EFFECT OF PLASMA PARTICLE DENSITIES WITH KAPPA DISTRIBUTION FUNCTION ON EMIC WAVES

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Abstract:- Electromagnetic ion-cyclotron (EMIC) waves have been studied by single particle approach. The effect of kappa distribution function on EMIC waves at different plasma particle densities, the dispersion relation, growth length, growth rate, parallel and perpendicular resonance energy of electromagnetic ion cyclotron waves in low β (ratio of plasma pressure to magnetic pressure) homogeneous plasma have been obtained. The effect of steepness of the kappa distribution and density has been studied of the ions. In these studies, we have presented some previous work but different investigation of kappa distribution function on EMIC waves in cold plasma. The dispersion relation, particle densities, growth rate and growth length of the EMIC waves are derived The wave is assumed to propagate parallel to the static magnetic field. The whole plasma is considered to consist of resonant and non-resonant particles. It is assumed that resonant particles participate in energy exchange with the wave whereas non-resonant particles participate in the oscillatory motion of the wave. The effect of kappa distribution function with plasma densities is to enhance the wave growth of EMIC waves with ions. The results are interpreted for the space plasma parameters appropriate to the auroral acceleration region of the earth magneto plasma.

Keywords: Electromagnetic ion cyclotron waves, Kappa distribution function, auroral acceleration region, Particle aspect approach.

I. Introduction

Electromagnetic ion cyclotron (EMIC) waves are transverse, low frequency (below the proton cyclotron frequency) waves in the range of 0.1-5Hz are seen on the ground as pc1-pc2 pulsations.¹ Electromagnetic ion cyclotron waves generated in the equatorial region of earth's magnetosphere as left-handed circularly polarized waves propagate field line guided towards the ionosphere.² Experiment evidence for naturally occurring ion cyclotron instability has been summarized by Cornwall³. It has been clearly pointed out that both particle precipitation and wave generation are observed to occur preferentially in the region of high density of cold electrons. Cuperman and Gombroff have stated that a number of authors have pointed out that the generation of electromagnetic ion cyclotron waves (EMIC) waves would pitch-angle scattered protons, subsequently reducing their fluxes.⁴ Recently work on electromagnetic ion cyclotron waves (EMIC) with ions plasma have been studied using particle aspect <u>analysis</u> and kappa distribution function with plasma densities. The EMIC wave propagation and the wave polarization have been determined using tri axial magnetic field data from the CRRES satellite, together with two orthogonal components of the wave electric field [Fraser]⁵. In the auroral acceleration region, heating and acceleration perpendicular to the magnetic field is a common feature in the auroral acceleration region. Jordanova have stated that ring current ion distribution are often unstable and can generate different classes of plasma wave like magneto sonic, Electromagnetic ion cyclotron (EMIC) waves etc in the equatorial magnetosphere. An energetic particle and a plasma wave in pitch angle scattering and energy exchange between the particle and wave occurs with the subsequent interaction.⁶

Electromagnetic ion cyclotron instability in the presence of cold plasma are discussed by Cornwall & Schulz and gave analytical expression for the growth rates as a function of wave frequency ω .⁷ An estimation provides a low plasma electron density of 40 cm⁻³ wave emission were confined to the following two frequency band, 1- low frequency wave below the local oxygen gyro frequency ($\Omega_0^+/2\pi=25Hz$), with a maximum activity at 8Hz and Local helium ($\Omega_{He}^+/2\pi=100Hz$), and hydrogen gyro frequencies peaked at above 130Hz.⁸

In the recovery phase of the storm, the plasma pause moves out of $1 \sim 6R_E(R_E \text{ is earth radius})$ in the recovery phase of the storm and the plasma pause is filled by ambipolar diffusion of cold ionosphere plasma. Thus the higher density plasma causing EMIC wave generation and proton precipitation envelops the stably trapped outer ring current proton.⁴ In addition, enhanced cold plasma density reduces the proton resonant energy to the energy range of the ring current where higher fluxes are available for resonance. The super thermal populations are well described by the so-called kappa (k) or generalized velocity distribution function (VDFs) as shown for the first time by vasyliunas (1968). Such Distribution have high energy tails deviated from a Maxwellian and decreasing as a power law in particle speed.⁹

