E_3 - E_1) \times \frac{y}{l} + (E_4 - E_{3-}E_{2-}E_1) \times \frac{(x \times y)}{l^2}$

4. Bi- non-linear variation in element

$$D=\frac{E(x)}{(1+v)(1-2v)}\begin{bmatrix}1-v & v & 0\\ v & 1-v & 0\\ 0 & 0 & \frac{1-v}{2}\end{bmatrix}$$

Where $E(x) = E_1 + (E_2 - E_1) \times \frac{x^2}{l^2} + (E_3 - E_1) \times \frac{y^2}{l^2} + (E_4 - E_{3-}E_{2-}E_1) \times \frac{(x^2 \times y^2)}{l^4}$

Element properties are assembled to get global properties of structure by using following system equation

$$[k][\delta] = [F]$$

Then boundary and loading condition imposed
Finally getting the result nodal unknowns and future calculate stress, strain etc

For 2Dimensional
A heterogeneous object with different material properties variation with two different materials is shown in Figure 2, 3, 4. The Young's modulus of material 1 is 1000 GPa and material 2 is 10 GPa, Poison ratio constant for both materials (υ) is 0.25 is considered. The dimensions of bar are 20x10x10 cm taken.

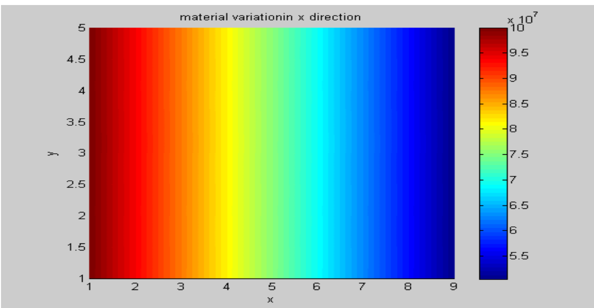


Figure 2.X direction material variation

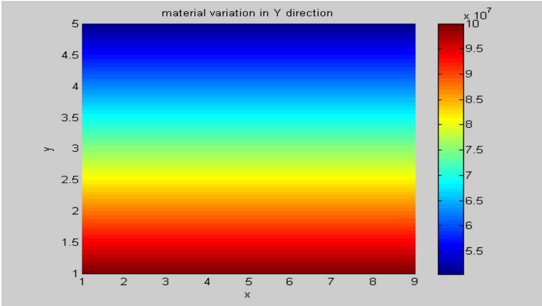


figure 3.Y direction material variation

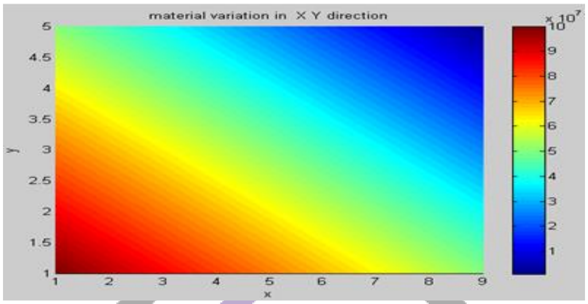


Figure 4. XY direction material distribution

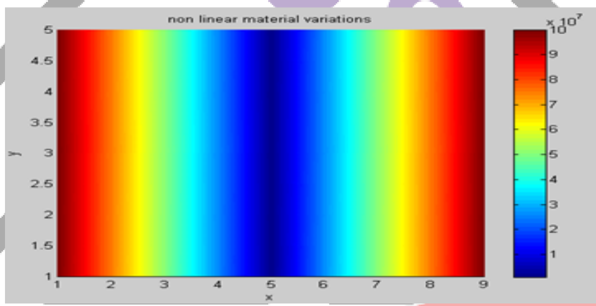


Figure 5. Non linear material distribution

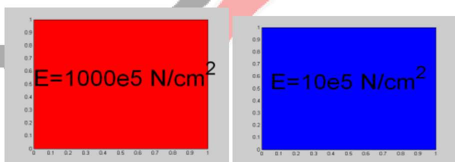


Figure 6. Model heterogeneous object

V. SIMULATION RESULT

A heterogeneous object is taken with two different materials. Heat resistant material increases its fraction gradually from one side to another (the blue line), while the metal material linearly reduces its fraction (the red line), as shown in figure (1) with young modulus of material 1=1000e5GPa ,young modulus for material 2=10e5GPa and poison ratio =0.25.As we discuss the solution technique for solving the heterogeneous object we have four types of element .average material property of domain(E is same for all