

# Tuning of PID Controllers for SOPDT Systems

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**Abstract**—The ease implementation, the simple mechanism and robustness of PID controllers has attracted the use of these controllers in the process industries. There are numerous tuning techniques are available for tuning of PID controllers. In this research work, a second-order plus dead time (SOPDT) model is developed and tuning of PID controller is done by using the root locus method. In this method poles are allocated in such a way that model poles are cancelled out by controller zeros, but exact cancellation is not possible. Since the controller cancels out the model poles which is nearest to its exact value, hence an approximation is done to cancel the model poles and to determine damping ratio and the time constant. In this Technique, the higher order system (oscillatory and non-oscillatory) with delayed input is reduced into a second order system and then tuning of the PID controller is done by using the root Locus method, and also the damping ratio and the time constant is calculated to improve the speed of the response. Here in this work, the Routh-Hurwitz criterion is also used to find the value and range of the dc gain k to design a stable system. Here a quantitative and comparative analysis of the original higher order systems and the SOPDT systems (modelled by Root-Locus technique), is done to compare the various parameters like, settling time, rise time, transient time, peak overshoot, peak time etc. and then the unit step response for both the systems has been plotted and analyzed.

**Index Terms**-SOPDT, PID controller, Ziegler Nichols, Root Locus technique, settling time, tuning, rise time, peak overshoot etc.

## I. INTRODUCTION:

PID controller as name suggests is a combination of 3 gain parameters, namely P-Proportional, I-Integral, Derivative gain parameters. These three elements control all the processes in the process industries [1]. A PID controller is an instrument/component/controller used in industrial control applications to regulate temperature, flow, pressure, speed, and other process variables. It is also known as three-term controller. The PID controller as a whole change the dc gain, increases the order of the system, reduces the steady state error, and improve the transient response of the system.

## II. OBJECTIVE AND MOTIVATION FOR TUNING OF PID CONTROLLERS:

Since the last few decades, a lot of research work has been done because of the growing popularity into process control industries. In very early nineties century Ziegler-Nichol [2] gave tuning procedure for PID controller, after that many methods have been used and developed to tune PID controller for getting good results. Because of high requirement of best tuning procedures which tune the plant in such a way that could provide optimized solution, many tuning methods have been developed so far in which some methods give better response for speed of the system and some show good response for stability. Thus, maximum methods are application oriented. Some important methods of tuning of PID controller are: Tuning method of PID controllers for Desired Damping coefficient [3], Tuning of PID controller by D partition rule [4], Tuning of PID controller using immune algorithm [5], etc.

## III Model Reduction Technique for SOPDT Process:

In this Section Model reduction Technique is used for modelling of higher order system into SOPDT system and subsequently Tuning of PID controller is done by using Root-Locus technique is discussed in Section IV. Tuning of PID controller by using the root locus method is simple and result oriented for any systems; whether it is the high order or low order, high delay time or low dead time, the oscillatory or non-oscillatory.

Several methods have been developed and used to find out PID controller parameters for SISO (single input single output) and multiple input multiple output (MIMO) systems [6-7], but maximum of these tuning methods are developed for any specific applications [8-11] hence can be used only for some applications, but the Root Locus technique is a general tuning method and can be used for several applications in process control industries.

Let us take a SOPDT (second order plus dead time) system for getting the best results after tuning. In this method, first, the higher order system is reduced into a second order system by using the model reduction technique. For this, if we put  $s=j\omega$ , then the complex variable is divided into two parts and then angle condition is applied. In the FOPDT model for the monotonic (non-oscillatory) system, we cannot generate peaks, but it is possible in case of the SOPDT system, also FOPDT have only real poles not imaginary poles. Hence, they are not able to generate peaks for oscillatory systems, so we are using second order plus dead time for PID tuning.

In SOPDT systems, closed loop poles have been selected according to the delay to time constant ratio, damping ratio and delay time model hence in this case we have more satisfactory results than in other methods.

#### IV Higher Order Reduction Method:

Let us take The SOPDT model process with the transfer function  $G(s)$  given in Eq. (3.14),

$$G(s) = \frac{1}{as^2 + bs + c} e^{-st_0} \quad (3.14)$$

Depending on the values of  $a$ ,  $b$ , and  $c$ , this model can be characterized into real or complex poles. Hence it is easy to represent both the systems non-oscillatory as well as oscillatory.

A PID controller can be represented by Eq. (3.15),

$$K(s) = K_p + \frac{K_I}{s} + K_D s \quad (3.15)$$

The aim is to calculate  $K_p$ ,  $K_I$  and  $K_D$  in such way that it improves the response of the system and will give a better result.

