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Comparison of Roman domination number, Total domination number and distance-2 domination number with radius of a trapezoid graph TG

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***Abstract*: Domination in graphs has been an extensively researched branch of graph theory. Graph theory is one of the most flourishing branches of modern mathematics and computer applications. Different types of dominations like roman domination, total domination and distance-2 domination in graphs have applications to several fields. In this paper we compare roman domination number, total domination number and distance-2 domination number with radius of a graph G.**

***Keywords*: Eccentricity, Distance, Radius of a graph, Roman domination number, Total domination number, Distance-2 domination number**

Introduction:

Graph theory is rapidly moving into the mainstream of mathematics mainly due to its applications in diverse fields which include biochemistry (genomics), electrical engineering (communications networks and coding theory), computer science (algorithms and computations) and operations research (scheduling). Although graph theory is one of the younger branches of mathematics, it is fundamental to a number of applied fields.

Few subjects in mathematics have as specified an origin as graph theory. Graph theory originated with the Konigsberg Bridge Problem, which Leonhard Euler (1707-1783) solved in 1736. Over the past sixty years, there has been a great deal of exploration in the area of graph theory. Its popularity has increased due to its many modern day applications and it has become the source of interest to many researchers. High-speed digital computer is one of the main reasons for the recent growth of interest in graph theory and its applications.

Now a days graphs are really important in different fields. Probably, more important than we think. Graphs are among the most ubiquitous models of both natural and human-made structures. They can be used to model many types of relations and process dynamics in Physical, Biological and Social systems.

Many problems of practical interest can be represented by graphs and can be solved using graph theory. In Architecture, bipartite graphs play an important role in finding minimum number of cross beams required to make a grid of beams rigid, when the joints provide no rigidity to the structure. The structure is rigid if and only if the corresponding bipartite graph is connected. But the smallest connected graph is a tree and the largest possible tree in the bipartite graph with m, n vertices has m+n-1 edges. Hence an mxn graph is rigid iff the corresponding bipartite graph is connected. The rigid bracing will have minimum cross beams iff the bipartite graph is a tree with m+n-1 cross beams.

So, one can confidently put forward that a mere act of thinking about a problem in terms of a graph will certainly suggest insights and probable solution methods.

Graph theory is branch of mathematics, which has becomes quite rich and interesting for several regions. In last three decades thousand of research articles have been published in graph theory. There are several areas in graph theory which have reserved good attention from mathematicians. Graphs are very convenient tool for representing the relationship among objects, which are represented by vertices. In their term relationships among vertices are represented by connections. In general, any mathematical objects involving points and connection among them can be called a graph. For a great diversity of problems such pictorial representations may lead to a solution. For example data basics map coloring web graph, physical net work and organic molecular as well as less tangible interactions occurring in social net works in a flow of a computer program.

Domination is a rapidly developing area of research in graph theory, and its various applications to ad-hock net work, distributed computing, social net works and web graph, partly explain the increased interest. Domination in graphs has been an extensively researched branch of graph theory. Graph theory is one of the most flourishing branches of modern mathematics and computer applications. The last 30 years have witnessed spectacular growth of graph theory due to its wide application to discrete optimization problems, combinatorial problems and classical algebraic problems. It has a very wide range of application to many fields like engineering, physical, social and biological sciences, linguistics ect., the theory of domination has been the nucleus of research activity in graph theory in recent time.

Domination in graphs has applications to several fields. Domination arises in facility location problem, where the number of facilities like hospitals, fire stations is fixed and one attempts to minimize the distance that a person needs to travel to get to the closest facility. A similar problem occurs when the maximum distance to a facility is fixed and one attempts to minimize the number of facilities necessary so that ever one is serviced. Concepts from domination also appear in problems involving finding sets of representatives in monitoring communication or electrical networking, and in land surveying like minimizing the number of places a surveyor must stand in order to take high measure mints for an entire region.

A subset D of V is said to be a dominating set of G if every vertex in V\D is adjacent to a vertex in D. A dominating set with minimum cardinality is said to be a minimum dominating set. The domination number $\gamma(G)$ of the graph G is the minimum cardinality of the dominating set in G.

A trapezoid graph (TG) consists of two horizontal lines L_1 (top line) and L_2 (bottom line) and a set of trapezoids $T = \{T_1, T_2, ..., T_n\}$ with corner points lying on these two lines. An undirected graph G with vertex set $V = \{v_1, v_2, ..., v_n\}$ and edge set $E = \{e_1, e_2, ..., e_m\}$ is called a TG when a trapezoid representation exists with trapezoid set T, such that any vertex $v_k \in V$ corresponds to a trapezoid $T_k \in T$ and an edge $(v_k, v_l) \in E$ iff T_k and T_l intersect.

Any trapezoid T_k within these two lines is known by four corner points a_k, b_k, c_k and d_k which represent the upper left, upper right, lower left and lower right corner points respectively. Without loss of generality, we assume that no two trapezoids share a same endpoint. It is assumed that $T_p < T_q$ or $v_p < v_q$ iff $a_p < a_q$.

