On (gg)*-closed sets & generalized ω-closed sets In Topological Spaces

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Abstract: In this paper we introduce generalization of generalized star closed sets ((gg*)-closed *sets*)& generalized ω-closed sets in topological spaces.

Keywords: Closed sets, Generalized closed set, (gg*)-closed sets, gg-open, g ω -closed sets.

1. Introduction

Closed sets are basic objects in a topological space. In 1970, N.Levine [3] initiated the study of g-closed sets . By Definition, a subset *S* of a topological space X is called generalized closed if $clA \subseteq U$ whenever $A \subseteq U$ and *U* is open. Generalized closed sets also proffer new properties of topological spaces and mainly are separation axioms weaker than T_1.In [1],Aull and Thron introduce several separation axioms between T_0 and T_1.

Furthermore,the study of generalized closed sets also provides new characterization of some known classes of spaces,for example the class of extremely disconnected spaces.Other new properties are Definitionined by variations of the property of submaximality.

In Section 2 , we follow a similar line to introduce generalized ω- closed sets by utilizing the ω-closure operator. We study *g*-closed sets and $g\omega$-closed setsin the spaces (X,τ) and (X,τ_ω). In particular, we show that a subset *A* of a space $(X,\tau)$is closed in (X,τ_ω) if and only if it is *g*-closed in (X,τ_ω) if and only if it is $g\omega$-closed in(X,τ_ω).

2. Preliminaries

Throughout this paper (X ,τ) denotes the topological space with no separation properties assumed.For a subset *A* of X , the closure of *A* and interior of *A* are denoted by *cl* (A) and *int* (A) respectively.

A subset *A* of a topological space X is called *α-open* [resp. semi-open, preopen,semi-preopen] if $A \subseteq int(cl(int\ A))$ [resp. $A \subseteq cl(int\ A)$, $A \subseteq int(cl\ A)$, $A \subseteq cl(int(cl\ A))$]. Moreover, *A* is said to be *α-closed* [resp. semi-closed, preclosed,semi-preclosed] if X / A is *α*-open [resp. semi-open, preopen, semi-preopen] or,equivalently, if $cl(int(cl\ A)) \subseteq A$ [resp. $int(cl\ A) \subseteq A$, $cl(int\ A) \subseteq A$, $int(cl(int\ A)) \subseteq A$].

Let (X, τ) be a topological space and let *A* be a subset of *X*. The closure of *A*, the interior of *A*, and the relative topology on *A* will be denoted by $cl_\tau(A)$ (*A*), int_τ (*A*), and τ_A, respectively. The ω-interior (ω-closure) of a subset *A* of a space (X, τ) is the interior (closure) of *A* in the space (X,τ_ω), and is denoted by $\text{int}_{\tau\omega}$ (*A*) (cl_{τ_ω} (*A*)).

Definition 2.1. A space (X,τ) is called

(a) locally countable [4] if each point $x \in X$ has a countable open neighborhood;
(b) anti-locally countable [2] if each nonempty open set is uncountable;
(c) $T_{1/2}$-space [10] if every *g*-closed set is closed (equivalently if every singleton is openor closed, see [30]).

Definition 2.2. A function $f : (X,\tau) \rightarrow (Y,\sigma)$ is called

(a) g-continuous [5] if $f^{-1}(V)$ is g-closed in (X,τ) for every closed set V of (Y,σ);
(b) g-irresolute [5] if $f^{-1}(V)$ is g-closed in (X,τ) for every g-closed set V of (Y,σ);
(c) ω-continuous [11] if $f^{-1}(V)$ is ω-open in (X,τ) for every open set V of (Y,σ);
(d) ω-irresolute [12] if $f^{-1}(V)$ is ω-open in (X,τ) for every ω-open set V of (Y,σ);

(e) α-continuous [31] if $f^{-1}(V)$ is α-set in (X, τ) for every open set V of (Y, σ).

Lemma 2.3 .[4] *Let A be a subset of a space* (X,τ). *Then,*

(a) $(\tau_\omega)_\omega = \tau_\omega$;
(b) $(\tau_A)_\omega = (\tau_\omega)_A$.