The advantage of present approach and study is to its suitability for dealing and determining the effect of kappa distribution on EMIC waves in magnetosphere and determine the property of the plasma in magnetosphere. Electromagnetic ion cyclotron (EMIC) waves are able to resonate with magneto spheric ions and electrons over a broad range of energies. The interactions lead to particle pitch-angle scattering and energy diffusion. Electromagnetic ion cyclotron waves excitation source is provided by the anisotropic pitch-angle distribution of ring current ions. Observations show that wave growth is favored at higher L-shell regions where the

ratio of plasma to cyclotron frequency is larger [Anderson 1992]¹⁰. The aim of this study is to investigate the generation of EMIC waves in magnetosphere and see the effect of kappa distribution in magnetosphere plasma.

II. Basic Trajectories

Taking the particle trajectory in the presence of EMIC waves, the dispersion relation, particle perturbed densities, energy, growth rate and growth length is derived for different kappa distribution indicesk.

The left handed circularly polarized EMIC wave in cold magnetized plasma having angular frequency ω is defined as.¹¹

When the system is commoving with the wave, the electric field vanishes. Thus the wave magnetic field has the form, $B = B \left[\cos K + z \right] x + B \left[\sin K + z \right] y$

$$\boldsymbol{B} = \boldsymbol{B}_{x} [\cos \boldsymbol{K}_{\Pi} \boldsymbol{z}] \boldsymbol{x} + \boldsymbol{B}_{y} [\sin \boldsymbol{K}_{\Pi} \boldsymbol{z}] \boldsymbol{y}$$
(1)

Where

$$B_{x} = B\cos(K_{\Pi}z - \omega t)$$

$$B_{y} = B\sin(K_{\Pi}z - \omega t)$$
(2)

Where the following conditions are valid,

$$Z^{wave} = Z^{lab} - (\mathscr{O}_{K_{\Pi}})t$$

$$V^{wave} = V^{lab} - (\mathscr{O}_{K_{\Pi}})$$
(3)

Thus the equation of ion motion in the wave is given as,

$$\frac{dv_i}{dt} = \frac{q_i}{m_i c} [(V_i \times B_0) + (V_i \times B)]$$
⁽⁴⁾

We take the cylindrical variables in velocity space as

$$egin{aligned} V_{ix} &= V_{\perp i} \cos \phi \ V_{iy} &= V_{\perp i} \sin \phi \ V_{iz} &= V_{\Pi i} \end{aligned}$$

Then the equation of motion is written as,

$$V_{\perp i} = V_{\Pi i0} + \delta V_{\perp i}$$
$$V_{\Pi i} = V_{\Pi i0} + \delta V_{\Pi i}$$
(5)

Where $V_{\Pi i}$ initial values at t=0, Substituting eq. (4) in eq. (5) we find the following set of equations,

$$\delta V_{\perp i} = \frac{h V_{\Pi} \left[h \Omega_i \left(V_{\Pi i 0} - \frac{\omega}{K_{\Pi}} \right) \right]}{[K_{\Pi} V_{\Pi i 0} - (\omega - \Omega_i)]} \times [\cos(K_{\Pi} z - \omega t - \psi) - \varepsilon \cos(K_{\Pi} z - \omega t - \psi - (K_{\Pi} V_{\Pi i 0} - (\omega - \Omega_i))t)]$$

$$\delta V_{\Pi i} = \frac{-hV_{\perp 0}\Omega_i}{[\kappa V_{\Pi} - (\omega - \Omega)]} \times [\cos(K_{\Pi}z - \omega t - \psi) - \varepsilon \cos(K_{\Pi}z - \omega t - \psi - (K_{\Pi}V_{\Pi i0} - (\omega - \Omega_i))t)]$$

$$(6)$$

$$(6)$$

$$(7)$$

Where $Z = Z_0 + V_{\Pi}$ t and $\psi = \psi_0 - \omega t$ and $\mathcal{E}=0$ for the non-resonant and resonant particles.