The TG was established by Dagan. Corneil and Kamula established the same class but they refer to them as II (interval-interval) graphs. Felsner, Muller and Wernisch established an equivalent characterization for TG. After that, it has been broadly studied by many researchers. The class of TG is the generalization of the classes of IG and PG. It is very interesting that if $a_p = b_p$ and $c_p = d_p$ then the corresponding trapezoid T_p reduces to straight line. So, in that way, if each trapezoids cut down to a straight lines the corresponding TG reduces to a PG. Similarly, if $a_p = c_p$ and $b_p = d_p$, for every p, then the TG reduces to an IG. It is to be noted that any trapezoid T_u and the corresponding node u are one and same thing.

Preliminaries:

Let G be a graph. An edge which joins the same vertex is called a loop. An edge that joins two distinct vertices is called a link. If two or more edges of G have the same end vertices, then these edges are called parallel edges or multiple edges. A graph is said to be a simple graph if it has no loops and no parallel edges.

A graph G is said to be finite if both its vertex set and edge sets are finite, otherwise it is called an infinite graph. The number of edges incident with a vertex v of a graph is called the degree of v and is denoted by d(v). A graph is said to be connected if there is a path between every pair of vertices, otherwise it is said to be disconnected graph. Neighborhood of a vertex $v \in V$ is a set consisting all vertices adjacent to v (including v), it is denoted by nbd [v]. i.e., nbd [v] = {the set of all vertices adjacent to v} $\cup$ {v}.

A subset S of V is called an independent set of G if no two vertices in S are adjacent. A maximum independent set of G is an independent set whose cardinality is largest among all independent sets of G. A subset D of V is said to be a dominating set of G if every vertex in V-D is adjacent to a vertex in D. A dominating set with minimum cardinality is said to be a minimum dominating set. The domination number $\gamma(G)$ of the graph G is the minimum cardinality of the dominating set in G. A total dominating set D of a graph G is a dominating set in which every vertex is adjacent to some vertex in it. The total domination number $\gamma_t(G)$ of the graph G is the minimum cardinality of the total dominating set. The distance between two vertices u and v of a graph is the length of the shortest path (path of minimum length) between them and is denoted by d_G of (u,v).

A dominating set D of a graph G = (V, E) is roman dominating set, if S and T are two subsets of D and satisfying the condition that every vertex u in S is adjacent to exactly one to one a vertex v in V − D as well as adjacent to some vertex in T . The roman domination number of γ_{rds} (G) of G is the minimum cardinality of a roman dominating set of G.

An independent set $\boldsymbol{S}$ of vertices in a graph is an efficient dominating set when each vertex not in $\boldsymbol{S}$ is adjacent to exactly one vertex in $\boldsymbol{S}$. A set $S \subset V$ is a dominating set if $N\,S = V$, that is, every vertex in $V \backslash S$ is adjacent to some vertex in S. The domination number (G) is the minimum cardinality of a dominating set in G.

A subset S of V is called a domination set in G if every vertex in V-S is adjacent to at least one vertex in S. A dominating set is said to be Total Dominating set if every vertex in V is adjacent to at least one vertex in S. Minimum cardinality taken over all total dominating set is called as total domination number and is denoted by $\gamma_t(G)$.

A set D of vertices in a graph G = (V,E) is a distance - 2 dominating set if every vertex in V-D is within distance 2 of at least one vertex in D. The distance -2 domination number of G equals the minimum cardinality of a distance 2- dominating set in G.

Distance between Two Vertices in a graph G is number of edges in a shortest path between Vertex u and Vertex v. If there are multiple paths connecting two vertices, then the shortest path is considered as the distance between the two vertices. Denoted as d(u,v).

Eccentricity of a Vertex is the maximum distance between a vertex to all other vertices is considered as the eccentricity of vertex. Denoted as e(V). The distance from a particular vertex to all other vertices in the graph is taken and among those distances, the eccentricity is the highest of distances.

Radius of a Connected Graph G is the minimum eccentricity from all the vertices is considered as the radius of the Graph G. The minimum among all the maximum distances between a vertex to all other vertices is considered as the radius of the Graph G. denoted as r(G). From all the eccentricities of the vertices in a graph, the radius of the connected graph is the minimum of all those eccentricities.

Let G be a graph and v be a vertex of G. The eccentricity of the vertex v is the maximum distance from v to any vertex. That is, e(v)=max{d(v, w):w in V(G)}. The radius of G is the minimum eccentricity among the vertices of G. Therefore, radius(G)=min{e(v):v in V(G)}.

Distance between u and v in the graph G is the length of the shortest $u - v$ path in G if such a path exists. If no $u - v$ path exist, define $d(u, v) = \infty$.

For each vertex v in a graph G, the eccentricity, $e(v)$, is the distance from v to the vertex furthest from v, that is, $e(v) = \max \{d(v, u) \mid u \text{ in } V(G)\}$.

Radius rad G of a connected graph G is defined as rad G = $\min \{e(v) \mid v \text{ in } V(G)\}$.

A trapezoid graph (TG) consists of two horizontal lines L_1 (top line) and L_2 (bottom line) and a set of trapezoids $T = \{T_1, T_2, \dots, T_n\}$ with corner points lying on these two lines. An undirected graph G with vertex set $V = \{v_1, v_2, \dots, v_n\}$ and edge set $E = \{e_1, e_2, \dots, e_m\}$ is called a TG when a trapezoid representation exists with trapezoid set T, such that any vertex $v_k \in V$ corresponds to a trapezoid $T_k \in T$ and an edge $(v_k, v_l) \in E$ iff T_k and T_l intersect.