Definition 2.4.

(1) generalized closed set (g-closed) [3] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X .

(2) Semi generalized closed [6] if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is Semi open in X .

(3) generalized semi closed [8] if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X .

(4) generalized α -closed (gα -closed)[7] if $\alpha - cl(A) \subseteq U$ whenever $A \subseteq U$ and U is α -open in X .

(5) α generalized closed (α g-closed) [9] if $\alpha - cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X .

(6) generalized semi pre closed (gsp-closed)[13] if $spcl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X .

(7) generalized pre closed (gp-closed)[14] if $pcl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X .

(8) regular semi-open[15] if there is a regular open set U such that $U \subseteq A \subseteq cl(U)$.

(9) regular open set[16] if $A = \text{int}(cl(A))$.

(10) regular closed set if $A = cl(\text{int}(A))$.

(11) t-set [17] iff $\text{int}(A) = \text{int}(cl(A))$.

(12) regular generalized closed set (rg-closed)[18]if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular open in X .

(13)generalized pre-regular closed (gpr-closed)[19] if $pcl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular open in X .

(14) generalized semi-pre regular closed (gspr-closed)[20]if $spcl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular open in X .

(15) generalized star pre closed (g*p-closed)[21]if $pcl(A) \subseteq U$ whenever $A \subseteq U$ and U is g-open open in X .

(16) regular generalized α -closed($rg\alpha$ -closed)[22]if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular α -open in X .

(17)generalized α -closed($g\alpha$ -closed)[23]if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is α - open in X .

(18) generalization of generalized closed set (gg-closed)[24]if $gcl(A) \subseteq U$ whenever and U is regular semi open in X .

(19) A topological space X is said to be locally indiscrete if every open subset is closed.

(20) R*-closed set [25] if $rcl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular semi open in X .

Definition 2.5. A space X is said to be submaximal if every dense subset of X is open.A Space X is α -sub maximality (resp. *g*-submaximal, *sg*-submaximal) if every dense subset is α -open (resp g-open,sg-open)[26].Obviously every submaximal space is g-submaximal[27],that if ($X, \alpha(X)$) is g-submaximal,then ($X, \alpha(X)$) is also sg-submaximal .

Remark 2.6. Every semi-preclosed set is sg-closed and every preclosed set is gα -closed [28].

Definition 2.7. Let S be a subset of a space X .A resolution of S is a pair $<E_1, E_2>$ of disjoint dense subsets of S. The subset S is said to be resolvable if it possesses a resolution, otherwise S is said to be irresolvable.

Definition 2.8. Let S ne a subset of a space X ,then S is called strongly irresolvable ,if every open subspace of S is irresolvable.

Remark 2.9. If $<E_1, E_2>$ is a resolution of S then E_1 and E_2 are condense in X .i.e. have empty interior.

Lemma 2.10 Every space X has a unique decomposition $X = F \cup G$ where F is closed and resolvable and G is open and hereditarily irresolvable.This decomposition is called Hewitt decomposition of X .[29][26]

Theorem 2.11. For a space X with Hewitt decomposition $X = F \cup G$ .Then the following are equivalent.[32]

(1) every semi-preclosed subset of is X is sg-closed set.

(2) $X_1 \cap sclA \subseteq spclA$ for each $A \subseteq X_1$

(3). $X_1 \subseteq \mathrm{int}(clG)$

(4) $X \approx Y \oplus Z$,where is locally indiscrete and Z is strongly irresolvable.

(5) every preclosed subset of X is gα -closed

(6) X is g-submaximal with respect to $\alpha(X)$.

3.(gg)*-closed sets

Definition 3.1. A subset A of a topological space (X, τ) is called generalization of generalized star closed sets (gg)*-closed if $rcl(A) \subseteq U$ whenever $A \subseteq U$ and U is gg-open.

Proposition 3.2. Every regular closed set is (gg)*-closed.

Proof: Let A be a regular closed set in X such that $A \subseteq U$ and U is gg-open.

