Particle Swarm Optimization from Animation to Recent Trends: A Systematic Review

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Abstract: Particle Swarm Optimization (PSO) is a natural phenomenon computing technique which came into existence in mid-1990 and belongs to swarm intelligence (SI) family. Many researchers tried to modify the core algorithm while other tried hybridization by combining best features of other algorithm. In this comprehensive investigation paper, a systematic and chronological effort has been made for literature review from 1983 to 2019 which fits the need of researcher from starting with all the developments like historical development, addition of new parameters, tuning or refinement of parameters and its variants for different optimization problems with constraints, multi-objectives. In addition this paper also covers the literature survey of parallel PSO, its hybrids, communication topology and for multi-objective problems strategy used for parallel computing is covered in detail. This review will give a correct direction for future study and open problems to many researchers.

Index Terms—Particle Swarm Optimization, hybridization, refinement, multi-objective, Stagnation, GPU, CUDA, Cloud computing.

I. INTRODUCTION

Nature computing paradigms are the correct way to solve real world problems due to dynamic in nature, noisy or multi dimension problems. Many species solve their complex tasks in nature by (PSO) emerged from biological research and simulation on swarming animals. Reeves [1] in 1983 firstly attempted in computer animation to show natural phenomena by generating moving particles to predefined locations with initial velocity and having characteristics like texture, colour, lifetime and angular movements. It was widely used for special effects and natural looking. The first computer simulation and movies related work was given by Craig Reynolds [2] about simulating bird swarms in 1986. He also added orientation and communication in them. The result was some simulated swarm of who's the individual he called as Boids [3] directed by three simple rules. Heppner F and Grenander [4] in 1990 reviewed Reynolds work for doing more detailed bird flock animation and studies. They introduced the concept of need of roosting or swarming, which results in too realistic nature like. Hoffmeye J[5] a biologist in 1994 studied the semiotics and defined the swarm in concept of algorithm. This algorithm being in Ontogeny group are co-operative in nature. The first review paper was given by Parsopoulos and Vrahatis [6] in 2002 with the analysis on the effectiveness on different versions of PSO in finding global optima, strategies for local optima prevention, identification of multiple minimizers have been mentioned along with various approaches has been presented and the ability of PSO in noisy and dynamic environments has been examined.Hu et al. [7] in 2004 reviewed the conventional PSO having various methods to evaluate the parameters of velocity update law, second section comprises of discrete PSO having different versions for overcoming combination optimization issues, third section deals with multi-objective PSO versions were explained and final section deals with applications. A further analysis research was conducted by Song and Gu [8] in 2004 in two sections on improvement in performance and implementations. In first section a variety of papers focussed on modification on parameters, convergence rate and diversity whereas other section emphasis was on PSO applications for multi objective technique, neural network training and electronics were addressed. A review was also presented by Poli et al. [9] in 2007 on study on particle dynamics, its topologies, dynamic environment and analysis on theoretical aspects on deterministic and stochastic techniques. Another review was published in 2007 by Banks et al. [10] discusses the growth of PSO algorithm and improvements to avoid stagnation of swarm and addresses dynamic situations. Again a second part review by Banks et al.[11] in 2008 acknowledge on latest research on hybridization of PSO, combinational issues on PSO, multi criteria and restricted optimization PSO along with its applications was presented. Poli [12] in 2008 introduced a review paper covering on applications of PSO in various fields and found that majority of paper were on image and video,distribution networks, control engineering, power system, scheduling, electronics, signal processing, communication networks, design, biomedical etc. in the decreasing order of number of publications. Sedighizadeh and Masehian [13] in 2009 presented a brief history of PSO, classification of PSO, its variants and their applications. Thangaraj et al. [14] in 2010 presented the review of hybrid of PSO and GA, hybrid of PSO and differential evolution, hybrid of PSO with other techniques and tested few on benchmark functions. Zhang et al.[15] in 2015conducted a survey on PSO with modifications, topology, and hybridization of PSO with various algorithms, analysis of PSO with various parameters, parallel implementation and applications in eight fields and pointed out less analysis on theoretical view. The provisions of this paper is organised in seven sections including that of introduction with recent trends. Section 1 deals with historical background of PSO, motivation for developing a PSO algorithm along with literature reviews. Section 2 provides the brief overview, development and refinement of PSO. Section 3 gives a brief description on hybrid PSO techniques. Section 4 deals with constrained optimization problems of PSO. Section 5 gives various techniques to solve multi-objective optimization problems. Section 6 deals with parallel implementation of PSO and is further divided into sub-categories of multi-core, GPU computing, cloud computing, its hybrids and multi-objective problems using PSO. Finally section 7 provides the conclusion with discussion and future direction.
II. PARTICLE SWARM OPTIMIZATION AND ITS REFINEMENT

III. Kennedy and Eberhart [16] in 1995 took the interesting task and start working on the leftover work of Reynolds to show the social behaviour in swarms and make a realistic goal oriented. It uses simply mimic swarm behaviour in birds flocking, fish schooling or swarms of bees in solving optimization problems. Another version "Lbest" of PSO developed by Eberhart and Kennedy [17] considering only the information between close neighbourhood of two, six and examined its effects on the convergence. Eberhart et al. [18] in 1996 proposed a velocity clamping strategy to control the velocity of the particles. Kennedy [19] in 1997 performed analysis of PSO algorithm for social interaction with new four types of models (full model, cognition only, social only and selfless model) and suggested cognition only and social only performed well but did not want to replace the core algorithm as it will result in premature convergence. Kennedy and Eberhart [20] suggested in 1997 a first discrete binary PSO, where trajectories vary in the terms of variation in probability of each bit by zero or one value. In starting PSO was without inertia weight but in 1998, Shi and Eberhart [21] introduced momentum in the velocity equation by multiplying constant inertia weight for better exploration and exploitation and this algorithm was called standard PSO. PSO is initialized by initial solutions of the particles moving in the search space, each particle is represented by a position and velocity and keeps updating as follows

\[ x_j(k+1) = x_j(k) + v_j(k+1) \]

\[ v_j(k+1) = \omega v_j(k) + c_1 r_1(p_j(k) - x_j(k)) + c_2 r_2(g(k) - x_j(k)) \]

Where, \( j = 1, 2, 3 \ldots i \)

\( k+1 \) denotes next iteration, 
\( k \) is the current iteration number, 
\( v_j \) denotes velocity and position of the j th particle, 
\( \omega \) is Inertia weight factor, 
\( c_1, c_2 \) are acceleration factors, 
\( p_j \) is personal best of particle j, 
g is the global best of the swarm, 
\( r_1, r_2 \) are random numbers between 0 and 1.

Ozcan and Mohan [22,23] in 1998, 1999 observed that the particles surf on sine wave instead of flying and made many simplifications and removed the stochastic element of the algorithm. The optimal strategy of using inertia weight w by Eberhart and Shi [24] in 1999 from 0.9 to 0.4 in a linearly decreasing way improved exploration and optimal global minimum. Maurice Clerc [25] in 1999, modified the PSO by derived a constraint coefficient which operates without Vmax. Kennedy[26]in1999 worked on four topologies and concluded that topology effects the swarm’s performance but also dependent on objective function. Suganthan [27] in 1999 introduced dynamic topology to start searching with local best topology and gradually increasing the neighborhood size till by the end of iterations entire swarm is completely linked and showed good results with this method. Eberhart and Shi [28] in 2000 suggested two approaches using constriction factor \( \chi \) and inertia weight \( \omega \) are used and found to be mathematically equivalent and the initial value of \( \omega \) to be set at 0.9 and reducing it linearly to 0.4 for better exploration and exploitation. Carlisle and Dozier [29] in 2000, proposed this technique to forget the former experience by periodically resetting particles memory and replacing their best fitness value and position with the current position and fitness value. Kennedy [30] in 2000, performed cluster analysis of best performer particles but with high computational cost and real time. Van Den Berg and Engelbrecht [31] in 2001 proposed to divide the solution vector into sub vectors and a separate PSO is used to optimize individual smaller vectors. The optimal swarm size depends on the problem when the search region is more complex. Carlisle and Dozier [32, 34] in 2001, 2002 proposed to divide the particles into 'sentry’ to monitor the environment and inform all other particles to reset memory. It slows down the particles and difficult to track moving optimum. In 2001 Carlisle and Dozier [33] proposed with constricted model with a global neighborhood of 30 particles that will be changed asynchronously but without improving the optimizer from initial configuration. Later in 2002 Clerc and Kennedy [35] suggested constriction factor \( \chi \), which alleviates the requirement of velocity clamping by showing better results with new velocity equation as:

\[ v_j(k+1) = \chi v_j(k) + \phi_1 (p_j(k) - x_j(k)) + \phi_2 (g(k) - x_j(k)) \]

\[ X = \frac{\phi^2}{2 - \phi \sqrt{\phi^2 - 4\phi}} \]  
\[ \phi = \phi_1 + \phi_2, \phi > 4 \]

Ratnaweera et al.[36] in 2002, proposed to decrease c1 linearly with time while c2 to be increased linearly. Kennedy and Mendes [37] in 2002 suggested best version of PSO with inertia weight and the other with a factor of constriction and suggested a small neighborhood is suitable for complex problems and large for simple problems. Van Den Berg and Engelbrecht [38] in 2002 proposed Guaranteed Convergence PSO (GCPSO) for avoiding swarm stagnation problem with different velocity update equation, within a radius with random search around global position. Fan [39] in 2002 introduced an adaptive scaling term as an efficient speedup strategy for better control and convergence. Trellea [40] in 2003 analysed the convergence boundaries, convergence point and parameter setting procedure with inertia weight \( \omega = 0.6 \) and \( \phi_1 = \phi_2 = 1.7 \), resulting in good performance of PSO. Zheng et al.[41] in 2003 observed by increasing inertia weights also yields in good results. Ratnaweera et al.[42] in 2004 proposed the value of \( \phi_1 \) is decreased and \( \phi_2 \) is increased for better exploration and exploitation, further the author modified by using adaptation rule to reinitialize the velocity of particles. Chatterjee and Sastry [43] in 2006 introduced non-linear variation of inertia weight with the particle old velocity for improvement in the speed of convergence and fine tuning in search space. Bratton and Kennedy [44] in 2007 defined a standard PSO as the baseline for the researchers as common grounding to work from. Arumugam et al.[45] in 2008 proposed lower acceleration coefficients and greater inertia weights, if the personal best are not comparable to global best of the particles. Chen and Zhao [46] in 2009 suggested an adaptive variable swarm size and periodic partial increasing or declining of
particles in the form of ladder function. Nakagawa et al. [47] in 2009 proposed by adding a random number to the particle velocity depending upon the distance from the global best location for the velocity control. Van Den Berg and Engelbrecht [48] in 2010 proposed a mutation operator used to avoid the stagnation for the local convergence problem in standard PSO. Nickabadi et al. [49] in 2011 proposed adaptive change in inertia weight during the exploration and depends on improved present fitness value with the previous iteration value, only then has a linear relationship. Bansal et al. [50] in 2011 proposed 15 different strategy for inertia weight in PSO and suggested chaotic inertia weight strategy is best suited for accuracy and that random inertia weight is best suited for efficiency. Engelbrecht [51] in 2012 advocated through study is to initialize particles to zero or close to zero without imposing a personal best bound. Schmitt and Wangsa [52] in 2013 suggested to reinitialize the velocity in each dimension whenever there is a stagnation. Cleighorn and Engelbrecht [53] in 2014 suggested from the first order stability analysis that topology does not effects the boundary of convergence but may affect the speed of convergence and divergence. Van Zyl and Engelbrecht [54] in 2015 proposed initialization strategy called seed set which is a set of randomly generated n-dimensional orthogonal unit vectors obtained from modified Gram Schmit method and is suited to high dimension problems. This strategy forces the swarm within the subspace of search space for better exploration. Bonyadi and Michalewicz [55] in 2016 investigated the behavior of particles and suggested during exploration, the particles oscillate in different patterns in four classes based on the maximum oscillations frequency and the boundaries do not depend on the number of dimensions. Liu et al. [56] in 2016 studied the effect of 198 regular topology with 09 different number of particles and devised formulae to help to choose optimal topology parameters. Bonyadi and Michalewicz [57] in 2017 examined the relationship between the base frequency F’ and the correlation between the particle position, particles with smaller’ F’ will exhibit smooth trajectories and larger’ F’ values are prone to more oscillations with large steps between positions. Oldewage [58] in 2018 proposed the initialization strategy which forces the swarm on sub space of search space for exploration rather than entire search space with optimal number of seed set size depending upon dimensionality. Shi et al. [59] in 2018 proposed new strategy called Oscillatory PSO which uses a particle to drive into oscillatory trajectories in complete search space. The cognitive and social factors are made to sum to unit, thus ensuring that particles converge toward the weighted sum between current global best and particle wise best solutions. Inertia weight is selected to ensure a complex roots are obtained from PSO update equation. Oldewage et al. [60] in 2019 investigated different particle swarm movement patterns behavior are highly influenced by inertia weight and acceleration coefficients in higher dimensions space. Parameter configuration with inertia weight ω=0.9694 and c1=c2=0.099381 gives smooth particle trajectories and restrict unwanted roaming behavior due to initial velocity explosion in range and even in higher dimensions. Sun et al. [61] in 2019 proposed the random sampling of control parameters with immediate particle updating strategy is used along with stochastic correction approach on each dimension and also take information from other particles for better convergence and accuracy.

