

APPLICATION TO FUZZY ASSIGNMENT PROBLEM FOR MAXIMUM AND MINIMUM AMOUNT OF RAINFALL IN FOUR DIFFERENT COUNTRIES IN FOUR CONSECUTIVE YEARS

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ABSTRACT: In this paper fuzzy assignment problem is solved for highest and lowest amount rainfall in four countries for four consecutive years. The problem is converted into crisp assignment by p-norm ranking and later solved by Hungarian method for minimizing and Method of one's assignment for maximizing.

Keywords: Fuzzy Assignment Problem, MOA method

1. INTRODUCTION:

An assignment problem which is special type of linear programming is a well-studied optimization problem in management science. The main objective of this is to assign given number of persons to equal number of jobs on one to one basis in such way so as to minimize total cost of performing that task or to maximize the total profit of allocation. In the year 1955, most widely used Hungarian method for solving assignment problem was developed by Harold W. Kuhn[15]. Fuzzy set theory was introduced by L.A. Zadeh[16] in the year 1965 in order to handle ambiguity. In 1985, the fuzzy assignment problem is proposed by Chen[13]. In 2012, Hadi Basirzadeh[2] introduced a new approach to APs namely, Ones Assignment Method, for solving a wide range of such problems.

This problem is organized as follows: In section 2 we review the basic definitions. In section 3 solving the fuzzy assignment problem using the Hungarian method is explained. In section 4 an other method called Method on One's Assignment (MOA) is explained. In section 5 the results between Hungarian method for minimization and Method on One's Assignment (MOA) for maximization is discussed with a numerical example.

2. PRELIMINARIES:

In this section some basic definitions are presented

Definition 2.1[4]: The characteristic function μ_A of a crisp set. A is contained or equal to X assigns a value either 0 or 1 to each member in X. This function can be generalized to a function $\mu_{\tilde{A}}$ such that the value assigned to the element of the universal set X fall within a specified range. i.e. $\mu_{\tilde{A}}: X \rightarrow [0,1]$. The assigned value indicates the membership grade of the element in the set A. The function $\mu_{\tilde{A}}$ is called the membership function and the set $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) : x \in X\}$ defined by $\mu_{\tilde{A}}$ for each X is called a fuzzy set.

Definition 2.2[4]: A fuzzy number $\tilde{A} = (a, b, c, d)$ is said to be a trapezoidal fuzzy number if its membership function is given by

$$\mu_{\tilde{A}} = \begin{cases} 0 & , x \in (-\infty, a] \cup [d, \infty) \\ \frac{x-a}{b-a} & , a \leq x \leq b \\ 1 & , b \leq x \leq c \\ \frac{x-d}{c-d} & , c \leq x \leq d \end{cases}$$

Definition 2.3[1]: A fuzzy set $\tilde{A}$ defined on the Universal set of real numbers $\mathbb{R}$ is said to be generalized trapezoidal fuzzy number if its membership function has the following characteristics:

1. $\mu_{\tilde{A}}: \mathbb{R} \rightarrow [0, w]$ is continuous.
2. $\mu_{\tilde{A}}(x) = 0$ for all $x \in (-\infty, a] \cup [d, \infty)$
3. $\mu_{\tilde{A}}$ is strictly increasing on $[a, b]$ and strictly decreasing on $[c, d]$.
4. $\mu_{\tilde{A}}(x) = w$ for all $x \in [b, c]$ where $0 < w \leq 1$.

Definition 2.4[12]: A generalized trapezoidal fuzzy number $\tilde{A} = (a, b, c, d, w)$ is said to be a non-normal trapezoidal fuzzy number if its membership function is given by

$$\mu_{\tilde{A}} = \begin{cases} 0 & , x \in (-\infty, a] \cup [d, \infty) \\ w \left(\frac{x-b}{a-b} \right) & , a \leq x \leq b \\ w & , b \leq x \leq c \\ w \left(\frac{x-c}{d-c} \right) & , c \leq x \leq d \end{cases}$$

The generalized trapezoidal fuzzy number is said to be a normal trapezoidal fuzzy number if $w = 1$.