In Eq. (3.14) we put  $s = j\omega$ , then divide into two parts that is real and imaginary part. We need four equations for finding out four unknowns  $a$ ,  $b$ ,  $c$ ,  $t_0$ . For this we are calculating phase of the gain  $G(s)$  at two nonzero different frequency points  $\omega_b$  and  $\omega_c$  such that  $\angle G(j\omega_b) = -\frac{\pi}{2}$  and  $\angle G(j\omega_c) = -\pi$ . Here we are taking two non-zero frequency points on negative real axis as we are considering the poles are located in negative half of  $s$ -plane for the stable system.

By putting  $s = j\omega$  in Eq. (3.14) we have,

$$G(j\omega) = \frac{1}{-a\omega^2 + j\omega b + c} e^{-j\omega t_0} \quad (3.16)$$

Calculating The real and imaginary parts we will have,

Real part,

$$G_R(\omega) = \frac{1}{[(c - a\omega^2)^2 + b^2\omega^2]} [(c - a\omega^2)\cos(\omega t_0) + b\omega \sin(\omega t_0)] \quad (3.17)$$

Imaginary part,

$$G_I(\omega) = \frac{-1}{[(c - a\omega^2)^2 + b^2\omega^2]} [(c - a\omega^2)\sin(\omega t_0) + b\omega \cos(\omega t_0)] \quad (3.18)$$

Calculating the phase angle of  $G(s)$ ,

$$\angle G(j\omega) = -\tan^{-1}\left[\frac{b\omega}{(c - a\omega^2)}\right] - \omega t_0 \quad (3.19)$$

In Eq.(3.19), putting the condition,  $\angle G(j\omega_b) = -\frac{\pi}{2}$  and  $\angle G(j\omega_c) = -\pi$  and solving the trans-dental Eq.(3.19) we can calculate  $\omega_b$  and  $\omega_c$ .

With condition,  $\angle G(j\omega_c) = -\pi$ , we get the equation,

$$(c - a\omega_c^2)\sin(\omega_c t_0) + b\omega_c \cos(\omega_c t_0) = 0 \quad (3.20)$$

And with condition  $\angle G(j\omega_b) = -\frac{\pi}{2}$ , we get the equation,

$$(c - a\omega_b^2)\cos(\omega_b t_0) + b\omega_b \sin(\omega_b t_0) = 0 \quad (3.21)$$

Eq. (3.19), (3.20) and (3.21) are useful to calculate  $\omega_b$  and  $\omega_c$ .

$$\text{Now magnitude of } G(j\omega) = |G(j\omega)| = \frac{1}{\sqrt{[(c - a\omega^2)^2 + b^2\omega^2]}} \quad (3.22)$$

we have to calculate the 4 variables  $a$ ,  $b$ ,  $c$  and  $t_0$  so we need 4 equations.

Now calculating the magnitude of  $G(j\omega)$  at  $\omega_c$  and  $\omega_b$ , we get,

$$|G(j\omega_c)| = \frac{1}{\sqrt{[(c - a\omega_c^2)^2 + b^2\omega_c^2]}} \quad (3.23)$$

$$|G(j\omega_b)| = \frac{1}{\sqrt{[(c - a\omega_b^2)^2 + b^2\omega_b^2]}} \quad (3.24)$$

From Eq. (3.20) we have,

$$(c - a\omega_c^2)\sin(\omega_c t_0) = -b\omega_c \cos(\omega_c t_0) \quad (3.25)$$

From Eq. (3.25) we get,

$$\sin^2(\omega_c t_0) = \cos^2(\omega_c t_0) \left[ \frac{b^2\omega_c^2}{(c - a\omega_c^2)^2} \right] \quad (3.26)$$

Using the identity,  $\sin^2(\omega_c t_0) + \cos^2(\omega_c t_0) = 1$ , we have,

$$(c - a\omega_c^2) = -\cos(\omega_c t_0) \sqrt{[(c - a\omega_c^2)^2 + b^2\omega_c^2]} \quad (3.27)$$

From Eq (3.23),

$$(c - a\omega_c^2) = \frac{1}{|G(j\omega_c)|} \cos(\omega_c t_0) \quad (3.28)$$

From Eq. (3.21) we have,

$$(c - a\omega_b^2)\cos(\omega_b t_0) = -b\omega_b \sin(\omega_b t_0) \quad (3.29)$$

From above equation we get,

$$\cos^2(\omega_b t_0) = \sin^2(\omega_b t_0) \left[ \frac{b^2\omega_b^2}{(c - a\omega_b^2)^2} \right] \quad (3.30)$$