III. HYBRIDS OF PARTICLE SWARM OPTIMIZATION

Hybrid models of Particle swarm optimization (PSO) has been developed whose objective is to combine the good characteristics of various algorithms in order to minimize their individual weakness. Following are the different PSO based hybrid approaches which is used to refine the properties of PSO to achieve global values.

**Discrete PSO**-For binary problems first discrete version [62] was developed by Kennedy and Eberhart in 1997 by changing the velocity to the likelihood that each bit will be in one state or another.

**GA Selection based PSO**-Angeline [63] in 1998, applied GA tournament based selection criterion which replaced velocity and position of worst performing particle with good performers particles velocity and position. This improved the local search capabilities of PSO.

**Fuzzy PSO**-Shi and Eberhart [64] in 2001 suggested this algorithm where PSO is used along with a specific selection technique employing replication, mutation, reproduction, evaluation and selection operation to describe the parameters.

**PSO-GA**-Lovbjerg et al. [65] Presented in 2001 introduced breeding between particles in different sub populations which result in faster convergence and a better optimal solutions.

**Dissipative PSO**-In 2002 Xie et al. [66] suggested this algorithm to overcome the problem of entrapment in local minima, a dissipative system was implemented which uses negative entropy and produces craziness between particles resulting to come out of stagnant stage.

**Multi-phase Discrete PSO**-In 2002 Al-Kazemi and Mohan [67] suggested this hybrid version to fewer objective function with small swarm by using three coefficients, the values of which were set either 1 or 0-1 depending upon the level of optimization and personal best position was replaced with the previous value.

**Evolutionary PSO**-Miranda and Fonseca [68] proposed this algorithm in 2002 having hybrid characteristics of EA and of PSO. In EPSO there is replication, mutation, reproduction, evaluation and selection of particles which produce new solutions in search space.

**Attractive-Repulsive PSO**-This algorithm [69] was suggested by Riget et al. in 2002 which has two phases as attractive and repulsive with two operators as addition and subtraction which are used to update PSO equations to avoid premature convergence.

**Niche PSO**-In 2002 Britis et al. [70] devised a technique in which GCPSEO is run and only those particles are separated out as multiple sub swarms whose fitness do not show any change during running of algorithm. These multiple sub swarms explore and exploit the search space simultaneously with the PSO.

**Spatial Extension PSO**-Krink et al. [71] in 2002 proposed three strategies: Random direction change, Realistic bounce and Random velocity change to avoid collision in particle system. The last two strategies were helpful in multi-modal functions.

**Stretching PSO (SPSO)**–Parsopoulos and Vrahatis [72] in 2002 suggested to use deflection, stretching and repulsion technique in standard PSO. First two technique transform the objective function by using the already found minimum points and repulsion technique avoids the particle to go to already found minimum points. So more global minimum points can be found out.

**NBest PSO**-Bris et al. [73] in 2002 proposed to use standard PSO for finding multiple solutions by incorporating the idea of shrinking neighborhood for solving unconstrained optimization problems.
Barebone PSO (BBPSO) - Kennedy [74] in 2003 proposed a novel variant without velocity update and uses Gaussian sampling based on the gbest and pbest knowledge is used in place of canonical formula.

Gaussian PSO (GPSO) - A new hybrid algorithm was developed by Secrest et al. [75] in 2003 which uses probability distribution of the moving swarm having a Gaussian distance from the global and local best.

Fitness to Distance Ratio PSO - Peram et al. [76] in 2003 suggested a new concept in the algorithm each particle tracks that particle in the neighborhood having better fitness value instead of attracting towards global best particle. Regarding the search direction it utilizes relative fitness and the distance of other particles.

Dynamic Double PSO (DDPSO) - In 2004 Cui [77] et al. proposed a way for guaranteed convergence to global minima using convergence analysis and the position of particles are set dynamically with constraints.

Fully Informed PSO (FIPS) - Mendes et al. [78] developed a strategy in 2004 about a topology that all the particles are equally informed.

Hybrid Gradient Descent PSO - Noel et al. proposed a method [79] in 2004 for using the gradient information for faster convergence to global minima by employing random size and avoiding calculations of local neighborhood.

Quantum Delta PSO - Sun et al. [80] developed an algorithm in 2004 where the concept is taken from quantum physics which obeys uncertainty principle (position and velocity cannot be measured simultaneously) but the motion of particles is quantum in nature and it has only one parameter to control.

Unified PSO (UPSO) - Parsopoulos and Vrahatis [81] in 2004 proposed a technique which only uses the features of Gbest and Lbest and its velocity updating is done in two parts depending on the information.

Co-operative PSO (CPSO) - Van den Berg and Engelbrecht [82] in 2004 proposed to use co-operative behavior to improve original PSO and that several swarms be used to optimize the different components of the solution vector.

Species based PSO (SPSP) - Li [83] in 2004 proposed on the bases in there similarity of species of sub-populations of swarm where each species is grouped around a dominant particle called species seed and are identified as neighborhood best for that group in each iteration. Finally after many successive iteration many local minima are obtained from which global minima can be identified.

Kalman PSO (KPSO) - Monson and Seppi [84] in 2004 suggested to use Kalman filter to update the particle location thereby improving exploration for fast converge to optimal solutions.

Parallel PSO (PPSO) - Chang et al. [85] suggested in 2005 a technique in which fitness was calculated for each particle independently and efficiency of strategy communication was calculated. A hybrid strategy is considered to be good where sub optimal is provided and correlation is unknown.

Angle Modulated PSO (AMPSO) - Again in 2005, Pam para et al. [86] developed an algorithm in which a bit string is generated by using trigonometric functions. It changes high dimension problem to four dimension problem and makes it highly efficient in operation and also saves memory.

Exploring Extended PSO (EEPSO) - In 2005 Poli et al. [87] proposed the regular use of Genetic Programming and can play a specialist position update role with PSO in special domain problems.

Hierarchical PSO - Stefan and Martin [88] in 2005 introduced a new method in which particles are arranged in a hierarchical way depending on the fitness value. Good particles are on the top of hierarchy and have more, influence on the swarm.

New PSO (NPSO) - Yang and Simon [89] in 2005 suggested that each particle adjust its position from self previous worse position and its group previous worst to get best optimal solution. No changes are made in velocity and position equations, only uses the term worst position is used rather best position. This technique tries to get rid away from the worst instead of coming closer to best position.

Interactive PSO - Madar et al. proposed in 2005 that procedure of IPSO [90] is same that of PSO with a difference that best particle selection is done by the user in every iteration. Interactive PSO is different from Interactive EC from information sharing point of view.

Neural PSO - Duo et al. suggested a technique [91] in 2005 to combine feed forward neural network with PSO to acquire good learning in movements by the best particles previously in the search space.

Perturbation PSO - Yan et al. in 2005 developed an algorithm [92] which keeps on changing velocity and position equations keeping the existing equations of PSO for other particles.

Principal component PSO (PCPSO) - Voss et al. in 2005 propose a strategy [93] in which particles are flown simultaneously in two different dimension of search spaces to reduce the time in higher dimensions.

Opposition based PSO (OPSO) - Tizhoosh in 2005 proposed a novel concept [94] by considering counter estimates, opposite numbers, anti-chromosomes, counter actions and opposite's weights in machine learning algorithms has proven to be effective method by making revolutionary jumps in starting as time saving.

Fuzzy Adaptive Turbulence PSO (FATPSO) - Hongbo et al. [95] Proposed in 2005 that premature convergence can be effectively avoided by using minimum threshold velocity to control the velocity parameter which is adaptively tuned in TPSO algorithm.

Adaptive PSO Guided by Acceleration Information - In 2006 Zeng et al. [96] proposed to add acceleration term to the equations of position and velocity for updating, thus making PSO fast and efficient.

Comprehensive Learning PSO - In this technique Liang et al. [97] in 2006 the particle velocity is updated by analyzing the other particle velocity history information and the diversity of swarm is maintained.

PSO with Escape Velocity - A novel technique [98] is proposed by Zang et al. in 2006 equips the particles with the escape velocity to avoid to being stuck in local minima and increase the population diversity that outperforms PSO for high dimension and multi modal problems.

Genetic PSO - Yin [99] in 2006 proposed this novel technique which incorporates crossover and mutation features of GA in PSO.
Genetic Binary PSO-Sandri et al. in 2006 proposed [100] by keeping the swarms dynamic conditions in binary state for each particle and being treated as chromosome while the chain with the dimensional size.

Gregarious PSO-IN 2006 Pasupelti and Battiti [101] suggested a new technique in which particles uses only social knowledge and stochastic velocity vector is used in the search space. Self-setting of parameters is done by the integration to obtain the parameters.

Hybrid Discrete PSO-In 2006 Chandrasekaran et al. proposed [102] that each particle shows the job sequence as an optimal solution in job scheduling problem.

Hybrid Taguchi PSO-Roy and Ghosal in 2006 proposed [103] to select the intelligent particles only in Taguchi selection method with PSO.

Improved PSO-Zhao [104] in 2006 proposed to use PSO with Passive Congregation with harmony search and utilises a mechanism called fly-back for constraints.

Augmented Lagrangian PSO-Sedlaczek and Eberhart [105] in 2006 suggested this method for equal and unequal constraints by combining augmented lagrangian method with PSO for optimization problems.

Optimised PSO-Meisnner et al. [106] in 2006 proposed that bigger swarm optimize the parameters of smaller swarm while smaller swarms find a solution to a problem. Thus bigger swarm will move over time to an optimal point in search space.

Parallel Asynchronous PSO-Koh et al. in 2006 suggested a novel technique [107] which works on dynamic load balancing to calculate the processor run time with a master-slave approach. All function evaluations are carried by slave processors where master processor performs velocity update and convergence tasks are by the latest information.

Crazy and hill climbing PSO-In 2006 ozcan and Yilmaz [108] proposed to enhance a balance between discovery and extract by using craziness and hill climbing for optimizing multi modal functions.

Restricted velocity PSO (RVPSo)-Liu and Chen in 2006 developed [109] a restriction in velocity due to limited search space for unconstrained problems.

Self-organisation PSO-Jie [110] et al. proposed in 2006 that an extra feedback agent is required to improve swarm performance in next iteration and hence stagnation can be avoided.

Two Swarm PSO-Lie et al. [111] in 2006 suggested that two swarms are flown in different paths from each other by setting different parameters. One swarm will explore global and the other will perform local exploitation in feasible solution search space by using Roulette wheel selection mode.

Unconstrained PSO-Moore and Venayagamoorthy [112] in 2006 proposed not of using the constraints for position and velocity equations unlike the classic form of PSO.

Velocity Limited PSO-Xu and Chen [113] in 2006 suggested in this approach that only those particles who satisfy the constraints for velocity and position are considered otherwise discarded.

Adapted Dissipative PSO-Shen et al. [114] proposed an approach in 2007 by introducing adaptive mutation and adaptive inertia weight strategy into dissipative PSO which improve the swarm diversity and avoids stagnation problems.