Then $rcl(A) \subseteq U$ .Therefore A is (gg)*-closed.

Proposition 3.3. Every (gg)*-closed set is g-closed.

Proof: Let A be a (gg)*-closed set in X .Let U be an open set in X such that $A \subseteq U$ Since every open set is gg-open[24] and since A is (gg)*-closed , $rcl(A) \subseteq U$. But we have $cl(A) \subseteq rcl(A) \subseteq U$ Hence A is g-closed

Proposition 3.4. Every (gg)*closed set is gsp-closed

Proof: Let A be a (gg)*-closed set in X .Let U be a an open set in X such that $A \subseteq U$ Since every open set is gg-open[24] and since A is (gg)*-closed , $rcl(A) \subseteq U$. But we have $spcl(A) \subseteq rcl(A) \subseteq U$ Hence A is gsp-closed.

Proposition 3.5. Every (gg)*closed set is gp-closed.

Proof: Let A be a (gg)*-closed set in X .Let U be an open set in X such that $A \subseteq U$ Since every open set is gg-open[24] and since A is (gg)*-closed , $rcl(A) \subseteq U$. But we have $pcl(A) \subseteq rcl(A) \subseteq U$ Hence A is gp-closed.

Proposition 3.6. Every (gg)*closed set is gs-closed.

Proof : Let A be a (gg)*-closed set in X .Let U be an open set in X such that $A \subseteq U$ Since every open set is gg-open[24] and since A is (gg)*-closed $rcl(A) \subseteq U$. But we have $scl(A) \subseteq rcl(A) \subseteq U$ .Hence A is gs-closed.

Proposition 3.7. Every (gg)*closed set is αg -closed.

Proof: Let A be a (gg)*-closed set in X .Let U be an open set in X such that $A \subseteq U$ Since every open set is gg-open[24] and since A is (gg)*-closed $rcl(A) \subseteq U$. But we have $\alpha cl(A) \subseteq rcl(A) \subseteq U$ .Hence A is αg -closed.

Proposition 3.8. Every (gg)*closed set is rg-closed.

Proof: Let A be a (gg)*-closed set in X .Let U be an open set in X such that $A \subseteq U$ Since every regular open set is gg-open[24] and since A is (gg)*-closed , $rcl(A) \subseteq U$. But we have $cl(A) \subseteq rcl(A) \subseteq U$ .Hence A is rg-closed.

Proposition 3.9. Every (gg)*closed set is gpr-closed.

Proof: Let A be a (gg)*-closed set in X .Let U be an open set in X such that $A \subseteq U$ Since every regular open set is gg-open[24] and A is (gg)*-closed , $rcl(A) \subseteq U$. But we have $pcl(A) \subseteq rcl(A) \subseteq U$.
Hence A is gpr-closed.

Proposition 3.10. Every (gg)*closed set is gspr-closed.

Proof: Let A be a (gg)*-closed set in X .Let U be a regular open set in X such that $A \subseteq U$.
Since every regular open set is gg-open[24] and since A is (gg)*-closed , $rcl(A) \subseteq U$. But we have $spcl(A) \subseteq rcl(A) \subseteq U$ Hence A is gspr-closed.

Proposition 3.11. Every (gg)*closed set is g*p-closed.

Proof: Let A be a (gg)*-closed set in X .Let U be a regular open set in X such that $A \subseteq U$.
Since every g- open set is gg-open[24] and since A is (gg)*-closed , $rcl(A) \subseteq U$ But we have $pcl(A) \subseteq rcl(A) \subseteq U$
Hence A is g*p-closed.

Proposition 3.12. Every (gg)*closed set is g**-closed.

Proof: Let A be a (gg)*-closed set in X .Let U be a g*- open set in X such that $A \subseteq U$ Since every g*- open set is gg-open[24] and since A is (gg)*-closed , $rcl(A) \subseteq U$. But we have $cl(A) \subseteq rcl(A) \subseteq U$.
Hence A is g**-closed.