III. Density variation

We evaluate the density perturbation associated with the particle density, To determine the dispersion relation, Growth rate and growth length with consider a kappa distribution and density distribution as.¹¹

$$n_{i} = \frac{hV_{\perp i}\Omega_{i}K_{\Pi}F_{\kappa}(V_{\Pi i})}{[K_{\Pi}V_{\Pi i} - (\omega - \Omega_{i})]} \times \left[\cos x - \varepsilon \cos x_{0} + \varepsilon t \wedge \sin(x - \wedge t)\right]$$
(8)

Where

$$x = K_{\Pi} Z - \omega t - \Psi, \land = \left(K_{\Pi} V_{\Pi} - \left(\omega - \Omega_i \right) \right)$$

IV. Distribution Function:-

To investigate resonant and non-resonant energies, growth rate and growth length, we chose the kappa distribution function of the form as.¹²

$$F_{\kappa}(V_{i}) = \frac{1}{\pi^{3/2} V_{\perp i}^{2} V_{\perp i}^{2} V_{\perp i}^{2} \frac{\Gamma(\kappa+1)}{\kappa^{3/2} \Gamma(\kappa-\frac{1}{2})} \times \left(1 + \frac{V_{\Pi i}^{2}}{k V_{\Pi i}^{2}} + \frac{V_{\perp i}^{2}}{k V_{\perp i}^{2}}\right)^{-\kappa-1}$$
(9)

Where, κ is the distribution index.

The distribution function of bi-kappa is used as;

$$F_{\kappa}(V_{\Pi}) = \frac{1}{\pi^{\frac{1}{2}}} \frac{\Gamma(\kappa+1)}{\kappa^{\frac{3}{2}} \Gamma(\kappa-\frac{1}{2}) V_{T \amalg i}^{2}} \times [1 + \frac{V_{\Pi i}^{2} (\omega - \Omega_{i})^{2}}{K_{\Pi} V_{T \Pi i}^{2}}]^{-(\kappa+1)}$$
(10)

In equation (8) $V_{T\perp i}$ and $V_{T \Pi i}$ are thermal velocity.

$$V_{T\perp i}^{2} = \left[\frac{\kappa - \frac{3}{2}}{\kappa} \frac{2T_{\perp i}}{m_{i}}\right]$$

and

$$V_{T \amalg i}^{2} = \left[\frac{\kappa - \frac{3}{2}}{k} \frac{2T_{\amalg i}}{m_{i}}\right]$$

The kappa distribution function;

$$z_{\kappa}(\xi) = \frac{1}{\pi^{\frac{1}{2}}\kappa^{\frac{1}{2}}} \frac{\Gamma(\kappa+1)}{\Gamma(\kappa-\frac{1}{2})} \int_{-\infty}^{\infty} \frac{dx(1+\frac{x^{2}}{k})^{-\kappa}}{(x-\xi_{\kappa})}$$

Where

 $\xi = \left(rac{\omega - \Omega_i}{K_{\Pi}V_{T\Pi i}}
ight)$

V. Dispersion Relation

We consider the cold plasma dispersion relation for the EMIC wave as.¹³

$$\frac{c^2 k_{\Pi}^2}{\omega^2} = 1 + \frac{4\pi N_0 e^2}{m_i \Omega_i \left(\Omega_i - \omega\right)}$$
(11)

Where

$$\omega_{pi}^2 = \frac{4\pi N_0 e^2}{m_i}$$

The dispersion relation for electromagnetic ion cyclotron waves propagating in the direction of the external magnetic field in the system consisting of ion, electron and the non-drifting of ion, consisting of resonant and non resonant particles participate in the energy exchange with the wave. The cold plasma dispersion relation approximation is also a good approximation to the warm plasma dispersion relation, provided that plasma $CK/\omega >> 1$