Area extension PSO-Atyabi and Phon-Ammuaisuk [115] in 2007 introduced this technique for solving multi-robots task problem with large area by adding new elements in the equation which results in new velocity equation for correct direction and premature convergence is solved by adding hot area/zone. New credit assignment and boundary methods are used for avoiding the particles to stuck in the areas and communication limitation helped to solve real world problems.

Behaviour of Distance PSO-Wang and Qian [116] in 2007 proposed that the particle changes there flying behavior when guided by optimum of each particle and the optimum of the swarm. So individual can adapt themselves to search for best position more effectively.

Best Rotation PSO-Alviar et al. [117] in 2007 proposed this approach that a swarm population is divided into smaller swarms and the stagnation problem is avoided by forcefully exchanging these swarms from one local minima to another local minima and a periodic rotation is performed from particles of these sub swarms which makes better exploration in search space.

Rotation Invariant PSO -Wilke et al. [118] in 2007 proposed rotation invariant and uses random matrices rather than random diagonal matrices to disrupt the direction of movement in every iteration using exponential map method.

Combi national PSO-Jarbouria et al. [119] in 2007 suggested a new clustering technique based on this algorithm with each particle being represented as a string length ‘n’ where i th element of the string indicates the group number assigned to the object ‘i’ and an integer vector corresponds to feasible solution to the clustering problem. The performance of this algorithm depends upon the choice of parameters and initial population.

Co-operative Multiple PSO(CMPSO)-Felix et al. [120] proposed in 2007 this technique which works well in multi dimensions problems and is more efficient than conventional PSO.

Dual Layer PSO (DLPSO)-Subbramaniam et al. in 2007 [121] suggested a strategy that optimizes the neural network in an architectural layer and uses joint weights in neural network.

Dynamic and Adjustable PSO (DAPSO)-Liao et al. [122] proposed novel concept of keeping the distance of every particle to the best location is determined in order to adjust the velocity of next step for the diversity of particles.

Estimation of Distribution PSO-Kulkarani and Venayagamoorthy [123] in 2007 proposed (hybrid of EDA and PSO) Estimation of Distribution algorithm which uses stochastic models to locate the optimal solution areas during the optimization process. This increases the diversity and efficiency of PSO.

Evolutionary Iteration PSO (EIIPSO) -Lee [124] in 2007 pointed a new combination of PSO and Evolutionary programming to avoid the trapping of particles in local minima and provides a strength to PSO efficiency.

Evolutionary Programming PSO (EIPPSO)-Wei et al. [125] proposed in 2007 by combining the two algorithm (EP and PSO) gives more diversity among the particles to explore local and global minima and faster convergence.

Greedy PSO-Lamet al. proposed in 2007 a new concept of hybrid Evolutionary algorithm [126] of combining binary PSO with Greedy transform strategy. The greedy transform method was successfully tested on knapsack problem.
Heuristic PSO—Lam et al. [127] proposed in 2007 that rate of convergence to local optima is faster than conventional PSO. This algorithm avoid stagnation or premature convergence near the global minima by randomly re-initialization of the positions of particles. This algorithm is powerful and efficient due to joint strategy by heuristic updating and position re-initialization.

Map Reduce PSO—McNabb [128] et al. proposed in 2007 a novel technique to solve big data problems which is time consuming with conventional PSO but this technique runs a parallel PSO for computationally compressed functions.

Modified Binary PSO—Yuan and Zhao [129] in 2007 suggested this strategy to randomly produce the particles as binary vectors and to map the permutation space using the lowest value of position.

Novel Hybrid PSO—Li and Li [130] in 2007 proposed the hybrid of PSO and Harmony search for better exploration in high dimension problems resulting in increasing exploitation of PSO.

Predator Prey PSO—Jang [131] et al. in 2007 suggested a new concept in which predator follow prey and prey escapes from predator means avoiding to get trapped in local minima and move towards optimal global minima.

Quadratic Interpolation PSO (QIPSO)—Pant et al. [132] in 2007 proposed that the hybrid use of EA and PSO in which swarm leader is selected in every iteration and other partners are selected from remaining particles for cross over and an offspring be produced as Quadratic cross over. Finally, selected particle is only accepted if it is better than the current best particle of the swarm.

Shuffled Sub swarm PSO—Wang and Qian et al. [133] proposed in 2007 for a better diversity and performance of the swarm.

Trained PSO (TPSO)—Gheitanchi et al. [134] in 2007 proposed this technique to the ad-hoc communication networks to reduce completion complexity and time by training the particles.

2-D OTSU PSO—Wei et al. [135] in 2007 suggested to use optimal threshold selecting search with PSO for better performance. The threshold selecting method is used for image segmentation based on PSO is combined with two-dimension Otsu method.

Vertical PSO—Yang in 2007 [136] proposed that the particles can move to both global position and vertical direction to avoid stagnation or entrapped near to global points.

Opposition based PSO Cauchi Mutation—Wang et al. [137] in 2007 suggested opposition based learning strategy to every particle in each iteration and selecting the best position of particle for applying dynamic Cauchy mutation resulting in fast convergence in complex optimization problems.

Clonal PSO (CPSO)—Tan and Xiao no [138] in 2007 suggested a method which clones and mutates the best particles of specific generations and then best one is selected to continue evolving.

Hybrid combination of PSO and GA (HEA)—Yang [139] et al. in 2007 proposed two stage evolution strategy where PSO conducts evolution process and GA preserves diversity and is used to solve three unconstrained and three constrained problems.

Active Target PSO—Zhang [140] et al. suggested in 2008 a new term as active target which is complicated to calculate and is used along with best position and previous best position for velocity updating. This method retains diversity of PSO and does not trap in local minima.

Adaptive Mutation PSO—Pant [141] et al. in 2008 proposed a new concept of using beta distribution in adaptive mutation in two forms. One form uses best individual position in a swarm and the other for the best global position.

Co-operatively Coevolving Particle Swarms—Yao [142] in 2008 suggested that the bigger problems can be broken up into smaller ones with least values such that there inter dependence cooperation is generated.

Geometric PSO (GPSO)—Moragila et al. [143] propose in 2008 about the geometric between the PSO and Evolutionary algorithm and can be used for problems of continuous and combinatorial spaces.

Immune PSO (IPSO)—Lin et al. [144] in 2008 suggested to improve the mutation mechanism of immune algorithm with PSO. In some situations information is often transmitted as immune operator in the PSO.

Modified Genetic PSO (MGPSO)—Zhiming et al. [145] in 2008 proposed a combinational algorithm of Genetic PSO and Differential Evolution. Position updating is done by both algorithm for each particle and the better results become the reference for next position.

Orthogonal PSO (OPSO)—Ho et al. [146] in 2008 suggested the intelligent move method for velocity update in which divide and conquer approach is used for finding next particle position and this gives better results with large problems than conventional PSO.

Pursuit Escape PSO (PEPSO)—Higashitani et al. [147] in 2008 proposed to divide the swarm in two groups as escape group and pursuit group on behavior basis. First group results in intensification and the other group results in diversification thus making a perfect balance in algorithm.

Self-adaptive Velocity PSO (SAPSO)—In 2008 Lu and Chen [148] investigated the impact of constraints on PSO due to its lack of knowledge of feasible solution and used dynamic objective constraint handling technique as the integral part of this method.

Velocity Limited PSO (VLPPO)—Xu and Chen [149] in 2008 suggested to keep those particles which satisfy the constraints for velocity and position otherwise they are eliminated for further participating.

Frankenstein’s PSO(FPSO)—Montes de Oca et al. [150] in 2009 proposed to combine together various PSO variants to eliminate the various deficiencies. Velocity update from FIPS, inertia weight from decreasing inertia PSO (DIPSO), acceleration co-efficient and maximum velocity from self-organizing hierarchical PSO and time varying acceleration co-efficient PSO(HPSO-TVAC)and its topology.

Adaptive Particle swarm optimization (APSO)—Zhan et al. [151] in 2009 suggested an adaptive change in the inertia weight as per location of the particles, adaptive change in acceleration co-efficient as per the stage evolution and new update rules in the selected evolution stage to avoid the stagnation of the particles.

Regrouping PSO (RegPSO)—Evers and Ghalia [152] in 2009 proposed automated regrouping in swarms when early convergence is detected. Particles are regrouped in each dimension in proportion to the degree of uncertainty indicated by the overall deviation of each particle from globally best location.

Discrete PSO with Embedded GA Operators—Premlatha and Natarajan [153] in 2009 suggested that during stagnation of the particles GA operator initiates reproduction and named this method Discrete PSO (DPSO) with mutation—crossover.
Hybrid PSO - Li et al. [154] in 2010 suggested to combine three algorithm non-linear simplex method for fast convergence and incorporated Tabu search into PSO for Tabu attribute to local regions solutions.

Tabu List PSO (TL-PSO) - Nakano et al. [155] in 2010 proposed to store the history of Pbest particles in Tabu list which will be used only when particle is not performing well for updates and thus avoiding stagnation issues.

Cultural based PSO - Daneshyari and Yen [156] in 2010 proposed to find global minimum by using multiple evolution and multiple progresses simultaneously strategy as compared to conventional PSO.

Genetically improved PSO (GIPSO) - Abdel-Kader [157] in 2010 suggested this algorithm for k-means clustering and used to find initial kernel of cluster centroid solutions that are then used by k-means for local search.

Perturbed PSO – Xinchao [158] in 2010 proposed that the global best particle be mutated during the run of the algorithm to avoid premature convergence.

Improved Artificial Immune network PSO (IAINPSO) - Tang et al. [159] in 2010 proposed this technique based on the population fitness variation, could be used as convergence factor for faster convergence with high precision and less number of iterations.

Feedback Learning PSO (FLPSO) - Tang et al. [160] in 2011 proposed a four phase technique by using modeled quadratic inertia weight strategy in first step, then acceleration co-efficient from every particle history followed by fitness input information for every particle is being used to construct the learning probabilities and finally a system of elite stochastic learning is employed to optimize the solution.

Self-Adaptive Learning PSO (SALPSO) - Wang et al. [161] in 2011 suggested to use four velocity updates rules from different variants of PSO and at every fixed number of iteration best update rule is selected and applied to every particles depending on situations. It is compared with its previous values then update rule is selected for next iteration.

Combination of PSO and Tabu Search (TS) - Zhang et al. [162] in 2011 proposed to solve non-linear integer program using a combination of PSO and TS in which new heuristic principles were prepared to solve unsuccessful solutions.

Self-Learning PSO (SLPSO) - Changhe et al. [163] in 2012 suggested a different strategy of probability update and rule as that of SALPSO.

Orthogonal Learning PSO (OLPSO) - Zhan et al. [164] in 2011 proposed an orthogonal learning (OL) technique which direct the particles to travel in better directions by an effective model and can be applied to any topological structure for faster global convergence with high quality solutions.

Adaptive Fuzzy PSO - Juang et al. [165] in 2011 suggested the use of fuzzy set theory to adapt PSO acceleration coefficient for better optimal and accurate values.

Enhancing PSO using generalised Opposition - based learning - Wang et al. [166] in 2011 proposed an improved PSO algorithm that use generalized opposition based learning and cauchy mutation which helped the particles to escape local minima and results in faster convergence.

Hybrid of Genetic Simulated Annealing Ant Colony system with PSO - Chen and Chien [167] in 2011 presented this strategy to solve travelling sales man problem with percentage deviation in average solution is much better than existing techniques.

Multiple-Adaptive strategy for PSO (PSO-MAM) - Hu et al. [168] in 2012 put up another idea of using updating only the global particles by two techniques randomly either by mutation or gradient descent method at every iteration.

Self-learning PSO (SLPSO) - Li et al. [169] in 2012 proposed a novel algorithm in which each particle has a set of four strategy to deal with different conditions in the search space through an adaptive learning system at individual basis which, in turn allows the particle to select optimal strategy according to its own fitness.

Chaotic Particle swarm fuzzy clustering - Liu et al. [170] in 2012 proposed a hybrid of new chaotic PSO and gradient method. The new chaotic PSO is used to search fuzzy clustering model using exploring capabilities of fuzzy C-means for exploitation whereas gradient operator accelerates the convergence process in this technique.

Hybrid of opposition based chaotic GA/PSO - Dong et al. [171] in 2012 proposed the method by combining the advantages of GA, PSO and chaotic dynamics. The velocity and position updates were from PSO, selection, cross over, mutation from GA, and opposition based learning was done in chaotic hybrid algorithm for population initialization.