Using the identity,  $\sin^2(\omega_b t_0) + \cos^2(\omega_b t_0) = 1$ , and From Eq. (3.24) we get,

$$(c - a\omega_b^2) = \frac{1}{|G(j\omega_b)|} \sin(\omega_b t_0) \quad (3.31)$$

Similarly, From Eq. (3.20) & (3.23) and using identity,  $\sin^2(\omega_c t_0) + \cos^2(\omega_c t_0) = 1$

We have,

$$bw_c = \frac{1}{|G(jw_c)|} \sin(w_c t_0) \tag{3.32}$$

Similarly, we can calculate,

$$bw_b = \frac{1}{|G(jw_b)|} \cos(w_b t_0) \tag{3.33}$$

From Eq.(3.32) we can calculate unknown b as,

$$b = \frac{1}{|G(jw_c)| w_c} \sin(w_c t_0) \tag{3.34}$$

Now to calculate a, do Eq.(3.28)-(3.31),

$$a = \frac{1}{|w_c^2 - w_b^2|} \left[ \frac{\cos(w_c t_0)}{|G(jw_c)|} + \frac{\sin(w_b t_0)}{|G(jw_b)|} \right] \tag{3.35}$$

to calculate c, do, Eq. (3.28)  $\times w_b^2$  - Eq.(3.31)  $\times w_c^2$ ,

$$c = \frac{1}{|w_c^2 - w_b^2|} \left[ \frac{\cos(w_c t_0) w_b^2}{|G(jw_c)|} + \frac{\sin(w_b t_0) w_c^2}{|G(jw_b)|} \right] \tag{3.36}$$

to calculate,  $t_0$  we use Eq. (3.32) and (3.33),

$$b = \frac{1}{|G(jw_c)| w_c} \sin(w_c t_0) = \frac{1}{|G(jw_b)| w_b} \cos(w_b t_0) \tag{3.37}$$

From Eq. (3.37), Let us take  $A = \frac{\sin(w_c t_0)}{\cos(w_b t_0)} - \frac{w_c |G(jw_c)|}{w_b |G(jw_b)|}$  (3.38)

Now expanding,  $\sin(w_c t_0)$  and  $\cos(w_b t_0)$  upto second order polynomial and after solving with approximation we get a quadratic equation in  $t_0$  as given below,

$$-0.34[w_c^2 - Aw_b^2]t_0^2 + [1.7w_c + A(0.11)w_b]t_0 - A = 0 \tag{3.39}$$

From Eq. (3.39) we can calculate delay  $t_0$ , here all the unknown a, b, c,  $t_0$  should be positive as we require a stable reduced 2<sup>nd</sup> order system. Also delay can't be negative hence  $t_0$  always be positive.

**3.3 Tuning of PID Controller for SOPDT Process:**

For tuning of the PID controller, we will first calculate the range k (dc gain) at which system is stable, it can be done by using Routh-Hurwitz criterion.

From Eq. (3.15), PID controller transfer function

$$K(s) = K_p + \frac{K_I}{s} + K_D s$$

**3.3.1 Calculation of range of dc gain(k):**

There is a range of Dc gain k for which the system is stable and we want a stable system hence we only tune PID controller for the given system in the calculated range of gain k. Assume PID controller transfer function in terms of dc gain is given below,

$$K(s) = \frac{k}{s} (\alpha s^2 + \beta s + \mu) \tag{3.40}$$

where K(s) is the controller transfer function.

We choose the controller zeros which cancel out model poles that implies,  $\alpha = a$ ,  $\beta = b$  and  $\mu = c$ . Here a, b, c are the values which are calculated by using Eq. (3.34), (3.35) and (3.36).

Comparing Eq (3.40) & (3.15) we can write the proportional, integral and derivative constants of PID controller as,  $K_p = k\beta$ ,  $K_I = k\mu$  and  $K_D = k\alpha$

Refer to Fig. (3.1), the system is unity feedback hence  $H(s)=1$ , and assume process gain,

$$G(s) = \frac{e^{-st_0}}{s}$$

Now calculating open loop transfer function (without considering controller K(s)),

$$G(s)H(s) = \frac{e^{-st_0}}{s} \tag{3.41}$$

Now the characteristics Equation (refer to Fig. (3.1)) is,

$$1 + G(s)H(s)K(s) = 0$$

From Eq (3.40) & (3.41), we can write,

$$1 + \frac{e^{-st_0} k}{s} (\alpha s^2 + \beta s + \mu) = 0 \tag{3.42}$$

Here to calculate characteristics equation we approximate the delay term,

$$e^{-st_0} = \left[ \frac{(1 - \frac{st_0}{2})}{(1 + \frac{st_0}{2})} \right] \tag{3.43}$$