VI. Wave energy for EMIC waves by kappa distribution function as:

The perpendicular resonant energy and parallel resonant energy of the resonant ions are calculated by help of basic equation as.¹⁴ **6.1 Perpendicular resonant energy**

$$W_{r\perp i} = \frac{\pi^{\frac{3}{2}}B^{2}}{C^{2}K_{\Pi}^{2}\omega K_{\Pi}^{2}} \left[\frac{\Gamma(\kappa+1)}{\kappa^{\frac{3}{2}}\Gamma(\kappa-\frac{1}{2})} V_{T\Pi I}} \frac{4\pi N_{0}e^{2}}{m_{i}} \frac{T_{\perp i}}{T_{\Pi i}} \left(\frac{\omega-\Omega_{i}}{\Omega_{i}}\right) + 1 \right] \left[1 + \frac{(\omega-\Omega_{i})^{2}}{K_{\Pi}V_{T\Pi i}^{2}} \right]^{-\kappa-1}$$
(12)

6.2 Parallel resonant energy

$$W_{r\Pi i} = \frac{\pi^{\frac{3}{2}}B^{2}}{C^{2}K_{\Pi}^{2}\omega K_{\Pi}^{2}} \left[\frac{\Gamma(\kappa+1)}{\kappa^{\frac{3}{2}}\Gamma(\kappa-\frac{1}{2})} \frac{4\pi N_{0}e^{2}}{m_{i}} \frac{T_{\perp i}}{T_{\Pi i}} \left(\frac{\omega-\Omega_{i}}{\Omega_{i}}\right)^{2} \right] \left[1 + \frac{(\omega-\Omega_{i})^{2}}{K_{\Pi}V_{T\Pi i}^{2}} \right]^{-\kappa-1}$$
(13)

VII. Growth Rate for EMIC waves

The growth rate is obtained as-

$$\frac{\gamma}{\omega_{i}} = \frac{\frac{\Omega_{i}\pi^{\frac{3}{2}}}{K_{\Pi}V_{T\Pi i}} \left[\frac{\Gamma(\kappa+1)}{\kappa^{\frac{3}{2}}\Gamma(\kappa-\frac{1}{2})} \times \left(1-\frac{\omega}{\Omega_{i}}\right) \frac{T_{\perp i}}{T_{\Pi i}} - 1 \right] \times \left[1+\left(\frac{\omega-\Omega_{i}}{K_{\Pi}V_{T\Pi i}}\right)^{2}\right]^{-\kappa-1}}{\left(\frac{CK_{\Pi}m_{i}}{4\pi N_{0}e^{2}}\right)^{2} \left(\frac{2\Omega_{i}-\omega}{\Omega_{i}-\omega}\right) + \frac{1}{2}\frac{\omega^{2}}{(\Omega_{i}-\omega)^{2}}}$$
(14)

Here it is noticed that κ -Lorentz distribution has affected the growth rate through plasma densities and change in the energy for the electromagnetic waves propagating parallel to the ambient magnetic field. The κ -Lorentz distribution has been introduced as more suitable for modeling magnetized.

VIII. Growth Length for EMIC waves

The formula for the growth length is obtained as-

$$L_{g} = \frac{1}{\gamma \omega_{pi}^{2}} \left(-C^{2} K_{\Pi} \Omega_{i} + \frac{C^{4} K_{\Pi}^{3} + 2\omega_{pi}^{2} C^{2} K_{\Pi} \Omega_{i}}{\sqrt{C^{4} K_{\Pi}^{4} + 4\omega_{pi}^{2} C^{2} K_{\Pi}^{2} \Omega_{i}}} \right)$$
(15)

Here, it is noticed that κ has affected the growth length through the plasma particle densities as discussed for the electromagnetic wave propagating parallel to the magnetic field.