Opposition based Natural Discrete PSO (ONDPSO) - Khan et al. [172] in 2012 introduced a new method in which particles were encoded by Natural Encoding scheme and position updating is done by new designed updating rule and opposition based learning is used in this technique. The natural encoding scheme and position update rule used in this technique allows the individual term to use co different attributes related to them within the rule to be a disjunction of the values of those attributes.

Grey PSO - Leu and Yeh [173] in 2012 suggested two grey based parameter approaches, inertia weight and acceleration co-efficient. Each particle has its own inertia weight and acceleration co-efficient whose values are dependent on corresponding grey rational grade which is varying over iterations, those parameters are also varying. Even in the same iteration those parameters may be different for different particles. This strategy gives information about particle distribution in search space.

Mutation linear PSO (MLPSO) - Bonyadi et al. [174] in 2013 came with idea of multi-start PSO by combining the mutation operator with linear decreasing PSO (LPSO) to solve constraint problems.

Automatic Particle Injection PSO (APIPSO) - Elsayed et al. [175] in 2013 proposed to have good balance between exploration and exploitation by applying standard PSO in starting and linear decreasing PSO in latter phase. In order to prevent from trapping in local minima a mutation operator and instant injection of new particles is done.

Essential PSO queen (EPSOq) - Ktari and Chabchoub [176] in 2013 suggested to use essential and strong feature of Tabu search to form improved discrete PSO.

PSO Using Dimension Selection method - Jin et al. [177] in 2013 proposed to use random dimension selection instead of stochastic coefficient is the another way of using randomness. Modified PSO is developed using dimension selection method shows better results and randomness is correct and important.
Recombination based hybridisation of PSO and ABC Algorithm -Kiran and Gndz [178] in 2013 proposed of combining artificial bee colony (ABC) with PSO with five strategies and counters to update the solutions. Artificial agent perform the search process and counters are used to determine the rule that is selected by the bees. Depending upon the characteristic of problem, the artificial agent learn which in turn update the rule to find better solutions.

Novel Fuzzy PSO – Aminian and Teshnehlab [179] in 2013 proposed a novel technique in which inertia weight, cognitive and social co-efficient are adjusted by fuzzy logic for each particle separately.

PSO – AIN Hybrid - Liu et al. [180] in 2013 suggested that the entire population is divided into two types of subpopulations as Elite and some common sub populations. The best individual in the normal sub population will be memorized in the Elite sub population during the evolution process.

Hybrid strategy in Continuous Ant Colony optimization (ACO) and PSO-Huang et al.[181] in 2013 suggested to hybrid ACO and PSO for better searching capabilities of global minima without entrapping in local minima and introduced four types of hybridization. The sequence method with the enlarged pheromone table was better than that of other types due to diversified generation of new solutions.

Hybrid PSO-Simulated Annealing(SA) Approach- Jiang and Zou [182] in 2013 suggested an improved PSO based parameter method by modifying the fitness function in the conventional evolution process of support vector machines and then combined with SA global searching method which results in avoidance of local minima and better results.

Hybrid of PSO and Artificial Bee Colony(ABC)- El-Abd [183] in 2013 introduced one component based, and the PSO was improved with the ABC component and was tested on the CE13 test bed to improve the personal best of particles.

Blend of PSO variant in ABC – Sharma et al. [184] in 2013 suggested a technique called local-global variant ABC (LGABC) for balancing between exploration and exploitation in the search space of ABC with good optimal results after testing on bench marks.

Combining Differential Evolution(DE) and PSO – Maione and Punzi[185] in 2013 introduced a hybrid two step approach in which DE determine the fractional integral and derivative acts that meets the meets necessary time-domain specification and PSO identifies rational approximations of the irrational fractional operators.

Improved Quantum behaved PSO Simplex (IQPSOS) - Davoodi et al. [186] in 2014 proposed to combine QPSO which gives direction to feasible optimal region and Nelder-Mead simplex approach for local search in the global region.

Centripetal Accelerated PSO (CAPSO) - Beheshadi et al.[187] in 2014 proposed to combine improved PSO with Newton laws of Motion which accelerates the method which results in avoidance of local minima and learning.

Improved Quantum behaved PSO – Li and Xaiao [188] in 2014 proposed an encoding technique based on Qubits mentioned on Bloch sphere where each particle contained three groups of Bloch coordinates of qubits and all the three groups of coordinates were known as approximate solutions. Updating of particles is done by using the rotation of qubits about an axis on the Bloch sphere.

Linear Constraint Minimum Variance (LCMV) enabled by PSO- Darzi et al. [189] in 2014 introduced by integrating PSO, dynamic mutated AIS and Gravitational search algorithm into the LCMV method for boosting the weights of LCMV technique.

Restated Simulated Annealing PSO – Zheng et al.[190] in 2014 proposed a modified method to decompose structuring elements (SE) of a randomly form by collectively using restarted SA and PSO. Firstly, a modified recursive dilation union model having a different termination criterion is used to form an optimization problem and subsequently restarted simulated annealing PSO technique was adopted as search method.

Adaptive hybrid of PSO and Differential Evolution(DE) – Yu et al.[191] in 2014 proposed a novel algorithm by balancing the parameter of PSO and DE, an adaptive mutation is applied to the current population when population clustering is near to local optimal and thus diversity is also maintained.

Improved Accelerated PSO with DE-Wang et al. [192] in 2014 suggested a hybrid approach by using DE mutation operator to the accelerated PSO for solving optimization problems.

Co Swarm PSO with DE - Yadav and Deep [193] in 2014 proposed to hybridize the shrinking hyper sphere PSO with DE method by dividing the population in two sub swarms. First sub swarm use SHPSO and the other sub swarm uses the DE approach and are able to solve any real constrained optimization problems effectively.

Biogeography based PSO with Fuzzy Elitism – Guo et al. [194] in 2014 introduced to split the entire population into many sub groups where biogeography based optimization (BBO) is selected to search within group to find some solutions and uses a fuzzy method to pick few solutions as leader for global search which is done by PSO which selects individual and redirect to original groups for next iteration.

Enhanced Comprehensive learning PSO – Yu and Zhang [195] in 2014 proposed two enhancements of comprehensive learning PSO, first a perturbation term is added to each particle velocity update equation to achieve better exploitation. Normative knowledge about dimensional bounds of personnel best position is used to activate the perturbation based exploitation, second the learning probabilities of particle are determined dynamically based on not on rankings of personal best fitness values but also the particles exploitation progress to facilitate convergence.

Levy Flight PSO – Husein and Harun[196] in 2014 proposed to combine PSO with levy flight where limit value for each particle is specified and if there is no improvement in its self-solution at the end of current iteration, then limit will be increased. If this limit is exceeds by the particle, the particle is distributed again in search space by levy flight method to get rid of local minima.

Teaching and Peer-learning PSO(TPLPSO) – Lim and Isal[197] in 2014 proposed this technique consisting of teaching phase, the peer learning phase and the stagnation prevention strategy to improve PSO performance with high searching accuracy and convergence speed. The particle enters into teaching phase firstand updates its velocity to its best historical and best global positions. If the particles fails to boost its performance then it enters the peer-learning process where a particle is selected as guidance particle and eventually the last step is used to alleviate the premature convergence.

Random Drift PSO (RDPSO) - Sun et al.[198] in 2015 got motivated by the model of free electron in metals placed in magnetic field have a drift velocity with thermal motion leading to a minimum potential energy.
Social Learning PSO (SL-PSO)-Cheng and Jin[199] in 2015 proposed that social learning technique inspired by learning methods, which requires no fine tuning of control parameters and is performed on sorted swarm. In comparison to learning from the best historical positions, the particles learn from any good performer particles called demonstrators in the present swarm.

Heterogeneous Comprehensive learning PSO - Lynn and Suganthan[200] in 2015 proposed this technique where particles in a swarm will be allocated different search behaviors by randomly selecting velocity and position update rules from a behaviour pattern to efficiently address exploration-exploitation trade off.

PSO with Adaptive Inertia Weight using Bayesian Techniques -Zhang et al. [201] in 2015 proposed to apply Bayesian technique for better search ability in the exploitation of the past particle positions and for exploring Cauchy mutation for faster convergence rate with better solution.

Self-Regulating PSO - Tanweer et al. [202] pointed two learning strategies in 2015, the first one uses a self-regulating inertia weight which is used by the best particle for better exploration and second using the self-perception of the global search path used by the rest of the particles for exploitation in the solution space.

Enhanced Leader PSO (ELPSO) -Jorde hi et al. [203] in 2015 proposed a five stage successive mutation novel strategy which is applied to the swarm leader at every iteration for mitigating convergence problem.

PSO based on Two Swarm Evolution -Wang et al. [204] in 2015 suggested a new strategy by adopting linearly decreasing inertia weight to one swarm and random inertia weight to the other swarm. A random disturbance is added to the particle position at the stagnation point where it breaks the swarm into new escaped swarm from the local minima.

Feature selection Algorithm based on Bare bones PSO – Zhang [205] in 2015 proposed to find optimal feature subset in solving classification problems. An improved memory strategy has been put in place to update the local leaders in order to prevent the loss of outstanding genes in the particles and provides good balance of exploration and exploitation.

Chaotic Simulating Annealing PSO – Geng et al. [206] in 2015 introduced to search more appropriate parameter combination where robust v-support vector regression is used to estimate port throughput.

ABC –PSO hybrid – Vitorino et al.[207] in 2015 proposed a method based on ABC to create diversity using adaptive PSO, when all the PSO particles converge to a single location by switching between two pre-specified behaviours by using fuzzy rules depending upon the size of entire swarm.

Novel Self Adaptive PSO – Pornsing et al. [208] in 2016 suggested a novel technique by splitting the entire swarm into many sub-swarms thus allowing the particles to disperse the whole search space where the worst performer dies out and the best performer produces the offspring. Survival sub swarm adaptive PSO and Survival sub swarm adaptive PSO with velocity line bouncing approaches outperformed than the other algorithms.

Genetic Learning PSO (GLPSO)–Gong et al. [209] in 2016 develops a new framework by hybridizing PSO with another optimization technique called learning PSO which have two layers first for exemplar generation and the other for particles update by PSO algorithm. Genetic operators are used to produce exemplars from which particles learn and from the historical knowledge on the hunt for given particles gives guidance to evolution of exemplars. From past information exemplars are constructed which are well trained exemplars and diversified through crossover, mutation and selection process.

PSO with Inters warm Interactive Learning Strategy (IIILPSO)-Cheng et al.[210] in 2016 put up a new concept of interactive learning behaviour in which particles are divided into two swarms. If there is no substantial change in the fitness value then inter-swarm interactive learning strategy will start and check the best particle fitness values in both swarms. Softmax and Roulette method is used to classify them as learning swarm and learned swarm. Exploration with global search ability is increased by using velocity mutation operator and global best vibration technique.

Distribution-Guided Bare-bones PSO (DBPSO)–Zeng and Shen [211] in 2016 suggested that they can overcome the problem of being trapped in local optima by jumping and its probability is adaptively adjusted according to its current location.

Elite Promotion Quantum-Behaved PSO–Yang et al. [212] in 2016 proposed to use differential evolution operators to elite particles of the swarm for more local search and produce more global results efficiently for complex optimization problems.

Parallel Clustered PSO – Hoassn et al. [213] in 2016 proposed to combine PSO and K-means clustering which runs in parallel using MapReduce on the Hadoop platform and takes less time to compute.

Sophisticated PSO(Sop PSO) - Xia et al.[214] in 2017 suggested that this technique by using multilevel adaptation and purposeful detection. In Sop PSO a particle not only updates its learning model but also chooses its target that the particle learns from neighbours while adaptive strategy is applied in multi-level. Tabu search and local searching strategies are to jump the local minima.

Hierarchical Bare Bones PSO–Guo and Sato [215] in 2017 proposed that particles are separated into different groups and play different roles whereas group leader exchange information with the global particle and rest particles learn from the leaders. In the next iteration any particle can have better position from their group leaders.

New Social-Based Radius PSO – Munlin and Ananthanavith [216] in 2017 proposed to regroup the particles in a given radius of the circle and finds the agent particle which is best particle of the group for each local minima which helps to achieve global minima.