Main theorems:

Theorem 1: If $T = \{T_1, T_2, \dots, T_n\}$ be a trapezoid family and TG is a trapezoid graph corresponding to a trapezoid family 'T'. Then the Roman domination number is greater than the radius of a trapezoid graph TG

Proof: A dominating set D of a graph G = (V, E) is roman dominating set, if S and T are two subsets of D and satisfying the condition that every vertex u in S is adjacent to exactly one to one a vertex v in $V - D$ as well as adjacent to some vertex in T . The roman domination number of γ_{rds} (G) of G is the minimum cardinality of a roman dominating set of G.

Distance between Two Vertices in a graph G is number of edges in a shortest path between Vertex u and Vertex v. If there are multiple paths connecting two vertices, then the shortest path is considered as the distance between the two vertices. Denoted as d(u,v).

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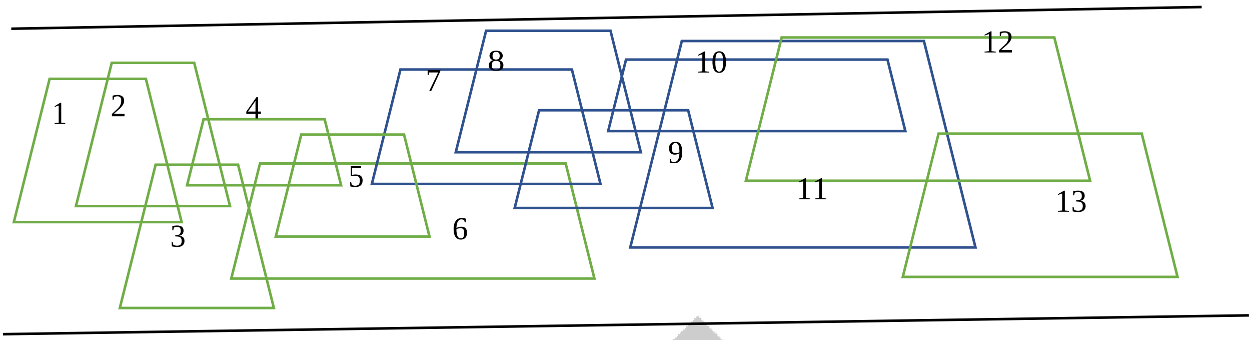
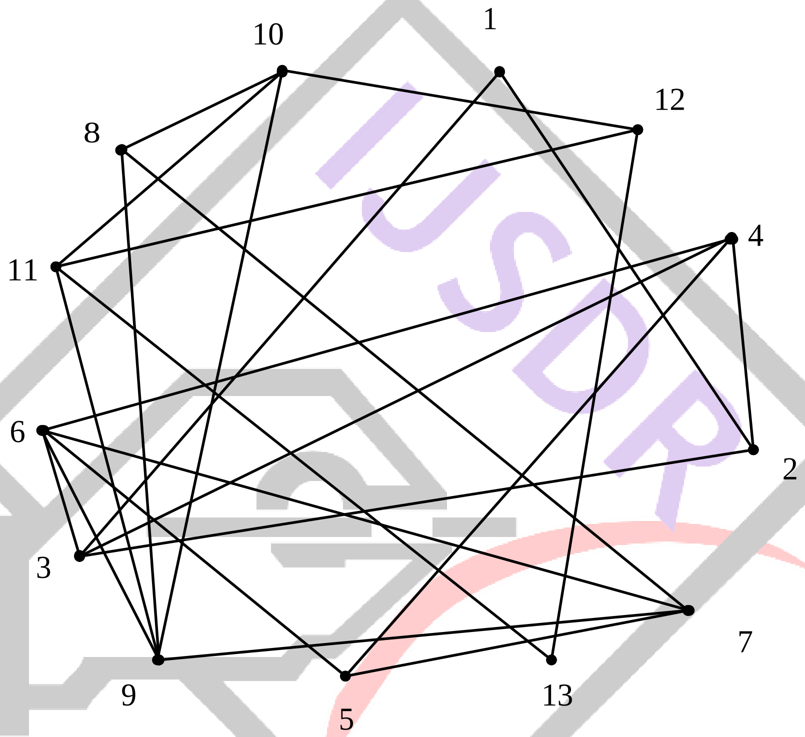
Let G be a graph and v be a vertex of G. The eccentricity of the vertex v is the maximum distance from v to any vertex. That is, e(v)=max{d(v,w):w in V(G)}. The radius of G is the minimum eccentricity among the vertices of G. Therefore, radius(G)=min{e(v):v in V(G)}.

Experimental problem





Find the Roman domination number of G

D is a dominating set v_1 dominates v_2 and v_3, v_3 dominates v_4 and v_6, v_6 dominates v_5, v_7, v_9, v_9 dominates v_{10} and v_{11}, v_{11} dominates v_{12}, v_{13}

S and T are two subsets of D

S= { v_1} and T={v_3, v_6, v_9, v_{11}}

Vertex v_1 in S is adjacent to exactly one vertex v_2 in V-D as well as adjacent to vertex v_3 in T.

S and T are two subsets of D and satisfying the condition that every vertex u in S is adjacent to exactly one to one a vertex v in V − D as well as adjacent to some vertex in T.

Roman dominating set of G is {v_1, v_3, v_6, v_9, v_{11}}

Roman domination number of G is 5.