element),average of young's modulus (E) will be taken in element ,Linearly variation of young's modulus (E) within element is taken in element, Non linearly variation of young's modulus (E) within element as taken in element. And we try to find out the actual variation in field variable (displacement in solid mechanics problem) while problem is same only selection of element is different. Before starting analysis we convergence our results and result is converge at 390 to 400 elements where the value comes 0.465 cm in the case of average E of the element case and the size of the element is 0.025 cm and linear 2 nodded element is taken . Which is more clearly shown in fig (7) and the difference in the result after analysis is shown in table 1.

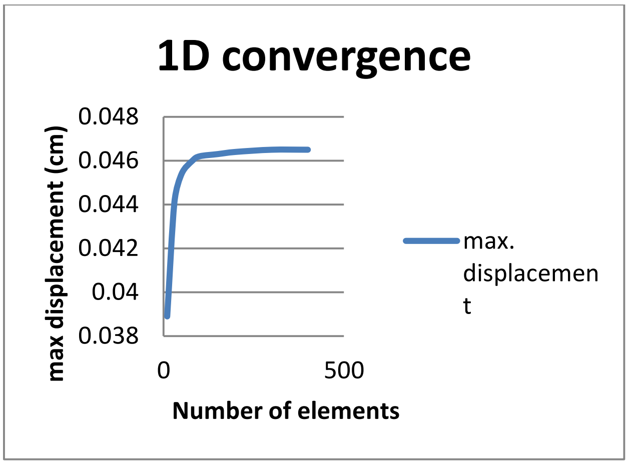


Figure 7 1Dimensional convergence

After analysis the difference in result is easily understandable .in first case the element property is taken constant 505e5GPa and the results is comes 0.0198 cm while if each and every element property is change according to variation than the result becomes 0.0465cm and also same in case of linear variation in element property in element 0.0465cm, but if case of non linear variation within element than result have variation and the result is .0462cm

Table 1Dimensional convergence

Element property variation	Maximum displacement (cm)
Average material property of whole domain	0.0198
Average material property in element	0.0465
Linear variation of material property in element	0.0465
Non linear variation of material property in element	0.0462

Two Dimensional Results (loading condition 1)

Results of average material property in element

The elastic analysis for four different type heterogeneous object are submitted to static load and the exact solution are obtained in case of average element (E). The contour result shows the variation in the field variable (X displacement) in four heterogeneous objects