4.Generalized ω-closed sets

Definition 4.1. A subset A of a space (X , τ) is called generalized ω-closed (briefly, $g\omega$-closed) if $cl_{\tau_\omega}(A) \subseteq U$ whenever $U \in \tau$ and $A \subseteq U$

We denote the family of all generalized ω-closed (generalized closed) subsets of a space (X , τ) by $G\omega C(X,\tau)(GC(X,\tau))$.

It is clear that if (X, τ) is a countable space, then $G\omega C(X,\tau) = P(X)$, where P(X) is the power set of X .

Proposition 4.2. Every g-closed set is gω-closed.

The proof follows immediately from the definitions and the fact that τ_ω is finer than τ for any space (X , τ). However, the converse is not true in general as the following example shows.

Example 4.3. Let $X = \{a,b,c\}$ with the topology $\tau = \{\phi, X, \{a\}, \{a,b\}\}$ and let $A = \{a\}$ .Then $A \in G\omega C(X,\tau)$. But $A \notin GC(X,\tau)$ since $A \subseteq A \in \tau$ and $cl_\tau(A) = X \not\subset A$.

Lemma 4.4. Let (A, τ_A) be an anti-locally countable subspace of a space (X , τ). Then $cl_\tau(A) = cl_{\tau\omega}(A)$.

Proof. We need to prove that $cl_\tau(A) \subseteq cl_{\tau_\omega}(A)$. Suppose that there exists $x \in cl_\tau(A) - cl_{\tau_\omega}(A)$. Then $x \notin cl_{\tau\omega}(A)$, and so there exists $W_x \in \tau_\omega$ such that $x \in W_x$ and $W_x \cap A = \Phi$ A is a nonempty countable open set in (A, τ_A)), which is a contradiction andthe result follows.

Corollary 4.5. Let (A, τ_A) be an anti-locally countable subspace of a space (X, τ). Then $A \in GC(X,\tau)$ if and only if $A \in G\omega C(X,\tau)$.

Theorem 4.6. *Let* (X , τ) be any space and $A \subseteq X$. Then the following are equivalent.

1) A is ω-closed in (X , τ) (equivalently A is closed in(, τ_ω)).

2) $A \in GC(X, \tau_\omega)$

3) $A \in G\omega C(X, \tau_\omega)$

1)Proof. (a)⇒(b). It follows from the fact that every closed set is g-closed.

(b)⇒(c). It is obvious by using Proposition 4.2.
(c)⇒(a). We show that $cl_{\tau_\omega}(A) \subseteq A$. Suppose that $x_0 \notin A$ Then $U = X - \{x_0\}$ is an ω - open set containing A. Since $A \in G\omega C(X, \tau_\omega)$, $cl_{(\tau_\omega)\omega}(A) = cl_{\tau_\omega}(A) \subseteq U$,and thus $x_0 \notin cl_{\tau_\omega}(A)$. Therefore, $cl_{\tau_\omega}(A) \subseteq A$, that is, A is closed in (X , τ)

In the same way, it can be shown that a subset A of a space (X , τ) is closed if and only if $cl_\tau(A) \subseteq U$ whenever $U \in \tau_\omega$ and $A \subseteq U$.
Proposition 4.7. If $A \in GC(X, \tau_\omega)$, then $A \in G\omega C(X, \tau)$ but not conversely.

Example 4.8. Let $X =$ **R** be the set of all real numbers with the topology $\tau = \{\phi, X, \{1\}\}$ and put $A =$ **R-Q**. Then A is an ω - open subset of (X, τ) such that $cl_{\tau_\omega}(A) =$ R-{1} $\not\subset A$ (i.e., $A \notin GC(X, \tau_\omega)$. However, $A \in G\omega C(X, \tau)$ since the only open set in (X, τ) containing A is X .

In Example 4.8, for a space (X , τ) the collections $GC(X, \tau)$ and $GC(X, \tau_\omega)$ are independent from each other.

Example 4.9. Conside $X =$ R with the usual topology τ_u. Put $A = (0,1) \cap Q$ Then $cl_{(\tau_u)\omega}(A) = A$ (A is countable) ,and so $A \in$ GC(R, $(\tau_u)_\omega$). On the other hand, $A \notin GC$ (R, τ_u)since $U = (0,1)$ is open in (R, τ_u) such that $A \subseteq U$ and $cl_{\tau_u}(A) = [0,1] \not\subset U$.