Now from Eq. (3.42) & (3.43) we can write the characteristics equation as given below,

$$(t_0 - at_0 k)s^3 + (2ak + 2-bt_0 k)s^2 + (2bk - t_0 ck)s + 2ck = 0 \tag{3.44}$$

Now applying Routh Hurwitz criteria and taking all the elements of first column in Routh table of the same sign we can show that the range of value of k for a stable system for tuning of PID controller is given below,

$$k \leq \frac{b}{t_0} \tag{3.45}$$

**3.3.2 Calculation of time constant( $\tau$ ):**

From Eq (3.14), The pole location can be calculated by solving the equation,

$$as^2 + bs + c = 0 \tag{3.46}$$

Calculating the poles' location,

$$s_{1,2} = -b \pm \sqrt{(b^2 - 4ac)}/2a \tag{3.47}$$

The speed of response of any process is inversely proportional to its equivalent time constant and it is given by the inverse of real part of the pole location,

$$\tau = \frac{1}{\text{Re}(\text{pole location})} \quad (3.48)$$

hence the value of the time constant depends on real part of the pole and will always be a positive value, and it depends on the value of  $(b^2 - 4ac)$ , hence from Eq. (3.47) it can be shown that, if,  $b^2 - 4ac=0$  then time constant,

$$\tau = \frac{2a}{b}$$

And the equivalent time constant [19] can be calculated as,

$$\frac{1}{\tau} = \begin{cases} \frac{b}{2a}, \text{ if } (b^2 - 4ac) \geq 0 \\ \frac{c}{\sqrt{b^2 - 4ac}}, \text{ if } (b^2 - 4ac) < 0 \end{cases} \quad (3.49)$$

### 3.3.3 Calculation of damping ratio( $\xi$ ):

Compare Eq. (3.46) with 2<sup>nd</sup> order system's general equation  $s^2 + 2\xi w_n s + w_n^2 = 0$  it can be shown that,

$$w_n = \sqrt{\frac{c}{a}} \text{ and,}$$

$$\xi = \frac{b}{2\sqrt{ac}} \text{ for complex poles (i.e., } (b^2 - 4ac < 0))$$

and for real poles ( $(b^2 - 4ac) \geq 0$ ) we are assuming a critically damped system ( $\xi=1$ ).

Hence Equivalent damping ratio [20] can be found out as,

$$\xi = \begin{cases} 1, \text{ if } (b^2 - 4ac) \geq 0 \\ \frac{b}{2\sqrt{ac}}, \text{ if } (b^2 - 4ac) < 0 \end{cases} \quad (3.50)$$

Here the model poles have to be cancelled out by controller zeros, but exact cancellation may not possible so we approximate the zero to the nearest of model poles. For a process with a damping ratio less than one, un-cancelled dynamics may provide the heavy oscillations so it is not desirable to create one more oscillatory term in the system, but we can choose the real part of the close loop pole. For monotonic (non-oscillatory) processes, un-cancelled dynamics do not create the process over oscillation so selection of close loop pole will give a better result. Based on this, we are considering 4 cases to calculate value of k in terms of  $\xi$  and it depends on the range of  $\xi$  values. Taking  $\xi=0.7071$  as reference here.

#### Case (1): $\xi > 0.7071$ :

In this case, both real and imaginary poles on the root locus can be chosen. Considering 2<sup>nd</sup> order delay time system general open loop transfer function,

$$G(s)H(s) = \frac{k w_n^2}{s(s + 2\xi w_n)} e^{-st_0} \quad (3.51)$$

Having pair of complex poles,  $s_{1,2} = -\xi w_n \pm j w_n \sqrt{1 - \xi^2}$

To find the dc gain using root locus method we use magnitude condition [20],

$$|G(s)H(s)| = 1 \quad (3.52)$$

Here we can calculate the approximate value of k as,

$$k = w_n e^{-\xi w_n t_0} \quad (3.53)$$

Here, the pair of complex poles and real pole should be on root locus, we can calculate the value of  $w_n$  by using phase condition of root locus [20] that is,

$$\angle G(s)H(s) = \pm(2n+1)\pi$$

or,  $-\tan^{-1} \frac{w}{2\xi w_n} - \frac{\pi}{2} - w t_0 = \pm(2n+1)\pi$

at  $w=w_n$  with the given value conditions of  $\xi$ , we have,

$$w_n = \frac{\cos^{-1}(\xi)}{t_0 \sqrt{1 - \xi^2}} \quad (3.54)$$

Putting  $\xi = 0.707$ , we have,

$$w_n = \frac{1.10}{t_0} \quad (3.55)$$

From Eq. (3.53) & (3.55), it can be shown that,

$$k = \frac{0.5}{t_0} \quad (3.56)$$

#### Case (2): $\xi \leq 0.7071$ & $0.15 < \frac{t_0}{\tau} < 1$ :

In this case there may be 2 real closed loop poles and a pair of complex poles.