Definition 2.5[13]: A non-normal fuzzy number $\mu_A = (a, b, c, d, w)_p$ is said to be a non-normal p-norm trapezoidal fuzzy number if its membership function is given by

$$\mu_{\tilde{A}} = \begin{cases} 0 & , x \in (-\infty, a] \cup [d, \infty) \\ w[1 - (\frac{x-b}{a-b})^p]^{\frac{1}{p}} & , a \leq x \leq b \\ w & , b \leq x \leq c \\ w[1 - (\frac{x-c}{d-c})^p]^{\frac{1}{p}} & , c \leq x \leq d \end{cases}$$

Where p is a positive integer

The left and right inverse functions $\mu_{\tilde{A}}^{-1}(x)$ are

$$L_{\tilde{A}}^{-1}(y) = b + (a-b)[1 - (\frac{y}{w})^p]^{\frac{1}{p}} \quad , 0 \leq y \leq w$$

And

$$R_{\tilde{A}}^{-1}(y) = c + (d-c)[1 - (\frac{y}{w})^p]^{\frac{1}{p}} \quad , 0 \leq y \leq w$$

2.2: Arithmetic Operations: [1]

Let $\tilde{A}_1 = (a_1, b_1, c_1, d_1; w_1)_p$ and $\tilde{A}_2 = (a_2, b_2, c_2, d_2; w_2)_p$ be two non-normal p-norm trapezoidal fuzzy numbers defined on the universal set of real numbers $\mathbb{R}$. Then the arithmetic operations between $\tilde{A}_1$ and $\tilde{A}_2$ are

- $\tilde{A}_1 \oplus \tilde{A}_2 = (a_1 + a_2, b_1 + b_2, c_1 + c_2, d_1 + d_2; w)_p$
- $\tilde{A}_1 \ominus \tilde{A}_2 = (a_1 - a_2, b_1 - b_2, c_1 - c_2, d_1 - d_2; w)_p$
- $\tilde{A}_1 \otimes \tilde{A}_2 = (a_1 \times a_2, b_1 \times b_2, c_1 \times c_2, d_1 \times d_2; w)_p$
- $\tilde{A}_1 \oslash \tilde{A}_2 = (a_1/d_2, b_1/c_2, c_1/b_2, d_1/a_2; w)_p$; where $w = \min(w_1, w_2)$
- $\lambda \tilde{A}_1 = \begin{cases} (\lambda a_1, \lambda b_1, \lambda c_1, \lambda d_1; w_1)_p & , \lambda \geq 0 \\ (\lambda d_1, \lambda c_1, \lambda b_1, \lambda a_1; w_1)_p & , \lambda < 0 \end{cases}$

While considering more than two non-normal p-norm trapezoidal fuzzy numbers let $w = \min(w_1, w_2)$.

2.3: Ranking function: [1]:

As fuzzy numbers are represented by possibility distributions, they may overlap with each other and hence it is not possible to order them. It is true that fuzzy numbers are frequently partial order and cannot be compared like real numbers. An efficient approach for comparing fuzzy numbers is by the use of a ranking function. A ranking function is a map from the set of fuzzy numbers $F(\mathbb{R})$ into a real line.

The Ranking function is:

$$d = \frac{w}{2} \left\{ (a - b - c + d) \left(\frac{r^{\left(\frac{1}{p}+1\right)\left(1/p\right)}}{pr^{\frac{2}{p}+1}} \right) + (b + c) \right\}$$

The ranking in $F(\mathbb{R})$ is defined as follows:

- $d(\tilde{A}, \tilde{B}) > 0$ iff $d(\tilde{A}, 0) > d(\tilde{B}, 0)$ iff $\tilde{B} < \tilde{A}$,
- $d(\tilde{A}, \tilde{B}) < 0$ iff $d(\tilde{A}, 0) < d(\tilde{B}, 0)$ iff $\tilde{B} > \tilde{A}$,
- $d(\tilde{A}, \tilde{B}) = 0$ iff $d(\tilde{A}, 0) = d(\tilde{B}, 0)$ iff $\tilde{B} = \tilde{A}$,

3. HUNGARIAN METHOD:

Step 1: As the given problem is maximal assignment problem, we first convert it into minimization problem by subtracting all the elements of the matrix from the highest element of the matrix.

Step 2: Subtract minimum element of each row from all the elements of that row.

Step 3: Next subtract the smallest element of each column from every element of that column

Step 4: Now we check whether zero assignment is possible or not. So starting with row 1 examine one by one row containing exactly one zero and mark that cell of zero. Cross all other zeros in the column in which the assignment has been made. When all rows are examined, follow the identical procedure for the columns. When the assignment is made in column, cross all zeros in the row in which the assignment is made. Continue this until all zeros are either assigned or crossed-out.

Step 5: If the solution is unbalanced find the minimum number from the remaining elements and subtract it to the other elements, and add to the intersecting elements. Continue this process till we get a balanced solution.

4. METHOD OF ONE'S ASSIGNMENT (MOA)[2]:

The algorithm is stated as follows:

Step1: In a minimization (maximization) problem, find the minimum (maximum) element of each row (say a_i) and write it on the right hand side of the matrix. Then divide each element of the i-th row of the matrix by a_i which will result in creation of at least one ones in each rows. In terms of ones for each row and column do assignment, otherwise go to step 2.

Step 2: Find the minimum (maximum) element of each column in the assignment matrix (say b_j) and write it below j-th column of the matrix. Then divide each element of the j-th column of the matrix by b_j which will result in creation of at least one ones in each column. In terms of ones make the assignment. If no feasible assignment can be achieved from step 1 and step 2 then go to step 3.