In Example 4.9, (R, τ_u) is anti-locally countable and $A = (0,1) \cap Q \in G\omega C$ (R, τ_u) $- GC$ (R, τ_u). Thus the condition that (A, τ_A) is anti-locally countable inCorollary 4.5 cannot be replaced by the condition that (X, τ) is anti-locally countable.

Theorem 4.10. Let (X, τ) be an anti-locally countable space. Then (X, τ) is a T_1-space ifand only if every $g\omega$-closed set is ω-closed.

Proof. We need to show the sufficiency part only. Let $x \in X$ and suppose that $\{x\}$ is not closed. Then $A = X - \{x\}$ is not open, and thus A is $g\omega$-closed (the only open set containing A is X). Therefore, by assumption, A is ω-closed, and thus $\{x\}$ is ω-open. So there exists $U \in \tau$ such that $x \in U$ and $U - \{x\}$ is countable. It follows that U is a nonempty countable open subset of (X, τ), a contradiction.

Proposition 4.11. *If* $A = \{A_\alpha : \alpha \in I\}$ is a locally finite collection of gω-closed sets of a space (X, τ), then $A = \bigcup_{\alpha \in I} A_\alpha$ is gω-closed (in particular, a finite union of gω-closed sets is gω-closed).
Proof. Let U be an open subset of (X, τ) such that A ⊆ U. Since $A_\alpha \in$ GωC(X, τ) and $A_\alpha \subseteq$ U for each α ∈ I, cl_{τ_ω} (A_α) ⊆ U. As τ_ω is a topology on X finer than τ, A is locally finite in (X, τ_ω). Therefore, $cl_{\tau_\omega}(A) = cl_{\tau_\omega}(\bigcup_{\alpha \in I} A_\alpha) = \bigcup_{\alpha \in I} cl_{\tau_\omega}(A_\alpha) \subseteq$ U. Thus, A is gω- closed in (X, τ).

Proposition 4.12. If A ∈ GωC(X, τ) and B is closed in (X, τ), then A ∩ B ∈ GωC(X, τ).

Proof: Let U be an open set in (X, τ) such that A ∩ B ⊆ U. Put W = X − B. Then A ⊆ U ∪ W ∈ τ. Since A ∈ GωC(X, τ), cl_{τ_ω} (A) ⊆ U ∪ W. Now, cl_{τ_ω} (A ∩ B) ⊆ cl_{τ_ω} (A) ∩ cl_{τ_ω} (B) ⊆ cl_{τ_ω} (A) ∩ cl_τ (B) = cl_{τ_ω} (A) ∩ B ⊆ (U ∪ W) ∩ B ⊆ U.

Lemma 4.13. (a) If A is an ω-open subset of a space (X, τ), then A − C is ω - open for every countable subset C of X.

(b) The open image of an ω-open set is ω-open.

Proof. Part (a) is clear. To prove part (b), let f : (X, τ) → (Y , σ) be an open function and let W be an ω-open subset of (X,τ). Let $y \in f(W)$. There exists $x \in W$ such that $y = f(x)$. Choose $U \in \tau$ such that $x \in U$ and $U - W = C$ is countable. Since f is open, $f(U)$ is open in (Y, σ) such that $y = f(x) \in f(U)$ and $f(U) - f(W) \subseteq f(U - W) = f(C)$ is countable. Therefore, $f(W)$ is ω -open in (Y, σ).