Primal-Dual Asynchronous PSO (pdfPSO) – Gbenga et al.[217] in 2017 suggested a novel technique by combining Asynchronous PSO and Primal dual interior point algorithm. This algorithm blends the explorative capabilities of PSO with the explorative capabilities of Primal dual approach thereby possessing a higher ability to avoid stagnation or stuck in local optima.

Movement PSO (MPSO)- Hudaib and Hwaitat[218] in 2018 proposed this algorithm that enhances the behavior of PSO by using random movement function to find more feasible points in the search space. This algorithm has good features like exploration, exploitation and local optima avoidance.

Centroid PSO – Anwar[219] in 2018 proposed a dubbed centroid PSO inspired by centre based sampling theorem which specifies that the centre area of the search space includes points of high probability closer to optimum solution for data classification problems.
Scout particle swarm optimization (ScPSO) – Koyuncu and Ceylan [220] in 2018 suggested an efficient technique by hybriding PSO and Artificial Bee Colony (ABC), along with adding a scout bee phase to standard PSO. The scout bee phase in ABC regenerates the useless particles that cannot improve their individual best positions and this process is operated through the parameter limit.

Fuzzy Controlled COBRA-fas (Co-operation of Biology based Algorithm) – Akhmedova et al. [221] in 2019 developed an strategy based on six optimization methods namely PSO, Wolf Pack search (WPS), Firefly Algorithm (FFA), Cuckoo Search Algorithm (CSA), Bat Algorithm (BA) and Fish School Search (FSS) for solving real valued unconstrained optimization problems and is better in both exploration and exploitation than any other bio inspired algorithm.

Diversity-Guided Multi-Mutation PSO (DMPSO) – Tian et al. [222] in 2019 proposed Opposition based learning is used to get the high quality initial particles acceleration along with self-regulating inertia weights with three mutation strategies (Gaussian, Cauchy and Chaotic) to maintain diversity of the whole swarm. An Auxiliary velocity-position update mechanism is applied to the global best particle for convergence.

Triple Archives PSO – Xia et al. [223] in 2019 proposed this model in which particles in three archives are used. First elite particles are documented in one archive while other particles which show quicker progress called profiteers are documented in another archive. Second when breeding, each dimension of a future exemplar for a particle we select a pair elite and profiteers from corresponding archive as two parents to produce the dimension value by genetic operators. Third, each particle executes a particular learning model as per the fitness of future exemplars. Finally the outstanding exemplar are saved in third achieve and reused by worse particles for better exploitation.

Fractional-order quantum PSO – Xu et al. [224] in 2019 proposed by using concepts of quantum mechanics and PSO with fractional calculus to achieve better global search ability. Grunwald-Letnikov is most frequently used fractional differential definition uses its discrete expression for its position updating of quantum behaved PSO.

Dual-Environmental Particle swarm optimizer – Zhang et al. [225] in 2019 proposed PSO variant that uses a weighted search centre based on top k-elite particles to guide the population. It averages their position rather than re-sampling fitness values of particles to achieve noise free environment.

IV. CONSTRAINED OPTIMIZATION PROBLEMS (COPs)

Many practical engineering optimization problems have constraints and require the solutions in that search space. PSO can easily solve such problems using certain strategy like static penalty, dynamic penalty, death penalty, MO approach, co-evolutionary, stochastic ranking, a-constrained, c-constrained, hybrids, Del Valle’s approach and Debs approach along with some modifications. Most commonly used are death penalty which is simple and parameter free whereas Debs approach is simple, derivative free and explore in infeasible regions also.

Trial and Error approach to constrained PSO-Hu and Eberhart [226] in 2002 proposed this method with two modifications, in first method the particles are initialized in feasible position and the other method use only those solutions who satisfy the constraints are used for local and global positions.

PSO for Constrained problems – Parsopoulos and Vrahatis [227] in 2002 introduced dynamic penalty functions for the three variants of PSO and compared with other EA and found good results.

Death Penalty PSO – Coath and Halgamuge [228] in 2003 proposed this approach in which initialization is carried out in feasible solution region and memories are updated, particles keep only feasible solution in memory. This approach is simple and parameter free.

Constraint handling Mechanism for PSO – Pulido and Coello [229] in 2004 presents a simple concept based on proximity of a particle to the feasible solution as a leader with a turbulence operator for exploration in search space.

Contrain PSO – Zavala et al. [230] in 2005 proposed a modified PSO which uses a ring topology and a mixture of feasibility and superiority in the selection of local best particle to maintain flexibility and exploration within the entire swarm.

Dynamic Multi-swarm PSO for Constraints - Liang and Suganthan in 2006 [231] proposed that the swarm is periodically divided into sub swarms and particles are selected at random. For better exploration the sub swarms search optimal solutions in constrained space.

Constrained PSO – Bochenek and Forys [232] in 2006 proposed controlled reflection technique for to address inequality constraints and particle trap strategy is used for equality constraints. If the particle is entrapped, then a penalty term is applied to the objective function to compel the trapped particle and the constraints become active at the optimum solution.

Constrained optimization via PSO (COPSO) – Aguirre et al. [233] in 2007 proposed to use Lbest(local best) PSO to evaluate the constraints and prepare an external file called Tolerant to perform a particle analysis. Lifetime of particles is developed by using the tolerant file with ring topology which maintains diversity.

Handling Constraints of PSO using small population size – Cabrera and Coello [234] in 2007 proposed to use leader selection scheme based on a distance of a solution to a feasible region along with a mutation operator to improve the exploration search using small population size of five.

PSO in constrained space – Flores and Mezura [235] in 2008 suggested a modified version of Debs approach, where calculating the sum of constraint infringements is performed differently for equality and inequality constraints and comparing infeasible solutions.

New Vector PSO – Sun et al. [236] in 2009 proposed a vector PSO technique to evaluate the constrained optimization problem in which one dimensional search methods were used to find a feasible position for each escaped particle.

Cooperation Comprehensive Learning PSO (CCLPSO) – Liang et al. [237] in 2010 presented a novel idea for solving constraints problem along with objective function where CLPSO was used either to satisfy constraints or optimize the objective and sequential quadratic programming was used for improvement in solution during the run.
Improved vector PSO – Sun et al. [238] in 2011 proposed for search of feasible position in a local region consisting of dimensions of the parent and current position of the escaped particle using multi-dimension search algorithm for solution.

Cultural Based constrained PSO – Daneshyari and Yen [239] proposed in 2012 to combine the objective function and constrained violation in four areas as belief space, specifically normative awareness, spatial awareness and temporal knowledge. With this information communication is good at personal, swarm and global level.

Extension of constrained PSO – Afsha r [240] in 2013 suggested that three constrained version of PSO should be based on identification and exclusion of the infeasible region of search space.

PSO based Hyper-Heuristic – Koulianas et al. [241] in 2014 proposed a PSO based on hyper-heuristic which worked as upper level algorithm and controlled a number of low level heuristics operating in a solution space. Solutions are represented on the basis of random keys and active schedules were developed using the priorities of activities which were changed by low level heuristic.

Constrained PSO – Singh et al. [242] in 2014 proposed this technique to detect a silent object in two phases. In first phase features such as multi-scale contrast, center-surround histogram and color spatial distribution were obtained and in next phase constrained PSO calculate an optimal weight vector to combine these features in order to obtain saliency map to distinguish the salient object from the image background.

Multi-Target PSO- Cui et al.[243] in 2014 presented a novel approach multi-target (m PSO) where each particle will search its target within given target depending on the radius of search, big radius will ignore close solution with better fitness whereas small radius will produce many targets and at end only few goals are left with threshold fitness value.

Hybrid PSO – Shou et al.[244] in 2015 suggested a solution to a problem of constrained pre-emptive resource project scheduling problem in which a maximum of one interruption per activity was allowed. Particle representation of four forms was used and two schedule generation schemes were used to decode the representations of particles. Peak cross over operator were used for particle updating for particle representations.

Constrained modified PSO (SASPSO 2011) – Tang et al. [245] in 2016 proposed the adaptive relaxation approach that would be combined with a feasibility based rule to tackle the constrained optimization problems of modified PSO (named as SASPSO 2011) so as to increase the diversity of solutions along with a parameter selection principle which guarantees the convergence.

Augmented Lagrange constrained PSO – Lu et al.[246] in 2017 proposed to optimize the objective function by combining the constrained PSO with the Augmented Lagrange multiplier (ALM) technique. Initially, a new particle swarm is produced each time and in order to avoid falling into a local minima, its value can be easily extracted from the previous generation which is saved and transferred to the next generation during the procedure.

Strongly Constrained space PSO – Ma et al. [247] in 2018 introduced a highly constrained particle swarm optimization method that would impose water balance constraint on the search for feasible regions and this technique pays importance of the water constraint and rest of the constraints would be using the constant penalty function approach to avoid the issue of feasible regions.

V. MULTI-OBJECTIVE PARTICLE SWARM OPTIMIZATION

In order to solve real world multi-objective or multi criteria issues we optimise a solution and create feasible solutions across a parento optimal front but due to the unconstrained nature of PSO, the technique is modified to achieve a number of elite non dominated solutions.

Multi-Objective PSO -Coello and Lechuga [248] in 2002 firstly proposed using the adaptive grid method to preserve the external file.

PSO Method in Multi-Objective Problems- Parsopoulos et al. [249] in 2002 suggested that three variants in weighted aggregation methods for multi-objective PSO. In linear aggregation function weights are fixed in objective function but in bang-bang aggregation function weights keep changing more than dynamic aggregation function in all the iterations.

Swarm Metaphor for Multi-objective Design optimization – Ray and Liew [250] in 2002 proposed to choose particles whose performance is good to be leaders and other particles select their leader randomly from the group leader where the leader with low followers is most likely to be selected.

Vector Evaluated PSO -Omkar et al. [251] in 2002 suggested a multi swarms strategy depending on the number of objectives. Every swarm has its own objective function to be optimized and the velocity update is done from the knowledge from other swarms. This strategy gives a number of parento front solutions.

Dynamic Neighbourhood PSO (DNPSO)-Another strategy by Hu et.al [252] in 2003 was proposed to use Nbest instead of current best and is the best particle in the specific neighborhood. In this way the selection of neighbors for the current particle is one objective and the other selection of their best.

Particle Swarm with Extended Memory -Hu et al. [253] in 2003 suggested for Multi-objective optimization to combine extended memory to DNPSO as the number of Parento front solution are limited and some best solution are lost.

Divided Range PSO-Ji et.al [254] proposed a multi objective PSO in 2004 where the particles are divided into sub-swarms for one objective function then discrete PSO is run for each sub-swarms till stopping criteria meets otherwise particles are again ordered again for the next objective function and the categorization take place again.

PSO with Passive Congregation (PSOPC) – He et al. [255] suggested interesting concept in 2004 about passive congregation (selfish behavior in information sharing and forms passive group) where this passive group is added to PSO to increase its efficiency.

Parento Optimality and PSO - Baumgartner et al. [256] in 2004 proposed that parento based approach generates a set of solutions satisfying the main objective without effecting the performance of other objectives.

Handling Multiple Objectives with PSO - Coello et al. [257] in 2004 suggested to use the mutation strategy to solve multi-objectives using PSO.
Improved PSO based Multi-Objective optimization - Sierra and Coello[258] in 2005 proposed the crowding distance and E- dominance for diversity and divided the population to be divided into small sub populations with different mutation operator from escaping the local minima.

Variable Neighbourhood PSO- Liu et al. [259] proposed in 2006 that in the multi-objective problems trapping in local minima can be escaped by local search repeatedly from starting point to local optimum till better than current value.

Two level of Non-dominated solution approach - Abido [260] in 2007 proposed to find non-dominated solutions at local set and global set levels in multi objectives PSO.

Scaleable Co-evolutionary Multi-Objective PSO - Zheng and Liu [261] in 2010 suggested to use decomposed decision variables and cooperative co-evolutionary sub swarms to solve multi-objective PSO.

Multi-Objective PSO based on Decomposition - Mart et al.[262] in 2011 proposed to use decomposition method to solve multi- objective problems.

Binary PSO hybrid with Artificial Immune network (AIN) – Ibrahim et al.[263] in first presented in 2011 the idea of topological reach area monitor and used a binary PSO hybridized with AIN to tackle multi-objective problem.