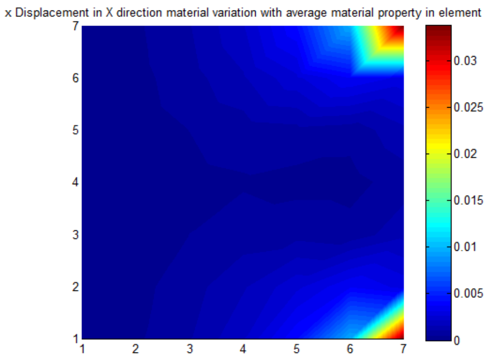


Figure 8.X material distribution

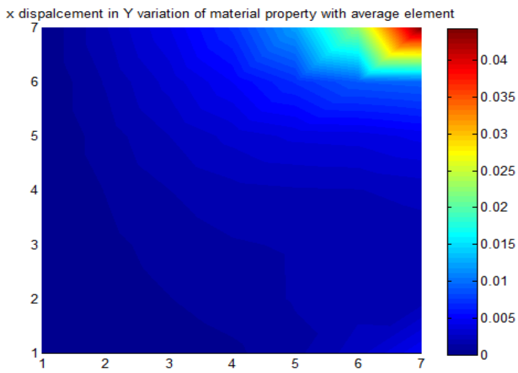
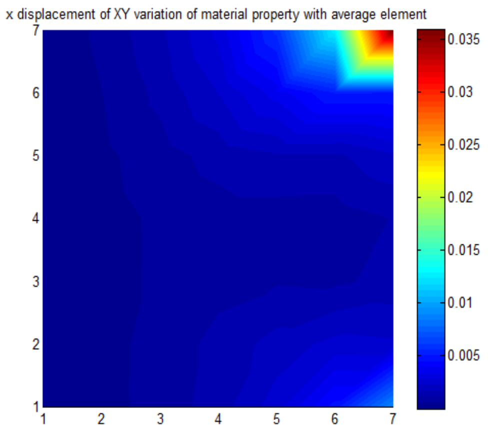


Figure 9. Y direction material distribution



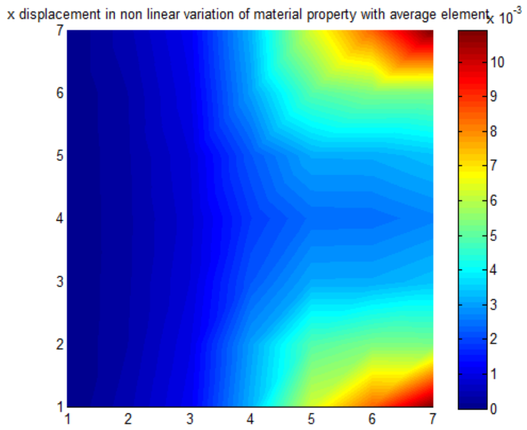


Fig (4.4.3) XY direction material distribution   Fig (4.4.4) Nonlinear material distribution

The elastic analysis for four different type heterogeneous object are submitted to static load and the exact solution are obtained in case of average element (E). The load point shows the variation in the field variable (X displacement) in four heterogeneous objects

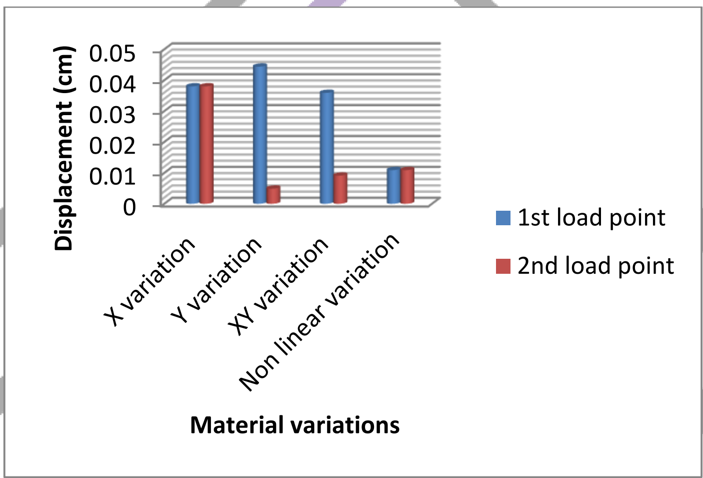


Fig (4.4.5) difference of maximum at load point

VI. Conclusion

The purpose of the study was to introduce heterogeneous object with Material based graded element finite element analysis. Usually the accuracy forecast of field variables in heterogeneous object is depending upon accurate representation of material property and mesh sizes. In our approach material property is calculate in every point in the domain and increasing the number of points can increase the accuracy of domain and considering the variation within the element by using this new type of element results improves which is seen load case 1 for different material variation.

References

[1] G.I.Kanel, M.F.Ivanov and A.N. Parhikov "Computer simulation of the heterogeneous material response to the impact loading" International journal impact engineering, vol.17, pp.455-464, 1995

[2] Xiang Houngjun, Shi Zhifei, Zhang Taotao "Elastic analysis of heterogeneous hollow cylinder" Mechanics Research communication 33,pp.681-691,2006.

[3] H.Khezrzadeh, M, Mofid "Tensile facture behavior of heterogeneous material based on fractal geometry" Theoretical and applied fracture mechanics 46,pp. 46-56,2006.