Theorem 4.14. Let (X, τ) and (Y , σ) be two topological spaces. Then $(\tau \times \sigma)_\omega \subseteq \tau_\omega \times \sigma_\omega$. Proof: Let $W \in (\tau \times \sigma)_\omega$ and (x, y) ∈ W. There exist U ∈ τ and V ∈ σ such that (x, y) ∈U× V and U × V − W = C is countable. Put $W_1 = (U \cap p_X(W)) - (p_X(C) - \{x\})$ and $W_2 = (V \cap p_Y(W)) - (p_Y(C) - \{y\})$, where $p_X : (X \times Y, \tau \times \sigma) \to (X, \tau)$ and $p_Y : (X \times Y, \tau \times \sigma) \to (Y, \sigma)$ are the natural projections. Then $W_1 \in \tau_\omega$, $W_2 \in \sigma_\omega$ (Lemma 4.13) and $(x,y) \in W_1 \times W_2 \subseteq W$.Thus $W \in \tau_\omega \times \sigma_\omega$.

Definition 4.15. A subset A of a space (X, τ) is called generalized ω-open (briefly, gω- open) if its complement X− A is gω-closed in (X, τ).

It is clear that a subset A of a space (X, τ) is gω-open if and only if F int_{τ_ω} (A),whenever F ⊆ A and F is closed in (X , τ).

Theorem 4.16. If A × B is a gω-open subset of ($X \times Y$, τ × σ), then A is gω-open in (X , τ) and B is gω-open in (Y , σ).

Proof. Let F_A be a closed subset of (X, τ) and let F_B be a closed subset of (Y , σ) such that $F_A \subseteq A$ and $F_B \subseteq B$. Then $F_A \times F_B$ is closed in ($X \times Y$, τ × σ) such that $F_A \times F_B \subseteq A \times B$.By assumption, A × B is gω-open in ($X \times Y$, τ × σ), and so $F_A \times F_B \subseteq \text{int}_{(\tau\times\sigma)_\omega}(A \times B) \subseteq \text{int}_{\tau_\omega}(A) \times \text{int}_{\sigma_\omega}(B)$ by using Theorem 4.14. Therefore, $F_A \subseteq \text{int}_{\tau_\omega}(A)$ and $F_B \subseteq \text{int}_{\sigma_\omega}$ (A),and the result follows.

The converse of the above theorem need not be true in general.

Example 4.17. Let X = Y = R with the usual topology τ_u. Let A = R − Q and B = (0, 3). Then A and B are ω-open subsets of (R, τ_u), while A × B is not gω-open in (R × R, $\tau_u \times \tau_u$) , since $\text{int}_{(\tau_u \times \tau_u)_\omega} A \times B = \phi$ and { $\sqrt{2}$ }× [1, 2] is a closed set in (r × r, $\tau_u \times \tau_u$) contained in A × B.

Theorem 4.18. Let (Y , τ_Y) be a subspace of a space (X , τ) and A ⊆ Y. Then the following hold.

(a) If A ∈ GωC(X , τ), then A ∈ GωC(Y , τ_Y).

(b) If A ∈ GωC(Y, τ_Y) and Y is ω-closed in ($X \times Y$, τ), then A ∈ GωC(X, τ).

Proof. (a) Let V be an open set of (Y , τ_Y) such that A ⊆ V . Then there exists an open set U ⊆ τ such that V = Y ∩ U. Since A GωC(X, τ) and A U, $cl_{\tau_\omega}(A) \subseteq U$. Now, $cl_{(\tau_Y)_\omega}(A) = cl_{(\tau_\omega)Y}(A) = cl_{\tau_\omega}(A) \cap Y \subseteq Y \cap U = V$. Therefore, A ∈ GωC(Y , τ_Y).

(b) Let A ⊆ U, where U ∈ τ. Then A ⊆ Y ∈ U ∈ τ_Y . Since A ∈ GωC(Y , τ_Y), $cl_{(\tau_Y)_\omega}(A) = cl_{(\tau_\omega)_Y}(A) = cl_{(\tau_\omega)}(A) \cap Y \subseteq Y \cap U$. Finally, $cl_{\tau_\omega}(A) = cl_{\tau_\omega}(A \cap Y) \subseteq cl_{\tau_\omega}(A) \cap cl_{\tau_\omega}(Y)$ = (Y is ω-closed) $cl_{\tau_\omega}(A) \cap Y \subseteq Y \cap U \subseteq U$. Thus A ∈ GωC(X, τ).

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