Step 3: Draw the minimum number of lines to cover all the ones of the matrix. If the number of drawn lines is exactly equal to n, then the complete assignment is obtained else the complete assignment is not possible and then go to step 4.

Step 4: Select the smallest (largest) element (say d_{ij}) which do not lie on any of the lines in the above matrix and divide each element of the uncovered rows or columns by d_{ij} . This will result in creating some new ones to this row or column. If still a complete optimal assignment is not achieved in the new matrix, then use step 4 and step 3 iteratively. By repeating the same procedure the optimal assignment will be obtained. Priority plays an important role in this method, when we want to assign the ones.

5. NUMERICAL PROBLEM:

Rainfall in Four countries namely India, USA, UK, and China are taken for sample. The rainfall caused due to precipitation, humidity, change in temperature and the monsoon set are taken for last four years say 1992-1995. With the help of trapezoidal assignment problem the year that has the minimum amount of rainfall is calculated and maximum amount of rainfall is calculated by method of one's assignment problem for each country.

$$\begin{bmatrix} (6,10,12,14) & (10,16,22,24) & (18,20,22,30) & (10,16,20,22) \\ (14,16,20,22) & (6,10,12,14) & (12,16,20,24) & (10,16,18,20) \\ (4,8,10,12) & (10,14,20,22) & (16,22,26,110) & (8,12,14,20) \\ (12,16,20,24) & (4,10,12,14) & (10,14,20,22) & (4,8,10,16) \end{bmatrix}$$

A. Hungarian Method

Using the Step 1 the value of $p = w = 1$ and solving the above trapezoidal fuzzy number and converting into minimization problem

$$\begin{bmatrix} 10.5 & 18 & 23.5 & 7 \\ 18 & 10.5 & 18 & 16 \\ 8.5 & 16.5 & 43.5 & 13.5 \\ 18 & 10 & 16.5 & 9.5 \end{bmatrix}$$

Using Step 2 and Step 3 reducing the row and column by Hungarian method

$$\begin{bmatrix} 0 & 7.5 & 6 & 6.5 \\ 7.5 & 0 & 0.5 & 5.5 \\ 0 & 8 & 28 & 5 \\ 8.5 & 0.5 & 0 & 0 \end{bmatrix}$$

Using Step 4 The balanced solution is obtained

$$\begin{bmatrix} \textcircled{0} & 2.5 & 1 & 1.5 \\ 12.5 & \textcircled{0} & 0.5 & 5.5 \\ 0 & 3 & 23 & \textcircled{0} \\ 13.5 & 0.5 & \textcircled{0} & 0 \end{bmatrix}$$

From the above we get that

India $\rightarrow$ 1992, USA $\rightarrow$ 1993, UK $\rightarrow$ 1995 and China $\rightarrow$ 1994

The optimization is

$$= (6,10,12,14) + (6,10,12,14) + (8,12,14,20) + (10,14,20,22) \\ = 51.$$

B. MOA Method

Using Step 1 the value of $p = w = 1$ and solving the above trapezoidal fuzzy number and converting into maximization problem

$$\begin{bmatrix} 10.5 & 18 & 23.5 & 7 \\ 18 & 10.5 & 18 & 16 \\ 8.5 & 16.5 & 43.5 & 13.5 \\ 18 & 10 & 16.5 & 9.5 \end{bmatrix}$$

Using Step 2 and Step 3 reducing row and column by MOA

$$\begin{bmatrix} 0.44 & 0.76 & 1 & 0.72 \\ 1 & 0.58 & 1 & 0.88 \\ 0.19 & 0.37 & 1 & 0.31 \\ 1 & 0.76 & 1 & 0.88 \end{bmatrix}$$

Using Step 4 The balanced solution

$$\begin{bmatrix} 0.44 & \textcircled{1} & 1 & 0.81 \\ 1 & 0.76 & 1 & \textcircled{1} \\ 0.19 & 0.48 & \textcircled{1} & 0.35 \\ \textcircled{1} & 0.72 & 1 & 0.88 \end{bmatrix}$$

From the above we get that

India $\rightarrow$ 1993, USA $\rightarrow$ 1995, UK $\rightarrow$ 1994 and China $\rightarrow$ 1992.

The optimization is

$$= (10,16,22,24) + (10,16,18,20) + (16,22,26,110) + (12,16,20,24) \\ = 95.5.$$

6. CONCLUSION:

In this paper we have solved the fuzzy assignment problem using Hungarian method for minimum amount of rainfall in each country and method of one's assignment (MOA) for maximum amount of rainfall in each country. This can further be extended to solve various amount of generalized assignment problem in various fields such as food production, surveys, health, finance and economics.

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