[4] Benidikt Helgason, Fulvia Taddei,Halldor Palsson, Enrico Schileo, Luca Cristofolini, Marco Viceconti, Sigurdur Brynjolfsson "A modified method for assigning material properties to FE models of bones" Medical Engineering & physics 30 (2008) 444-453.

[5] C.Provatidis, C.Vossou, E.Petropoulou, A.Balanika, G.Lyritis "A finite element analysis of a T12 vertebra in Two consecutive examination to evaluate the progress of osteoporosis" Medical engineering & Physics 31,pp. 632-641,2009.

[6] Uday V. Pise, Amba.D.Bhatt, R.Ksrivastava, Ravi Warkedkar "A B-spline based heterogeneous modeling and analysis of proximal femur with Graded element" Journal of biomechanics 42,pp. 1981-1988,2009.

[7] Pinghai Yang, Xiaoping Qian , "A B-spline-basedApproach to heterogeneous objects design and analysis" Computer-Aided design 39,pp. 95-111,2007.

[8] Y.K Siu, S.T. Tan, “Modeling the material grading ans structures of heterogeneous objects for layered manufacturing” Computer-aided design 34(2002) 705-716.
[9] Xiaoping Qian, Debasish Dutta, “Feature-based design for heterogeneous object” Computer aided-design 36(2004) 1263-1278,2004.
[10] X.Y Kou, S.T Tan, “Heterogeneous Object Modeling : A review” Computer aided-design 39(2007) 284-301.
[11] Anton Iulia-Alina, Panaitescu Mariana, Panaitescu Fanel-Viorel, “Applications In Matlab Using Finite Element Method”, Journal of Marine technology and Environment, Vol. 2 , Year 2015.
[12] Siu, Y.-K., and Tan, S.-T., “Source-based heterogeneous solid modeling”, Computer-Aided Design, 34(1), 2002, 41– 55.
[13] Kou, X.-Y., Tan, S.-T., “A hierarchical representation for heterogeneous object modeling”, Computer-Aided Design, 2004.
[14] Jackson, T. -R., “Analysis of functionally graded material object representation methods”, PhD thesis, Massachusetts Institute of Technology (MIT), Cambridge, MA, January 2000.
[15] Biswas, A., Shapiro, V., and Tsukanov, I., “Heterogeneous material modeling with distance fields”, Computer-Aided Geometric Design, 21(3), 215–42,2004.
[16] Chiu, W.-K., and Tan, S.-T., “Multiple material objects: from CAD representation to data format for rapid prototyping”, Computer-Aided Design, 32, 707–17,200.
[17] Kou X. Y., Tan S. T., Robust and efficient algorithms for rapid prototyping of heterogeneous objects, Rapid Prototyping Journal, 15(1), 5-18, 2009.
[18] Sun W., Jiang T., and Lin F., A processing algorithm for freeform fabrication of heterogeneous structures, Rapid Prototyping Journal, 10(5), 316-326, 2004.
[19] X.Y. Kou and S.T. Tan, “Heterogeneous Object Design: An Integrated CAX Perspective”, pp. 307-319.
[20] Y.H. You , X.Y. Kou, S.T. Tan “Adaptive meshing for finite element analysis of heterogeneous materials”, ACM dital lib, Volume 62 Issue C, Pages 176-189 , May 2015 .
[21] Xuanneng Gao, Haoming Zhu, Renhui Wang “Stability analysis of frame structures by bubble-function method”,Finite Elements in Analysis and Design, Volume 45, Issues 8–9, , Pages 495-500, June–July 2009.
[22] V. Gupta, K.-S. Kasana, and P. Tandon, “Reference based geometric modelling for heterogeneous objects,” Computer Aided Design & Applications, vol. 9(2), pp. 155-165,2012.
[23] V. Gupta, and P. Tandon, “Heterogeneous object modelling with material convolution surfaces,” Computer Aided Design, vol. 62, pp. 236-247, May 2015.
[24] Vikas Gupta et. al., “Rapid Prototyping of Heterogeneous Objects: Issues and Challenges”, Asian Review of Mechanical Engineering, Vol. 5 No. 1, pp. 24-29, 2016.