Find the radius of G

Find the distance between u ,v such that u,v are in V

d(1,1)=0	d(2,1)= 1	d(3,1)=1
d(1,2)=1	d(2,2)= 0	d(3,2)=1
d(1,3)=1	d(2,3)=1	d(3,3)=0
d(1,4)=2	d(2,4)=1	d(3,4)=1
d(1,5)=3	d(2,5)=3	d(3,5)=2

d(1,6)=2	d(2,6)=2	d(3,6)=1
d(1,7)=3	d(2,7)=3	d(3,7)=2
d(1,8)=4	d(2,8)=4	d(3,8)=3
d(1,9)=3	d(2,9)=3	d(3,9)=2
d(1,10)=4	d(2,10)=4	d(3,10)=3
d(1,11)=4	d(2,11)=4	d(3,11)=3
d(1,12)=4	d(2,12)=4	d(3,12)=3
d(1,13)=5	d(2,13)=5	d(3,13)=4

d(4,1)=2	d(5,1)=3	d(6,1)=2
d(4,2)=1	d(5,2)=3	d(6,2)=2
d(4,3)=1	d(5,3)=2	d(6,3)=1
d(4,4)=0	d(5,4)=1	d(6,4)=1
d(4,5)=1	d(5,5)=0	d(6,5)=1
d(4,6)=1	d(5,6)=1	d(6,6)=0
d(4,7)=2	d(5,7)=1	d(6,7)=1
d(4,8)=3	d(5,8)=2	d(6,8)=2
d(4,9)=2	d(5,9)=2	d(6,9)=1
d(4,10)=3	d(5,10)=3	d(6,10)=2
d(4,11)=3	d(5,11)=3	d(6,11)=2
d(4,12)=3	d(5,12)=2	d(6,12)=2
d(4,13)=4	d(5,13)=3	d(6,13)=3

d(7,1)=3	d(8,1)=4	d(9,1)=3
d(7,2)=3	d(8,2)=4	d(9,2)=3
d(7,3)=2	d(8,3)=3	d(9,3)=2
d(7,4)=2	d(8,4)=3	d(9,4)=2
d(7,5)=1	d(8,5)=2	d(9,5)=2
d(7,6)=1	d(8,6)=2	d(9,6)=1
d(7,7)=0	d(8,7)=1	d(9,7)=1
d(7,8)=1	d(8,8)=0	d(9,8)=1
d(7,9)=1	d(8,9)=1	d(9,9)=0
d(7,10)=2	d(8,10)=1	d(9,10)=1
d(7,11)=2	d(8,11)=2	d(9,11)=1
d(7,12)=1	d(8,12)=2	d(9,12)=2
d(7,13)=2	d(8,13)=3	d(9,13)=2

d(10,1)=4	d(11,1)=4	d(12,1)=4
d(10,2)=4	d(11,2)=4	d(12,2)=4
d(10,3)=3	d(11,3)=3	d(12,3)=3
d(10,4)=3	d(11,4)=3	d(12,4)=3
d(10,5)=3	d(11,5)=3	d(12,5)=2
d(10,6)=2	d(11,6)=2	d(12,6)=2
d(10,7)=2	d(11,7)=2	d(12,7)=1
d(10,8)=1	d(11,8)=2	d(12,8)=2
d(10,9)=1	d(11,9)=1	d(12,9)=2
d(10,10)=0	d(11,10)=1	d(12,10)=1
d(10,11)=1	d(11,11)=0	d(12,11)=1
d(10,12)=1	d(11,12)=1	d(12,12)=0
d(10,13)=2	d(11,13)=1	d(12,13)=1

d(13,1)=5
d(13,2)=5
d(13,3)=4
d(13,4)=4
d(13,5)=3
d(13,6)=3
d(13,7)=2
d(13,8)=3
d(13,9)=2
d(13,10)=2
d(13,11)=1
d(13,12)=1

d(13,13)=0

Find the eccentricity, $e(v)$ of G

$e(v)$ = max {d(v, u) | u in V(G)}.
$e(1)$ = max {d(1, u) | u in V(G)}= max { 0, 1, 1, 2, 3, 2, 3, 4, 3, 4, 4, 4, 5 }=5
$e(2)$ = max {d(2, u) | u in V(G)}= max { 1, 0, 1, 1, 3, 2, 3, 4, 3, 4, 4, 4, 5 }=5
$e(3)$ = max {d(3, u) | u in V(G)}= max { 1, 1, 0, 1, 2, 1, 2, 3, 2, 3, 3, 3, 4 }=4
$e(4)$ = max {d(4, u) | u in V(G)}= max { 2, 1, 1, 0, 1, 1, 2, 3, 2, 3, 3, 3, 4 }=4
$e(5)$ = max {d(5, u) | u in V(G)}= max { 3, 3, 2, 1, 0, 1, 1, 2, 2, 3, 3, 2, 3 }=3
$e(6)$ = max {d(6, u) | u in V(G)}= max { 2, 2, 1, 1, 1, 0, 1, 2, 1, 2, 2, 2, 3 }=3
$e(7)$ = max {d(7, u) | u in V(G)}= max { 3, 3, 2, 2, 1, 1, 0, 1, 1, 2, 2, 1, 2 }=3
$e(8)$ = max {d(8, u) | u in V(G)}= max { 4, 4, 3, 3, 2, 2, 1, 0, 1, 1, 2, 2, 3 }=4
$e(9)$ = max {d(9, u) | u in V(G)}= max { 3, 3, 2, 2, 2, 1, 1, 1, 0, 1, 1, 2, 2 }=3
$e(10)$ = max {d(10, u) | u in V(G)}= max { 4, 4, 3, 3, 3, 2, 2, 1, 1, 0, 1, 1, 2 }=4
$e(11)$ = max {d(11, u) | u in V(G)}= max { 4, 4, 3, 3, 3, 2, 2, 2, 1, 1, 0, 1, 1 }=4
$e(12)$ = max {d(12, u) | u in V(G)}= max { 4, 4, 3, 3, 2, 2, 1, 2, 2, 1, 1, 0, 1 }=4
$e(13)$ = max {d(13, u) | u in V(G)}= max { 5, 5, 4, 4, 3, 3, 2, 3, 2, 2, 1, 1, 0 }=5