IX. Results and discussion

Following plasma parameter are used which are relevant for the auroral acceleration region for the numerical evaluation of the dispersion relation, resonant energies and growth rate with the steepness of kappa distribution function. The characteristic of the EMIC waves, we have used the parameter of auroral acceleration region.¹⁴

B₀=4300nT, Ω_i=412sec⁻¹ ,V_{TII}=2x10⁹cm/
T_⊥/T_{IIi}=70 ,K_{II}=1-10x10⁻⁸cm⁻¹ ,n₀=1-5 cm⁻³

$$ω_{pi}^2 = 1.732 \times 10^6 \text{ sec}^{-2}$$

value of κ increases, then growth rate decreases low, at higher value of κ , the wave growth rate has decreasing according to increasing of κ , which may be due to the wave particle.



Fig.1(a) Variation of growth rate (γ/ω) versus wave vector $K_{II}x10^{-8}$ cm⁻¹ for different values of particle density n at distribution indices $\kappa=2$. Fig.1(b) Variation of growth rate (γ/ω) versus wave vector $K_{II}x10^{-8}$ cm⁻¹ for different values of particle density n at distribution indices $\kappa=4$.

Figure 1(a). Shows the variation of growth rate (γ/ω) versus wave vector $K_{II}x10^{-8}$ cm⁻¹ for different value of plasma particle density n=1,2,3,4 at kappa distribution indices κ =2. Here it is observed that increasing the value of particle density (n) as increases the growth rate of EMIC waves with wave vector K_{II} . Thus the steep kappa distribution enhanced the wave emission of EMIC waves in cold magnetized plasma. Figure 1(b) shows the variation of growth/damping rate versus wave vector K_{II} cm⁻¹ for different value of plasma particle density n=1,2,3,4 at kappa distribution indices κ =4. Here it is observed that the effect of increasing value of plasma particle density is to increases the growth rate. The effect becomes maximum at high density.



Fig. 2(a) Variation of growth Length (L_g) versus wave vector $K_{II}x10^{-8}$ cm⁻¹ for different value of densities n, at distribution indices κ =2, and temperature anisotropy= 70.

Fig 2(b) shows the variation of growth length (Lg)

Versus wave vector $K_{II} \times 10^{-8} \text{ cm}^{-1}$ for different value of densities n, at distribution indices $\kappa = 4$, and temperature anisotropy=70.

Figure 2(a) shows the variation of growth length (L_g earth radius) versus vector $K_{II}x10^{-8}cm^{-1}$ for different plasma particle density n=1,2,3,4 at the kappa distribution indices $\kappa=2$. In this graph we see that at high density growth length is less and by decreasing plasma particle density n as the growth length is increases. At low plasma particle density, the growth length is high. Figure 2(b) shows the variation of growth length (L_g earth radius) versus vector $K_{II}x10^{-8}cm^{-1}$ for different plasma particle density n=1,2,3,4 at the kappa distribution indices $\kappa=4$. In this graph also we see that at high density growth length is less and by decreasing plasma particle density n as the growth length is increases. At low plasma particle density growth length is less and by decreasing plasma particle density n as the growth length is increases. At low plasma particle density, the growth length is high.



Fig.3(a) Variation of perpendicular resonant energy versus wave vector K_{II} (cm⁻¹) at kappa distribution indices $\kappa=2$ for different densities n=1,2,3,4. Fig.3(b)Variation of perpendicular resonant energy versus wave vector K_{II} (cm⁻¹) at kappa distribution indices $\kappa=4$ for different densities n=1,2,3,4.

Fig. 3(a) shows the variation of perpendicular resonant (W_{rII} erg cm) versus wave vector ($K_{\perp}x10^{-8}$ cm⁻¹) for different plasma particle density n=1,2,3,4 at kappa distribution indices κ =2. It is observed that at lower plasma particle density n, perpendicular resonant energy is high and at higher density, perpendicular resonant energy is low. Fig. 3(b) shows the variation of perpendicular resonant energy (W_{rII} erg cm) versus wave vector ($K_{\perp}x10^{-8}$ cm⁻¹) for different plasma particle density n=1,2,3,4 at kappa distribution indices κ =4. It is observed that at lower plasma particle density n, perpendicular resonant energy is high and at higher density, perpendicular resonant energy is high and at higher density, perpendicular resonant energy is high and at higher density, perpendicular resonant energy is high and at higher density, perpendicular resonant energy is high and at higher density, perpendicular resonant energy is high and at higher density, perpendicular resonant energy is high and at higher density, perpendicular resonant energy is low.