Local search based hybrid PSO for Multi-Objective optimization - Mousa et al.[264] in 2012 proposed that by combining GA and PSO, the two character features of these algorithm can be used for Multi objective optimization. Firstly evolution of the particle is done to achieve non dominated solutions by initializing a set of random particles followed by a local search to explore more dominated solutions.

Trust Region (TR) algorithm for Multi-Objective optimization - El-Sawy et al.[265] in 2012 suggested multi-objective optimization problems be overcome by using trust region strategy based on local search (LS) technique, where a multi-objective problem is transformed into single objective optimization problem by using reference point method. For each reference point the TR method is used to determine a point on a Parento frontier and LS method is used to find more positions on parento-front.

Bare Bones Multi-Objective PSO- Zhang et al.[266] in 2012 proposed an algorithm with three features, namely particle update strategy that does not require tuning of control parameters, mutation search function operator and an particle diversity strategy to update global particle.

Fuzzy PSO for Multi-objective – Khan and Engelbrecht [267] in 2012 proposed to incorporate fuzzy logic in PSO to solve multi-objective problem where unified And-OR operator was used to combine the objective.

Multi-objective PSO (MOPSO) with K- Means – Qiu et al.[268] in 2013 proposed this technique with new Gbest selection method and a K-means algorithm. A Gbest particle is selected by using proportional distribution approach and a mutation operator is used to enhance exploration.

Co-evolutionary Multi-Swarm PSO for Multi-objective – Zhan et al.[269] in 2013 proposed this technique based on multiple population with multiple objectives (MPMO) by using an external shared database to share search information for others and by using two prototypes to improve the performance.

Multi-objective PSO (MOPSO) and Fuzzy Ant Colony Optimization (FACO)- Elloumi et al.[270 ] in 2014 introduced combination of best fuzzy Ant Colony particle and incorporate it as the best local particle of PSO to propose a new technique as hybrid MOPSO with FACO for solving multi-objective problems.

Multi objective hybrid Quantum PSO(QPSO)- Chen et al.[271] in 2014 suggested to use elitist hybrid QPSO with mutation where elitist method with crowding distance filtering was used to enhance the diversity and quantity of optimum solutions.

Multi objective planning using PSO – Ganguly [272] in 2014 proposed a PSO based multi objective planning algorithm with minimizing the three objectives simultaneously in order to achieve a set of non-dominated solutions.

Multi objective Reliability Redundancy problems using Extended Bare Bones PSO(BBPSO)– Zhang et al.[273] in 2014 proposed a two-step algorithm in wherein the Bare bones PSO multi objective PSO is developed and implemented to the first step in order to find a parento optimal set. This algorithm is the combination of Bare bones PSO and sensitivity dependent clustering to evaluate multi objective reliability redundancy allocation issues.

Improved Multi objective PSO with Preference Strategy – Cheng et al. [274] in 2015 proposed this strategy by using preference factors for some constraint space attributes. The performance of this technique was strengthened by using dynamic selection of global best, circular non dominated collection of particles and a mutation operator.

Co-operation of Biology related Algorithms for Constrained Multi-Objective Problems (COBRA-m) – Akhmedova and Semenkin [275] in 2015 proposed the cooperation work of five algorithm namely PSO, Wolf Pack search(WPS),Firefly Algorithm(FFA), Cuckoo Search Algorithm-CSA) and Bat Algorithm(BA) with the use of Pareto optimality theory to solve multi-objective optimization problems and to work effectively.

Multi swarm Comprehensive learning PSO to solve multi objective problems (CLPSO)- Xiang and Zhang [276] in 2017 proposed that each swarm focus on separate objective using CLPSO without learning from other swarms, mutation is only applied to elitists and modified differential evolution method is applied to some of the least crowded elitists.

External Archive-Guided Multi objective PSO – Zhu et al.[277] in 2017 proposed a novel algorithm where multi objective problems are converted into sub problems using decomposition method and then each particle is allocated accordingly to simplify the sub problem. This technique is designed for better exploration and the external archive is used from immune-based evolution strategy for speedup convergence.

Multi-objective PSO using Ring topology - Yue et al.[278] in 2018 proposed this technique to solve multi-modal multi-objective problems using ring topology and special crowding distance, where ring topology helps in finding much more parental-optimal solutions and special crowding distance considers the crowding distance both in decision and objective space to maintain multiple parento solutions.
Adaptive Gradient Multi objective PSO (AGMOPSO) – Han et al.[279] in 2018 suggested state of art technique in which stock ticker MOG method would update the archive for better convergence, local exploitation and self-adaptive flight parameters mechanism, by retaining the convergence and diversity of the balance according to the particle diversity information.

Self-organising RBF Neural network using Adaptive Gradient Multi objective PSO (AGMOPSO) – Han et al.[280] in 2019 proposed to optimize the structure and parameters of RBF Neural networks by developing AGMOPSO, then the AGMOPSO based self-organizing RBF Neural network can optimize the parameters (centers, width and weights) as well as network size.

Surrogate assisted PSO with Parento Active learning - Zhiming et al.[281] in 2019 proposed to save computational cost of multi-objective optimization problems. PSO is regarded as a sampler for the generation candidate solutions and the output is enhanced by preselecting results with the improved ε-PAL. A hybrid mutation sampling method based on simulated evolution is used to enhance the output of the sampler.

Multitasking Multi-swarm optimization (MTMSO) – Song et al.[282] in 2019 proposed to divide randomly the whole swarm into multiple task swarms for particular task and each swarm is further divided into sub swarms. Each task group works on dynamic multi-swarm optimization algorithm and probabilistic crossover of personnel best of particles from multiple task group is done for cross task knowledge. Task group and each group sub swarm are reformed periodically to maintain search diversity.

VI. Fig 1. Shows the distribution of all research papers published from 1995 to end of 2019 using Refinements in PSO, Hybrid PSO, COPs PSO and Multi-objective PSO

![Pie chart representing the distribution of published research papers using Refinements in PSO, Hybrid PSO, COPs PSO and Multi-objective PSO.](image)

**VII. PARALLEL IMPLEMENTATION**

The main problem in PSO implementation is its runtime when dealing with large optimization problems or in higher dimensions and parallel implementation is best suited to solve this problem. In parallel computing computations are carried out simultaneous. Multiple processing units of a single computer make independent operations in the inherently sequential framework of PSO by using the parallel sub-swarms to the different processors with the exchange of information between them. On the other hand multiple computers uses Grids, clouds and clusters [283] to perform the same task.

A. Multi core

Optimization problem can be solved speedily by splitting into parts and each part is computed simultaneously by a single or multiple machines by using parallel computing strategy with multi core or multiprocessor. There are various parallelization techniques like Hadoop MapReduce[284], Rparallel package [285], MATLAB Parallel Toolbox [286], OpenMP with C++ [287], Parallel computing module in Python [288], Julia- Parallel for and MapReduce [289], MPI [290] to access the multiple cores with one or more CPUs.

**Parallel Implementation** - Gies and Rahmat-Samii in 2003 firstly used parallel implementation technique and showed [291] eight fold improvement in performance using a system with 10 nodes for a parallel implementation over a serial one.

**Parallel Global Optimization with PSO** - Schutte et al.[292] in 2004 proposed parallel implementation on two types of problems. Firstly on large scale computational problems with inexpensive function assessments and secondly medium scale problems on bio-mechanical system recognition with computationally heavy function assessments. It uses a synchronous system based on a master-slave method.

**Parallel PSO accelerated by Asynchronous Evaluations** – Venter et al. [293] in 2005 proposed to use this parallel scheme based on the Message Passing Interface to provide master-slave implementation. The asynchronous algorithm updates all the design point knowledge fastly as point is available and begins the next iteration immediately for all the points to be evaluated.

**Parallel PSO with Communication strategies** – Chang et al. [294] in 2005 suggested three strategies based on data independence. The first strategy is intended for solution parameters that are independently or loosely correlated whereas second strategy applied to strongly correlated parameters and third strategy is applied to properties of unknown parameters.
Relative velocity updating in Parallel PSO – Chusananiputi et al.[295] in 2005 introduced synchronous implementation with relative velocity updating based on parallel relative PSO. In this technique after exploring the nearby slave, it send the best location and velocity updating to the master and master selects best velocity and the next move is decided accordingly.

Parallel Asynchronous PSO (PAPSO) – Koh et al.[296] in 2006 proposed a parallel asynchronous PSO that exhibits good parallel performance for large number of processors as well as good optimization performance. PAPSO gives 3 times faster than parallel synchronous PSO results in large number of processors, while retaining the overall optimization performance.

Parallel PSO using MapReduce (MRPSO) – McNabb et al.[297] in 2007 proposed a novel technique as MapReduce parallel programming model in Hadoop[298] where mapping phase particle is mapped and obtains updated velocity, position, pbest and reduced phase gbest is determined by collecting all the information details.

Two Phase Parallel PSO(PPPSO) – Liu et al.[299] in 2007 suggested individual orientation function for exploration and the overall orientation function for exploitation in parallel search area in second step.

Parallel Multi-Population PSO using OpenMP – Wang et al.[300] in 2008 suggested asynchronous version using OpenMP where the particles were ranked as per performance in fitness function, then sub-populations were generated and the best position in population and sub population is chosen for updating position and velocity equations.

Multiprocessor modelling of Parallel PSO – Waintraub et al.[301] in 2009 proposed this technique using master-slave approach and developed many PPSO using the enhanced network topologies implemented by a communication strategy in multiprocessor architectures.

Parallel PSO (PPSO) – Jeong et al.[302] in 2009 expressed this technique for PC clusters that shares the information with sub-swarms one by one using coarse grain topology in order to preserve diversity and avoid premature convergence.

Parallelization of PSO using Message Passing Interface (MPI) – Singhal et al.[303] in 2009 implemented asynchronous PSO using MPI commands on the multiple systems and this particle split the particle in such a fine way for each number of processors, and the processor with good result becomes the root processor at the end of each phase.

Synchronous Parallelization of PSO with digital pheromones – Kalivarapu et al.[304] in 2009 suggested to use multiple swarms in n-dimensions search space in parallel computing technique with new PSO version with digital pheromones, which increases the efficiency of this algorithm with lower time period in higher dimensions.

Agent based Parallel PSO – Lorion et al.[305] in 2009 introduced a coordination agent between swarms and other coordination swarm agents to disperse and manage a particle swarm on multiple interconnected processors.

Parallel Scalable hardware implementation of Asynchronous discrete PSO – Farmahani-Farmahani et al.[306] in 2010 proposed a PSO lined hardware for performing mathematical operation of algorithm with the notion of system on a programmable chip(master slave multi-processor) for discrete optimization problems. Sub-particle method is used to bring the benefit of full scalability and asynchronous PSO gives better efficiency for large and complex problems.

Communication latency tolerant Parallel PSO – Li and Wada[307] in 2011 suggested globally synchronized parallel PSO with delayed exchange parallelization which performs PSO on multiple environments by hiding communication latency. This method pose pone the best function fitness exchange to one loop later.

Parallel PSO implemented by multiple threads – Tu and Liang[308] in 2011 proposed that contact between sub groups be implemented through parallel computation models based on broadcast, star, migration and diffusion network topologies. Due to the complexity and difficulty of true parallel computation, multiple threads are used for simultaneous particle interaction.

Parallel PSO Clustering based on MapReduce – Alijarah and Ludwig[309] in 2012 proposed this algorithm for efficient clustering in three sub-modules. Sub-module first updates the particle swarm centroid in MapReduce and second sub module fitness calculations are for new centroid and third sub-module for personal best and global best centroid updating.

Particle Co-operative Micro-PSO – Parsopoulos[310] in 2012 proposed an algorithm based on decomposition of search space into smaller search spaces with smaller dimensions, using two types of computer systems as complex clusters and desk top multi core framework to test this method.

Twin PSO – Yu[311] in 2014 proposed the technique by integrating local search heuristic into PSO algorithm and this newly hybrid version is called Twin PSO and was applied to flow shop with a multiprocessors scheduling problem.

Parallel Multi-swarm algorithm based on Comprehensive learning PSO – Gulcu and Kodaz[312] in 2015 proposed multi swarm which work co-operatively and the local best get exchanged in each migration phase in order to ensure a variety of solutions.