Radius rad G of a connected graph G is defined as rad G = min {$e(v)$ | v in V(G)}

rad G = min {$e(1)$, $e(2)$, $e(3)$, $e(4)$, $e(5)$, $e(6)$, $e(7)$, $e(8)$, $e(9)$, $e(10)$, $e(11)$, $e(12)$, $e(13)$}
= min {5, 5, 4, 4, 3, 3, 3, 4, 3, 4, 4, 4, 5}
= 3

Roman domination number and Radius of G are 5 and 3 respectively
Roman domination number of G is greater than that of Radius of G 5>3
We proved that Roman domination number of a graph G is greater than Radius of G.

Theorem 2: If $T = \{T_1, T_2, \dots, T_n\}$ be a trapezoid family and TG is a trapezoid graph corresponding to a trapezoid family 'T'. Then the Total domination number is greater than the radius of a trapezoid graph TG

Proof: A subset S of V is called a domination set in G if every vertex in V-S is adjacent to at least one vertex in S. A dominating set is said to be Total Dominating set if every vertex in V is adjacent to at least one vertex in S. Minimum cardinality taken over all total dominating set is called as total domination number and is denoted by $\gamma_t(G)$.

Total domination in graphs was introduced by Cockayne, Dawes, and Hedetniemi and is now well studied in graph theory. The literature on this subject has been surveyed and detailed in the two excellent domination books by Haynes, Hedetniemi, and Slater who did an outstanding job of unifying results scattered through some 1200 domination papers at that time. A total dominating set, denoted TDS, of G with no isolated vertex is a set S of vertices of G such that every vertex is adjacent to a vertex in S. If no proper subset of S is a TDS of G, then S is a minimal TDS of G. Every graph without isolated vertices has a TDS, since S D V is such a set. The total domination number of G, denoted by $\gamma_t(G)$, is the minimum cardinality of a TDS.

Distance between Two Vertices in a graph G is number of edges in a shortest path between Vertex u and Vertex v. If there are multiple paths connecting two vertices, then the shortest path is considered as the distance between the two vertices. Denoted as d(u,v).

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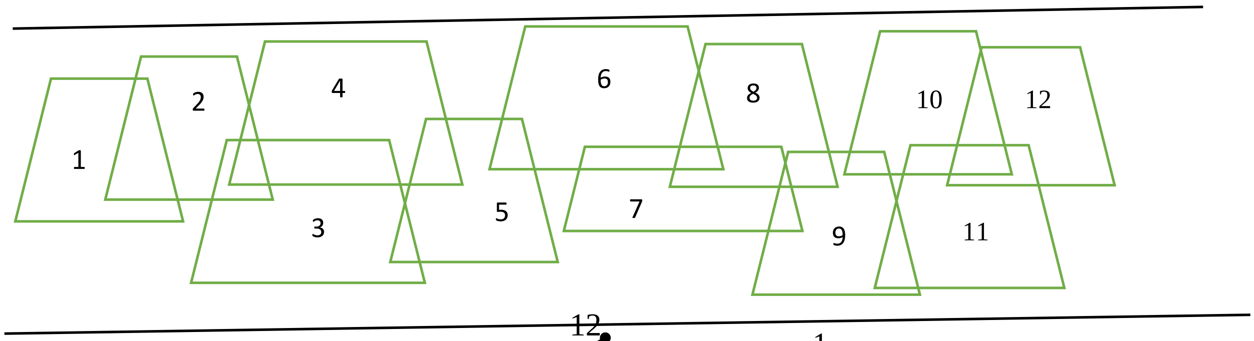
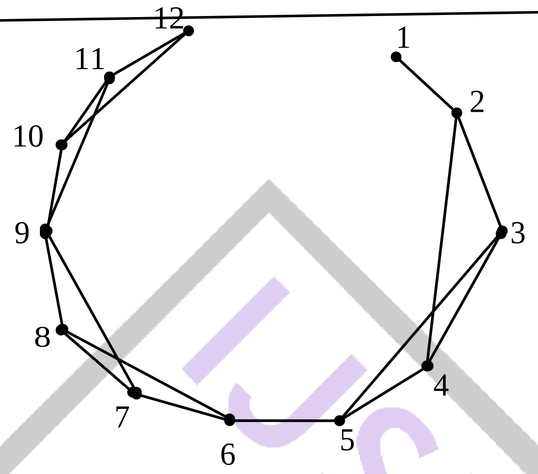
Let G be a graph and v be a vertex of G. The eccentricity of the vertex v is the maximum distance from v to any vertex. That is, e(v)=max{d(v,w):w in V(G)}. The radius of G is the minimum eccentricity among the vertices of G. Therefore, radius(G)=min{e(v):v in V(G)}.