Fig. 4(a) Variation of parallel resonant energy versus wave vector $K_{II} \times 10^{-8} \text{ cm}^{-1}$ at distribution indices $\kappa=2$ for different densities n and T=70. **Fig. 4(a)** Variation of parallel resonant energy versus wave vector $K_{II} \times 10^{-8} \text{ cm}^{-1}$ at distribution indices $\kappa=4$ for different densities n and T=70.

Fig. 4(a) show the variation of parallel resonant energy (W_{rII} erg cm) versus wave vector ($K_{II}x10^{-8}$ cm⁻¹) for different plasma particle density (n) at kappa distribution indices κ =2. Fig. 4(b) show the variation of parallel resonant energy (W_{rII} erg cm) versus wave vector ($K_{II}x10^{-8}$ cm⁻¹) for different plasma particle density (n) at kappa distribution indices κ =4. and at temperature anisotropy 70. Here it is observed that the effect of increasing the value of densities n, it is increases the parallel resonant energy W_{rII} for the particular value of κ =2 and κ =4. The kappa distribution with plasma densities effect of the magnetosphere enhance the parallel resonant energy by the EMIC wave in ion plasma. The heating is much more efficient when part of the wave energy is transfer at the ion-cyclotron frequency since the waves and particles can then interact resonantly [Lund 1999]¹⁵

Conclusions

In this paper we have studied the effect of kappa distribution function in the auroral acceleration region to discuss EMIC wave's emission. It is found that the effect of increasing the value off kappa distribution function to enhance the growth rate, growth length and resonant energy of the EMIC waves and also density. Electromagnetic ion cyclotron waves behavior studies for may be of importance in electromagnetic emission in the auroral acceleration region. The transversely accelerated ion's and their association with the EMIC waves in the auroral acceleration region have been recently reported by various workers in the analysis of FAST satellite.

Reference

- 1. Hajime Sugiyama, Satyavir Singh it al., J. Geophysical Research , 10.1002/2015JA021346.
- 2. G. Ahirwar, P Varma and M S Tiwari, Ind. J. of pure & Appl. Phys. 48, (2010).
- 3. J.M. Cornwall, H H Hilton and P F Mizera J. Geophys. Res.76 5220 (1971b)
- 4. S. Cuperman and L Gomberoff J. Plasma Phys. 18 391 (1977)
- 5. B.J. Fraser, singer, H.J., Hughes W. J., Wygant W. J., Anderson, R.R., Hu Y.D., 1996. J. Geophy. Res. 90, 347.
- 6. V.K. Jordanova, C J Farrugia, R M Thorne, G V Khazanov et, al., J.Geophys. Res. 1067 (2001).
- 7. J.M. Cornwall and M Schulz J. Geophys. Res. 76 7791 (1971).
- 8. O. Santolik, and M. Parrot, J. Geophy. Res. 101, 10639 (1996)
- 9. V.M. Vasyliunas, V Pierrard, M Lazar-2010 J. Geophy. Res. 73. 2839 (1968)
- B.J. Anderson, R. E. Erlandson, and L. J. Zanetti, J. Geophys. Res., 97(A3), 3089-3101,doi:10.1029/91JA02697, (1992).
- 11. K.D. Misra and M S Tiwari can. J. Phys. 57 1124 (1979).
- 12. G. Ahirwar . Internation journal of Sci. and Res. Publication, 2, 2250-3153 (2012).
- 13. C.F. Kennel C. F. and H E Petschek J. Geophys. 711 (1967).
- 14. G. Ahirwar, P Varma and M S Tiwari, Indian J. Phys. 80 (12), 1179-1187(2006).
- 15. E.J. Lund, Mobius, E., Klumpar, et al., 1999. Adv. Space Res. 23(10), 1721.