Parallel PSO using Message passing Interface – Zhang et al.[313] in 2015 proposed to combine the four versions of PSO, Global model PSO, Local model PSO, Bare bones PSO and Comprehensive Learning PSO, using the MPI to achieve high quality solutions as compared to serial versions of these four PSO variants.

Parallel PSO-Back Propagation Neural network based on MapReduce – Cao et al.[314] in 2016 proposed a parallel design realization method on PSO for optimizing BP neural network based on map-reduce on the Hadoop platform and PSO algorithm is used to optimization of the inertia weights and thresholds for the back propagation neural network.

Parallel Evolution of quantum behaved PSO- Tian et al.[315] in 2016 introduced the splitting of high dimension problem into sub-problems and get optimized individually with the intermittent communication resulting in high quality solutions.

Fine Grain Parallel PSO (FGPPSO) – Nedjah et al.[316] in 2017 proposed this technique for multi core and many core architectures along with serial implementation and the termination criterion was considered to be based on the accessibility of solution.

Parallel PSO for Multi core Environment – Abdullah et al.[317] in 2018 proposed Parallel PSO on multi core processing kernel to decrease the determination and transfer information easily among particles of shared area and to exchange information by random replacement strategy. This shared PSO technique is more effective than serial PSO and can avoid the reduction on test accuracy when applied to single core environment.
Adaptive Parallel PSO – Lai and Zhou [318] in 2018 suggested parallel PSO based on Osmosis and are capable to obtaining three parameters, such as migration interval, migration direction and migration rate which is helpful in determining the number of particles migrated from one sub population to another sub population.

B. GPU COMPUTING

In November 2006, NVIDIA a computer games corporation, launched a CUDA platform that enables programmers to write their own code using C programming language with NVIDIA extensions[319,320] and capable to compute big data parallel computations. GPU contains thousands of cores installed and the power of several CPUs in single processor. CUDA, OpenACC [321], Intel Xeon Phi bootable host processors, TPU [322], FPGA [323] are the GPU based parallelization approaches.

Fine-Grained Parallel PSO based on GPU-Acceleration – Li et al.[324] in 2007 first proposed the particles were mapped into textures on a graphics card and are calculated in parallel without Compute unified device architecture (CUDA) support and then implemented on CUDA.

GPU-based Parallel PSO – Zhou and Tan [325] in 2009 proposed a novel parallel way of running PSO on GPU based on the software platform of CUDA from NVIDIA [309]. The running speed of GPU is more than 1 times faster than the CPU and running time is also reduced. High dimension problems and large swarm population application are its special advantage in real optimizing problems.

Swarm’s Flight: Particle acceleration using C-CUDA - Veronese and Krohling [326] in 2009 suggested the implementation of PSO algorithm in C-CUDA, which showed high computing capabilities with lesser time on well-known bench marks as compared to C and MATLAB.

Parallel PSO based Particle filtering - Rymut and Kwolek [327] in 2010 proposed in their work that CUDA –capable GPU could accelerate object tracking algorithm using adaptive appearance models which performs with a factor of speed of 40 over CPU. The object tracking is done by PSO algorithm.

Evaluation of Parallel PSO within CUDA – Mussi et al.[328] in 2011 demonstrated a performance assessment of two versions of parallel algorithms with the sequential implementation of PSO over bench mark functions.

Collaborative multi-swarm PSO for task-making using GPU – Solomon et al. [329] in 2011 proposed collaborative multi-swarm PSO process on GPU utilizing multi-swarms instead of one, when applied to real world problems in a diverse distributed computing environment.

Parallelizing Euclidian PSO (pEPSO) in CUDA – Zhu et al.[330] in 2011 proposed this algorithm to use fine grain data parallelising to evaluate the fitness function with GPU for fast and better convergence.

Accelerating Parallel PSO via GPU (GPSO) – Hung and Wang [331] in 2012 suggested this approach by using thread pool model and implementing GPSO on a GPU. The GPU architecture with PSO approach significantly reduces the computational timing and high performance with good optimal results.

GPU based Parallel Co-operative PSO using C-CUDA – Kumar et al. [332] in 2013 suggested that computational time is reduced and huge computations would benefit from the GPU with Compute unified device architecture (CUDA). A thorough analysis of Parallel implementation of co-operative PSO and a comparative study on CPSO was performed in C and C-CUDA.

SIMT GPU Based PSO Approach - Awwad et al.[333] in 2013 suggested that CUDA GPU solution be computed to handle the topology problem and obtained a performance speed up factor of 392 over an implementation of the CPU for the large scale optimization problems.

CUDA based Co-operative Evolutionary multi-swarm optimizer – Souza et al.[334] in 2013 proposed this CUDA based technique to solve optimization problems. This method use master-slave swarm principle with the data sharing mechanism to accelerate of convergence.

Parallel GPU based implementation of High Dimension PSO – Calazan et al.[335] in 2013 proposed that each particle be implemented as a thread block and that each dimension be mapped onto a separate thread for a faster rate of convergence.

Parallel PSO – Chen et al. [336] in 2014 proposed an efficient PSO based technique to find optimal uniform designs with reference to the CCD requirement. A parallel computing technique based on the state of art graph processing unit (GPU) is used to accelerate computing.

FJSP based on CUDA Parallel Cellular PSO – Shenghui and Shuli [337] in 2014 proposed this algorithm by putting large number of GPU threads to each particle and using CA logic where each particle is taken as CA model. Calculation space is provided to each particle on respective thread and the number of threads in the GPU equals to the number of particles.

PSO Efficient implementation of GPU using Low Latency Memory – Silva and Filho [338] in 2015 developed this technique using the shared memory available on CUDA platform GPUs. Each dimension of every particle is identified as thread and implemented in parallel in GPU block with a maximum number of parallel threads allowed and use multiple sub-swarms. Each sub swarms Is executed in a GPU block with a aim of maximizing data alignments and avoiding instructions for bifurcations with two communication strategies and two topology.

Parallel PSO based on CUDA in the AWS Cloud – Li et al. [339] in 2015 proposed this algorithm to run all the operations in parallel for updating current position, velocity, best fitness and global fitness values. This algorithm speeds up 80 times as compared to PSO algorithm on CPU.

Parallel PSO approaches on GPU for constraint – Dali and Bouamama [340] in 2015 suggested two approaches to solve constraint problems, first one by parallel GPU-PSO for max-CSPs and the other by GPU distributed PSO for reducing the calculation time to explore the search space efficiently.

CUDA implementation on Standard PSO – Hussain et al.[341] in 2016 proposed the combined use of memory access, video RAM memory, which is more effective in simultaneously accessing memory through wrap threads for the standard PSO on the GPU based on CUDA and was found to be 46 times better than serial CPU.
Parallel PSO on GPU with trajectory optimization – Wu et al.[342] in 2016 presented the full implementation of PSO in parallel through GPU on CUDA platform and observed the impact of number of particles, dimensions, thread block size in the GPU and there interaction on computation time.

GPU-Based Parallel PSO for Graph Drawing – Que et al.[343] in 2017 developed two methods, one serial and then the other parallel, for undirected graph drawing. The serial PSO method was run on CPU with lesser time on small graphs whereas parallel PSO is run on GPU with lesser time on large graphs.

Adaptive PSO with heterogeneous multi-core parallelism and GPU acceleration – Wachowiak et al. [344] in 2017 adapted this parallelization method to heterogeneous parallel hardware containing multi-core technologies accelerated by GPU and Intel-Xeon Phi co-processors speeded up by vectorization. Task-parallel elements are performed with multi-core parallelism, and data-parallel components are run through GPU co-processing.

Gpu based Parallel Road Network method using PSO- Wan et al. [345] in 2018 developed this technique based on the basis of two stages feature such as computing and matching relationship identification using data partition and task partition methodologies, to fully utilize GPU threads. This method can easily handle massive data with good efficiency.

MS2 PSO – Tangerloni et al.[346] in 2018 proposed effective parallel and distributed execution of the PSO based Parallel Estimation (PA) for the estimation of biochemical system response constants. MS2 PSO is based on Master-Slave distributed computing in which master process during offloads time consuming calculations. Each Slave uses cup SODA which allows to run parallel on the cores of the GPU to calculate the fitness values for optimization.

Performance evaluation of PSO,GA based on GPU – Kawano et al.[347] in 2018 proposed to execute PSO,GA using the original code on the processor against the modified algorithm where the certain process of the algorithm are integrated on video card to compare the execution time. Also graphical interface was made for both algorithm to facilitate the process of handling the parameters.

Integrated motor optimization and Route planning for EV using GPU – Roberge et al.[348] in 2019 proposed PSO and the Bellman-Ford (BF) routing for minimizing energy consumption for electric vehicle (EV) and are implemented in CUDA. PSO is used to calculate magnetic flux settings for an Induction motor for various operating points and losses are also calculated prior to trip. BF is used to calculate optimized routes and Parento front of routes are prepared.

C.CLOUD COMPUTING USING PSO

Cloud computing technologies provide method to deal with massive data, delivering a flexible, pay-as-you-go for [349,350] and are needed for high performance complex application. Cloud computing lets user’s applications dynamically have as many computed services at specified locations (currently US east1 a-d for Amazon [351]) as and needed. Applications may select the storage centers to store their data (Amazon S3) [352] at global locations. These platforms are called Infrastructure as a service (IaaS), Platform as service (PaaS) and Software as a service (SaaS). Cloud computing has four layered architecture such as data centre layer, network layer, infrastructure layer and application layer. There are four cloud types such as Public cloud, Private cloud, hybrid cloud and community cloud. PSO has been widely used to solve all problems of cloud computing issues such as task scheduling, Energy optimization, Load balancing and workflow scheduling problems to get efficient solutions over the different virtual machines on the cloud environment.

PSO based heuristics for Scheduling workflow in cloud computing – Pandey et al.[353] in 2010 proposed to reduce the overall cost of implementing work flow on cloud computing systems by varying communication cost between resources and the cost of implementation of compute resources. The findings showed triple saving in cost as compared to Best Resource Selection (BRS) heuristic.

Discrete PSO in cloud workflow allocation – Wu et al. [354] in 2010 proposed that applications should be configured between cloud services providers by integrating cost of data transmission and processing cost at an optimum cost to user. Moreover this algorithm is not better for larger search space.

Set-based discrete PSO in cloud workflow scheduling – Chen and Zhang[355] in 2012 proposed to optimize the user defined Qos constraints such as make span, consumer cost and reliability on a standalone basis.

Sort based PSO in Cloud computing – Guo et al. [356] in 2012 implemented a small position value rule by sorting all the dimensions in place as per the actual value and giving an integer value to each dimension of the rank number and mapping this value to the cloud resource index . A single objective was formed by combining the time for data transmitting and the customer price.

PSO for energy aware virtual machine placement optimization – Wang et al.[357] in 2013 proposed for lowering the energy consumption of a virtualized data centres by means of virtual machine placement optimization while keeping in view the necessary requirements of cloud services. An improved version of PSO is used by reshaping parameters and operators, then adopting an energy aware fitness strategy and coding scheme.

Round based PSO in cloud computing – Rodriguez and Buyyua [358] in 2014 proposed to round the real number to integer number in order to demonstrate the resource index upon which the workflow was planned but does not indicate the characteristics of resources.

Energy efficient resource allocation of Virtual machine – Xiong and Xu [359] in 2014 suggested this algorithm by using Energy efficient resource allocation model and PSO method in cloud data centre to reduce the energy consumption. The fitness function of PSO is expressed as the total Euclidean distance to obtain the optimum point between resource allocation and energy usage.

Task based model load balancing using PSO (TBSLB-PSO) – Ramezani et al. [360] in 2014 suggested for system load balancing by migrating extra tasks from an overloaded virtual machine (VM) instead of shifting the whole VM overload. Instead PSO is used to move the extra tasks to the new VM hosts to reduce the downtime, expense and memory space involved with the process.

Renumber strategy enhanced PSO in cloud computing – Li et al. [361] in 2015 suggested a number technique using the metric price per unit time to record resources and thus make learning between the particles more effective.
Cloudlet scheduling with PSO – Al-Olimat et al.[362] in 2015 proposed a hybrid of PSO and Simulated Annealing is implemented inside the CloudSim is being used to minimize the makespan and maximize the resource usage.