Experimental problem





Find the total domination number of G

A subset S of V is called a domination set in G if every vertex in V-S is adjacent to at least one vertex in S. A dominating set is said to be Total Dominating set if every vertex in V is adjacent to at least one vertex in S.

D is a dominating set v_2 dominates v_1, v_4 dominates v_3, v_6 dominates v_7 and v_{10} dominates v_{11} and $v_{12.}$ D is a total dominating set every vertex in V is adjacent to at least one vertex in S. vertex v_2 is adjacent to v_4, v_4 is adjacent to v_5, v_5 is adjacent to v_6, v_6 is adjacent to v_8, v_8 is adjacent to v_9, v_9 is adjacent to v_{10}.

Total dominating set is $\{v_2, v_4, v_5, v_6, v_8, v_9, v_{10}\}$
Total domination number of G is 7

Find the radius of G

Find the distance between u ,v such that u,v are in V

d(1,1)=0	d(2,1)= 1	d(3,1)=2
d(1,2)=1	d(2,2)= 0	d(3,2)=1
d(1,3)=2	d(2,3)=1	d(3,3)=0
d(1,4)=2	d(2,4)=1	d(3,4)=1
d(1,5)=3	d(2,5)=2	d(3,5)=1
d(1,6)=4	d(2,6)=3	d(3,6)=2
d(1,7)=5	d(2,7)=4	d(3,7)=3
d(1,8)=5	d(2,8)=4	d(3,8)=3
d(1,9)=6	d(2,9)=5	d(3,9)=4
d(1,10)=7	d(2,10)=6	d(3,10)=5
d(1,11)=7	d(2,11)=6	d(3,11)=5
d(1,12)=8	d(2,12)=7	d(3,12)=6

d(4,1)=2	d(5,1)=3	d(6,1)=4
d(4,2)=1	d(5,2)=2	d(6,2)=3
d(4,3)=1	d(5,3)=1	d(6,3)=2
d(4,4)=0	d(5,4)=1	d(6,4)=2
d(4,5)=1	d(5,5)=0	d(6,5)=1
d(4,6)=2	d(5,6)=1	d(6,6)=0
d(4,7)=3	d(5,7)=2	d(6,7)=1
d(4,8)=3	d(5,8)=2	d(6,8)=1
d(4,9)=4	d(5,9)=3	d(6,9)=2
d(4,10)=5	d(5,10)=4	d(6,10)=3

d(4,11)=5	d(5,11)=4	d(6,11)=3
d(4,12)=6	d(5,12)=5	d(6,12)=4

d(7,1)=5	d(8,1)=5	d(9,1)=6
d(7,2)=4	d(8,2)=4	d(9,2)=5
d(7,3)=3	d(8,3)=3	d(9,3)=4
d(7,4)=3	d(8,4)=3	d(9,4)=4
d(7,5)=2	d(8,5)=2	d(9,5)=3
d(7,6)=1	d(8,6)=1	d(9,6)=2
d(7,7)=0	d(8,7)=1	d(9,7)=1
d(7,8)=1	d(8,8)=0	d(9,8)=1
d(7,9)=1	d(8,9)=1	d(9,9)=0
d(7,10)=2	d(8,10)=2	d(9,10)=1
d(7,11)=2	d(8,11)=2	d(9,11)=1
d(7,12)=3	d(8,12)=3	d(9,12)=2

d(10,1)=7	d(11,1)=7	d(12,1)=8
d(10,2)=6	d(11,2)=6	d(12,2)=7
d(10,3)=5	d(11,3)=5	d(12,3)=6
d(10,4)=5	d(11,4)=5	d(12,4)=6
d(10,5)=4	d(11,5)=4	d(12,5)=5
d(10,6)=3	d(11,6)=3	d(12,6)=4
d(10,7)=2	d(11,7)=2	d(12,7)=3
d(10,8)=2	d(11,8)=2	d(12,8)=3
d(10,9)=1	d(11,9)=1	d(12,9)=2
d(10,10)=0	d(11,10)=1	d(12,10)=1
d(10,11)=1	d(11,11)=0	d(12,11)=1
d(10,12)=1	d(11,12)=1	d(12,12)=0

Find the eccentricity, $e(v)$ of G

$e(v)$ = max {d(v, u) | u in V(G)}.
$e(1)$ = max {d(1, u) | u in V(G)}= max {0, 1, 2, 2, 3, 4, 5, 5, 6, 7, 7, 8}=8
$e(2)$ = max {d(2, u) | u in V(G)}= max {1, 0, 1, 1, 2, 3, 4, 4, 5, 6, 6, 7}=7
$e(3)$ = max {d(3, u) | u in V(G)}= max {2, 1, 0, 1, 1, 2, 3, 3, 4, 5, 5, 6}=6
$e(4)$ = max {d(4, u) | u in V(G)}= max {2, 1, 1, 0, 1, 2, 3, 3, 4, 5, 5, 6}=6
$e(5)$ = max {d(5, u) | u in V(G)}= max {3, 2, 1, 1, 0, 1, 2, 2, 3, 4, 4, 5}=5
$e(6)$ = max {d(6, u) | u in V(G)}= max {4, 3, 2, 2, 1, 0, 1, 1, 2, 3, 3, 4}=4
$e(7)$ = max {d(7, u) | u in V(G)}= max {5, 4, 3, 3, 2, 1, 0, 1, 1, 2, 2, 3}=5
$e(8)$ = max {d(8, u) | u in V(G)}= max {5, 4, 3, 3, 2, 1, 1, 0, 1, 2, 2, 3}=5
$e(9)$ = max {d(9, u) | u in V(G)}= max {6, 5, 4, 4, 3, 2, 1, 1, 0, 1, 1, 2}=6
$e(10)$ = max {d(10, u) | u in V(G)}= max {7, 6, 5, 5, 4, 3, 2, 2, 1, 0, 1, 1}=7
$e(11)$ = max {d(11, u) | u in V(G)}= max {7, 6, 5, 5, 4, 3, 2, 2, 1, 1, 0, 1}=7
$e(12)$ = max {d(12, u) | u in V(G)}= max {8, 7, 6, 6, 5, 4, 3, 3, 2, 1, 1, 0}=8

Radius rad G of a connected graph G is defined as rad G = min {$e(v)$ | v in V(G)}

rad G = min {$e(1)$, $e(2)$, $e(3)$, $e(4)$, $e(5)$, $e(6)$, $e(7)$, $e(8)$, $e(9)$, $e(10)$, $e(11)$, $e(12)$}
= min {8, 7, 6, 6, 5, 4, 5, 5, 6, 7, 7, 8}
= 5

Total domination number and Radius of G are 7 and 5 respectively
Total domination number of G is greater than that of Radius of G 7>5
We proved that Total domination number of a graph G is greater than Radius of G.

Theorem 3: If $T = \{T_1, T_2, \ldots, T_n\}$ be a trapezoid family and TG is a trapezoid graph corresponding to a trapezoid family 'T'. Then the distance-2 domination number is less than the radius of a trapezoid graph TG

Proof: A set D of vertices in a graph G = (V, E) is a distance - 2 dominating set if every vertex in V-D is within distance 2 of at least one vertex in D. The distance -2 domination number of G equals the minimum cardinality of a distance 2- dominating set in G.

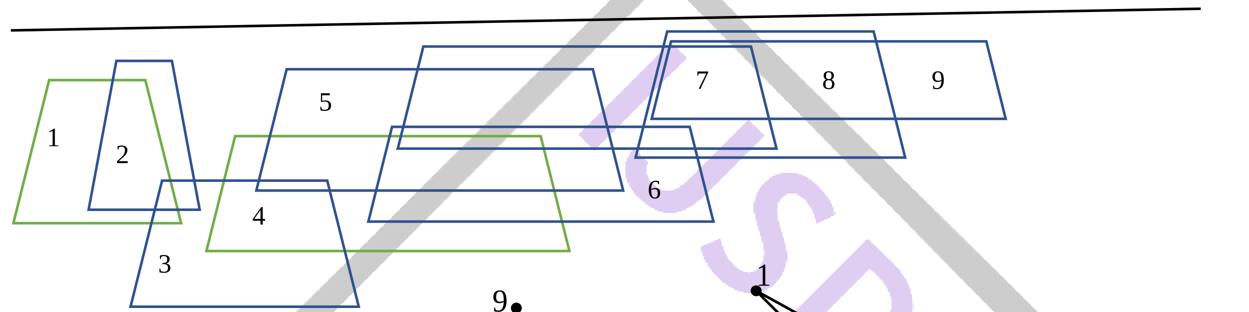
Distance between Two Vertices in a graph G is number of edges in a shortest path between Vertex u and Vertex v. If there are multiple paths connecting two vertices, then the shortest path is considered as the distance between the two vertices. Denoted as d(u,v).

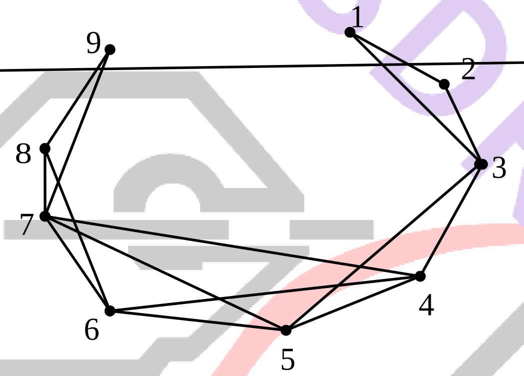
Eccentricity of a Vertex is the maximum distance between a vertex to all other vertices is considered as the eccentricity of vertex. Denoted as e(V). The distance from a particular vertex to all other vertices in the graph is taken and among those distances, the eccentricity is the highest of distances.

Radius of a Connected Graph G is the minimum eccentricity from all the vertices is considered as the radius of the Graph G. The minimum among all the maximum distances between a vertex to all other vertices is considered as the radius of the Graph G. denoted as r(G). From all the eccentricities of the vertices in a graph, the radius of the connected graph is the minimum of all those eccentricities.

Let G be a graph and v be a vertex of G. The eccentricity of the vertex v is the maximum distance from v to any vertex. That is, e(v)=max{d(v,w):w in V(G)}. The radius of G is the minimum eccentricity among the vertices of G. Therefore, radius(G)=min{e(v):v in V(G)}.