Dynamic Power saving Resource allocation using PSO (DPRA) – Chou et al. [363] in 2018 suggested this mechanism based on PSO which consider the energy consideration of physical machine (PM) and virtual machine (VM) and also take care of energy efficiency of air-conditioners, total electricity bill, VM migration, and number of shut downs of VMs.

Quantum PSO (QPSO) Based Load Balancing – Sivakumar et al. [364] in 2019 proposed to decrease the traffic surrounded by the incoming requests to the server which is protected by firewalls, sends to the load balancer that acts as reverse substitute and distributes network transversely to servers. This algorithm consider data dependences in cloud environment and data intensive workflow features.

D. HYBRID PSO USING PARALLEL IMPLEMENTATION

Parallel hybrid Moving boundary PSO(hmp PSO) – Zhang et al. [365] in 2009 proposed that this algorithm consists of three components global bpsos, local bpsos and direct local search by Nelder-Mead method. The hardware for parallel implementation is a LINUX cluster consisting of 96 dual processor dual-core Operton. This hybrid model improves the efficiency and avoid premature convergence to local minima’s.

Parallel PSO with Genetic Migration – Jin and Lu [366] in 2012 proposed coarse grain parallel PSO on GPU and implemented genetic strategy for communication using selection, crossover and mutation operators on the particles and then after competition of relocation between swarms, new swarms are then run on PSO.

Comparison Parallel GA and PSO – Roberge et al. [367] in 2013 proposed the hybrid GA and PSO to shorten the time of execution for the solutions using the single programming. By using parallel implementation on multi-core CPUs, a real time path planning for UAV is possible with a quasi-linear speeds up of 7.3 to 8 cores with low execution time.

Hybrid method focused on neighbourhood search and PSO for parallel machine (VNPSO) – Chen et al.[368] in 2013 proposed this algorithm to multi stage problem and formulated as a mixed integer linear programming model. The algorithm describe both independent sequence and sequence dependent initialization time.

Parallel Co-operative Evolution based PSO (PCCPSO) – Yuan et al.[369] in 2015 proposed this technique to address the issue of conditional nonlinear optimal perturbation (CNOP) problem. A hybrid using improvement in PSO with Tabu search algorithm was used and then parallelizing was performed.

Spark based Parallel Co-operative Evolutionary PSO – Cao et al.[370] in 2016 introduced a hybrid algorithm by combing probability distribution functions (Gaussian, Cauchy and Lcvm distribution functions) with the global and local version of PSO and implemented in parallel on spark platform to solve high dimension problems.

Parallel Quantum based PSO with Neighbourhood search – Long et al.[371] in 2016 suggested to use global search and local search neighbourhood strategy in quantum behaved PSO and employ parallel strategy to minimize runtime and increased the diversity of the population.

Hybrid of Multi Swarm PSO and GA – Franz and Thulasiraman[372] in 2016 proposed this algorithm by parallelizing the hybrid algorithm on an accelerating processing unit (APU) which is a hybrid multi core computer to improve performance and close coupling between GPU and CPU.

Hybrid Iterative Trimmed singular value decomposition (TSVD) and Parallel PSO – Ge et al.[373] in 2016 suggested the latest inversion algorithm can achieve desirable results for signals with noise ratio larger than 10 could be obtained by reversing the relaxation time (T1) and transversal time (T2) range in a low field nuclear magnetic resonance to obtain optimum truncated location with high computational speed.

Multi-Core Parallel PSO (PPSO) – Peng et al.[374] in 2017 proposed three multi-core parallel PSO algorithms (PPSO ring, PPSO share) based on Fork/Join mechanism and congruence in Java for exchange of information among the threads (sub-swarms). Fork/Join mechanism allocates threads to separate CPU cores, while synchronization and communication mechanisms are used to exchange information between the threads.

Hybrid GA-PSO in Cloud Computing – Manasrah and Ali[375] in 2018 proposed to allocate the task to resources efficiently and to insure the fair distribution of the workload among the available virtual machines in order to reduce the makespan and the processing costs of the workflow applications with minimum time in the cloud computing environments.

Hybrid GA-PSO in Cloud Computing – Senthil et al. [376] in 2019 proposed to combine GA and PSO to minimize the execution time for task scheduling. Initially GA will randomly generate the population and encoding the chromosomes is done with mapping task and matched resources. Fitness is calculated and elite are divided into two halves. GA is applied to best elite first half. The consequence is anew population following the introduction of crossover and mutation. With PSO, pbest and gbest are evaluated with every iteration for particle position and velocity. Results are combined both of GA and PSO. Finally optimal solution results are sorted based on fitness values and global best.

6.5 MULTI-OBJECTIVE PARTICLE SWARM OPTIMIZATION USING PARALLEL IMPLEMENTATION

Parallel Vector evaluated PSO (VEPSO) – Vlachogiannis and Lee [377] in 2005 implemented this algorithm which contains equal number of objective functions and number of swarms with the same number of PCs working in parallel for solving the many-objective problems in short processing time with precise results.

Parallel PSO for multi-objective problems – Fan and Chan [378] in 2009 developed the idea based upon the concept of parent-dominance as well as parallel computing in which after standard PSO run, each swarm shares a set number of crowded member after the migration phase. Fixed number of non-dominated solutions were obtained by external archive that which continues to be modified updating after each process.

GPU based Parallel multi-objective PSO – Zhou and Tan [379] in 2011 firstly proposed this technique for optimizing parallel multi-objective optimization problems via PSO using the GPU is much more effective in run time and speed spectrum from 3.74 to 7.92 times as compared to CPU sequential platform.
MOPSO implemented in parallel on GPU using OpenCL and CUDA – Arun et al.[380] in 2011 implemented this technique on the popular GPU frameworks which results in 90% improvement in performance as compared to sequential implementation.

Multi-objective parallel PSO – Soares et al.[381] in 2013 proposed to solve the dual objective of Vehicle to grid scheduling by applying parallel computing parento weights to multi-objective parallel PSO.

Multi objective Parallel PSO-SA (P-PSOSA)- Khoshahval et al.[382] in 2014 proposed two separate fitness function were taking into account the maximization of multiplication factor maximizing and reduction of power peaking factor objectives simultaneously. thus achieving near global core pattern.

Parallel multi-objective PSO based on Decomposition – Li et al.[383] in 2015 proposed both MPI and OPENMP be used to apply the algorithm using a combination of distributed and shared memory programming models.

Weighted sum approach using Parallel PSO – Borges et al.[384] in 2016 implemented parallel PSO to the non-linear multi-objective combinational resources scheduling problem of distributed energy in which the weighted sum of two objectives forms a single objective function.

Workload Distributor with a Resource Allocator (WDRA) – Alsubhai and Gaudiot[385] in 2017 proposed to combine workload distribution, core scaling, and thread allocation into a many-objective optimization problem using PSO in order to minimize the execution time, energy consumption, under peak power and peak CPU temperature constraints.

Scalable Parallel co-operative Co-evolutionary PSO - Atashpender et al. [386] in 2018 firstly suggested this variant of speed constrained many-objective PSO and evaluating scalability in the terms of number of variables and parallelization. This method gives high computation speed ups and higher convergence speed with quality solutions.

Parallel Multi-objective PSO for large swarm and high dimensions (MOPSO) – Hussian and Fujimoto[387] in 2018 introduced parallel implementation of MOPSO on a GPU based CUDA architecture using coalescing memory access, pseudo random number generator, thrust library, atomic function, parallel archiving and so on. This implementation uses master slave model provide up to 182 times speedup as compared to CPU MOPSO.

Bi-Objective PSO – Varshney and Singh[388] in 2018 proposed this technique in which two swarms are used one for each objective such that information of one swarm has been used to update the velocity of the other and both swarm co-operate each other to get better optimal solutions and is better in terms of reliability and execution time in cloud computing environment.

Parallelized Multi-objective Cultural algorithm PSO (CAPSO) – Stanley et al. [389] in 2019 proposed a parallelized hybrid optimization system by combining elements from cultural algorithm (CA), PSO and Vector Evaluated Genetic algorithm (VEGA). This algorithm works by dividing the search space within multiple swarms joined by sharing of CA knowledge among themselves.


Fig 2. Pie chart represents the distribution of the published research papers using PSO Parallel Implementation.
VII. Conclusion and future directions

PSO is computationally an intensive method and suffers with long run time while solving real world large optimization problems. From the initial introduction of this algorithm this has been recognised that the key drawback of this technique is its ability to converge premature or stagnation. Many strategies were adopted to solve this issue by controlling the velocity of particles, using the probability to achieve global minima, reward/punish penalty, hybrid algorithms, etc. to overcome this problem without affecting the exploration of search space. The review article focus on PSO based algorithmic approach, communication topologies and parameter setting based approaches, hybridized approaches and multi-objective approach for its robustness and efficient for solving large optimization problems. For the present review paper 1050 research papers were studied from IEEE Journals, IEEE Transactions on Evolutionary Computation, Nature Computing, Soft computing, IEEE Access, Springer Nature, IEEE Transactions on Parallel Distribution, Proceedings of the IEEE International Conference on Control and Automation, Int. IEEE Conf. on Systems, Man, and Cyber; IEEE International Conference on Machine Learning and Cyber, Neuro Computing, Journal of Innovative Computing, Applied soft computing, IEEE Latin America Transactions, IEEE Symposium Series on Computational Intelligence (SSCI), : IEEE Symposium Series on Computational Intelligence, International journal of Communication Technologies and Applications, The Scientific World Journal, and more. In the review we found that search effectiveness and convergence speed and run time are influenced by different strategy. This paper gives a through survey with more emphasis from its development, improvements from its basic form, different techniques derived from this algorithm are introduced. A number of researchers motivated PSO as single objective optimizer and extended it to multi-objective and constrained optimization. The review will help the researchers to pick the correct constraint handling technique for real-life optimization issues.

The publication chronological review of parallel PSO based on the parallelization strategy suggests descending order strategy as MPI, GPU, Multi-core, OpenMP, Hadoop and MATLAB and then cloud computing. Literature survey on communication based suggests master-slave is most popular parallelization approach then coarse grained and finally fine grained approach. Hybrid approaches has little share as compared parallel PSO parallelization strategy. Multi-objective using parallelization strategy has also low share but is very effective in the real life optimization problems as this reduces computational time significantly. After doing intensive study on PSO, it is suggested for future directions for detailed theoretical analysis of existing and new parameters, exploration, particle diversity, premature convergence and application based modeling with single smart architecture.

REFERENCES:


74. [74] J. Kennedy, Bare bones particle swarms, in Proceedings of the IEEE Swarm Intelligence Symposium (SIS ’03), pp. 80–87, 2003, Indianapolis, Ind, USA


Meissner, M., Schmucker, M. and Schneider, G. Optimized Particle Swarm Optimization (OPSO) and its application to artificial neural network training, BMC Bioinformatics, 7:125, 2006.


McNabb, A., W., Monson, C., K. and Seppi, K., D, MRPSO: Map Reduce particle swarm optimization, Proc. of the 9th annual Conf. on Genetic and evolutionary Comput., pp. 177 -177, 2007.


137. [137] Hui Wang, Yong Liu, Sanyou Zeng , Hui Li and Changhe Li, Opposition based Particle Swarm Algorithm with Cauchy Mutation, IEEE, 2007.


247.[247] Lejun Ma, Huan Wang, Baohong Lu, Changjun Qi, Application of strongly constrained space Particle swarm optimization to optimal operation of a Reservoir system, Sustainability 2018,10,4445, 2018.

293.[293] Venter, G., Sobieszczanski-Sobieski, J, A Parallel Particle Swarm Optimization algorithm accelerated by asynchronous evaluations, 6th world congresses of structural and multidisciplinary optimization, 2005.
Daolio, F., Cagnoni, S., Shi, H.; Jiao, L.

Parallel GPU Research on FJSP based on CUDA parallel cellular computing, Omar Ibrahim Al Saif, 2009.


nVIDIA , nVIDIA CUDA Programming Guide v.2.3. nVIDIA Corporation, Santa Clara, 2009.


Parallel particle swarm optimization with QPSO based Load balancing mechanism in Cloud environment, 2016.


Yunkio Kawano,Fevrier Valdez, Oscar Castillo, Performance evaluation of optimization algorithms based on GPU using CUDA Architecture, IEEE Latin American conference on Computational Intelligence, 2018, Mexio


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