Experimental Problem





Find the distance-2 domination number of G

A set D of vertices in a graph G = (V, E) is a distance - 2 dominating set if every vertex in V-D is within distance 2 of at least one vertex in D.

Find the distance between vertex v_5 and all other vertices in G

$d_G(v_1, v_5)=2$
$d_G(v_2, v_5)=2$
$d_G(v_3, v_5)=1$
$d_G(v_4, v_5)=1$
$d_G(v_5, v_5)=0$
$d_G(v_6, v_5)=1$
$d_G(v_7, v_5)=1$
$d_G(v_8, v_5)=2$
$d_G(v_9, v_5)=2$

Every vertex in G is within distance 2 from the vertex v_5 in D so D is a distance-2 dominating set.

Distance-2 dominating set of G is {5}
Distance-2 domination number of G is 1.

Find the radius of G

Find the distance between u ,v such that u,v are in V

d(1,1)=0	d(2,1)=1	d(3,1)=1
d(1,2)=1	d(2,2)=0	d(3,2)=1
d(1,3)=1	d(2,3)=1	d(3,3)=0
d(1,4)=2	d(2,4)=2	d(3,4)=1
d(1,5)=2	d(2,5)=2	d(3,5)=1
d(1,6)=3	d(2,6)=3	d(3,6)=2
d(1,7)=3	d(2,7)=3	d(3,7)=2
d(1,8)=4	d(2,8)=4	d(3,8)=3
d(1,9)=4	d(2,9)=4	d(3,9)=3
d(4,1)=2	d(5,1)=2	d(6,1)=3
d(4,2)=2	d(5,2)=2	d(6,2)=3
d(4,3)=1	d(5,3)=1	d(6,3)=2
d(4,4)=0	d(5,4)=1	d(6,4)=1
d(4,5)=1	d(5,5)=0	d(6,5)=1
d(4,6)=1	d(5,6)=1	d(6,6)=0
d(4,7)=1	d(5,7)=1	d(6,7)=1
d(4,8)=2	d(5,8)=2	d(6,8)=1
d(4,9)=2	d(5,9)=2	d(6,9)=2
d(7,1)=3	d(8,1)=4	d(9,1)=4
d(7,2)=3	d(8,2)=4	d(9,2)=4
d(7,3)=2	d(8,3)=3	d(9,3)=3
d(7,4)=1	d(8,4)=2	d(9,4)=2
d(7,5)=1	d(8,5)=2	d(9,5)=2
d(7,6)=1	d(8,6)=1	d(9,6)=2
d(7,7)=0	d(8,7)=1	d(9,7)=1
d(7,8)=1	d(8,8)=0	d(9,8)=1
d(7,9)=1	d(8,9)=1	d(9,9)=0

Find the eccentricity, $e(v)$ of G

$e(v) = \max \{d(v, u) \mid u \text{ in } V(G)\}$.

$e(1) = \max \{d(1, u) \mid u \text{ in } V(G)\} = \max\{0, 1, 1, 2, 2, 3, 3, 4, 4\}=4$

$e(2) = \max \{d(2, u) \mid u \text{ in } V(G)\} = \max\{1, 0, 1, 2, 2, 3, 3, 4, 4\}=4$

$e(3) = \max \{d(3, u) \mid u \text{ in } V(G)\} = \max\{1, 1, 0, 1, 1, 2, 2, 3, 3\}=3$

$e(4) = \max \{d(4, u) \mid u \text{ in } V(G)\} = \max\{2, 2, 1, 0, 1, 1, 1, 2, 2\}=2$

$e(5) = \max \{d(5, u) \mid u \text{ in } V(G)\} = \max\{2, 2, 1, 1, 0, 1, 1, 2, 2\}=2$

$e(6) = \max \{d(6, u) \mid u \text{ in } V(G)\} = \max\{3, 3, 2, 1, 1, 0, 1, 1, 2\}=3$

$e(7) = \max \{d(7, u) \mid u \text{ in } V(G)\} = \max\{3, 3, 2, 1, 1, 1, 0, 1, 1\}=3$

$e(8) = \max \{d(8, u) \mid u \text{ in } V(G)\} = \max\{4, 4, 3, 2, 2, 1, 1, 0, 1\}=4$

$e(9) = \max \{d(9, u) \mid u \text{ in } V(G)\} = \max\{4, 4, 3, 2, 2, 2, 1, 1, 0\}=4$

Radius rad G of a connected graph G is defined as rad G = min $\{e(v) \mid v \text{ in } V(G)\}$

rad G = min $\{e(1), e(2), e(3), e(4), e(5), e(6), e(7), e(8), e(9)\}$

= min {4, 4, 3, 2, 2, 3, 3, 4, 4}

= 2

Distance-2 domination number and Radius of G are 1 and 2 respectively

Distance-2 domination number of G is less than that of Radius of G 1<2

We proved that Distance-2 domination number of a graph G is less than Radius of G.

Conclusion:

We proved that Roman domination number of a graph G is greater than Radius of G.

We proved that Total domination number of a graph G is greater than Radius of G.

We proved that Distance-2 domination number of a graph G is less than Radius of G.

By this we concluded that Roman Domination Number and Total Domination Number of a graph G are greater than Radius of G, Distance-2 Domination Number of a graph G is less than Radius of G.

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