$$\because 0 < \xi < 1 \Rightarrow (b^2 - 4ac) < 0$$

$$\Rightarrow \frac{1}{\tau} = \frac{c}{\sqrt{b^2 - 4ac}}$$

For example, take  $\xi=0.5$ ,

$$\because \xi = \frac{b}{2\sqrt{ac}} \Rightarrow \frac{b}{2\sqrt{ac}} = 0.5 \Rightarrow b^2 = ac$$

From above discussion it is clear that,

$$\frac{1}{\tau} = \sqrt{\frac{c}{a}}$$

hence compare Eq. (3.46) with a 2<sup>nd</sup> order general equation, we get,

$$w_n = \frac{1}{\tau} = \sqrt{\frac{c}{a}}$$

Now from Eq. (3.53) we can calculate,

$$k = \frac{1}{\tau} e^{-\frac{t_0}{\tau}} \tag{3.57}$$

**Case (3):**  $\frac{t_0}{\tau} > 1$ : In this case since ratio of delay to time constant is greater than 1, so the value of k is slightly greater than that in case 1.

From Eq. (3.54) & (3.55), it can be shown that,

$$w_n = \frac{1.10}{t_0}$$

and also, we have,

$$w_n = \frac{1}{\tau}$$

$$\Rightarrow w_n = \frac{1.10}{t_0} = \frac{1}{\tau} \Rightarrow \frac{t_0}{\tau} = 1.10 \text{ (is greater than 1)}$$

From Eq. (3.57), we can calculate the approximate value of k as,

$$k = \frac{0.6}{t_0} \tag{3.58}$$

that is slightly greater than the value of k calculated in case 1.

**Case (4):**  $0.05 < \frac{t_0}{\tau} < 0.15$ :

In this case the poles are complex  $\Rightarrow 0 < \xi < 1$

For example, take  $\xi = 0.5$  &  $\frac{t_0}{\tau} = 0.10$

From Eq. (3.57), we can calculate the approximate value of k as,

$$k = \frac{0.4}{t_0} \tag{3.59}$$

**V RESULTS AND DISCUSSION:**

In this section some examples have been taken and will demonstrate how to use Root -Locus techniques. Also, a comparative and quantitative analysis is done for original higher order system (for which PID tuning is done) and the reduced 2<sup>nd</sup> order system in terms of different parameters like, settling time, peak time, stability, overshoot and undershoot etc. These quantitative parameters are shown in table for a comparison. The simulation results of step response for both the system are also plotted for a comparative study and a better understanding.

**Root-Locus Method for SOPDT systems:** In this section the higher order system is first modeled to reduced 2<sup>nd</sup> order PDT system then tuning is done using Root-Locus technique. Following examples are used to demonstrate the method.

**Example 1:** Let us consider a non-oscillatory system,

$$G(s) = \frac{1}{(s + 1)^2(s + 2)} e^{-0.3s}$$

Here two points are calculated by the method is  $w_b = 3.5$  and  $w_c = 1.56$ , and  $|G(jw_b)| = 0.0187$  and  $|G(jw_c)| = 0.1148$ .

The model of the process or the reduced 2<sup>nd</sup> order system is given by,

$$G_1(s) = \frac{1}{5.276s^2 + 4.469s + 18.040} e^{-0.595s}$$

And the PID parameter is calculated as,

$$K(s) = 1.425 + \frac{5.754}{s} + 1.683s$$

**Quantitative and comparative analysis**

The comparative analysis of both systems is shown below in table 1,

Table 1: Response values for unit step input

Parameters	Response values for Higher order system G(s)	Response values for reduced 2 <sup>nd</sup> order system G <sub>1</sub> (s)
Rise time	3.6132	0.6720
Transient Time	6.7730	5.8622
Settling Time	6.7730	5.8622
Overshoot (%)	11	7.533
Undershoot (%)	0	0
Peak	0.500	0.0818
Peak time	12.8484	2.392

From the above table the stability of the reduced 2nd order system is more because settling time of the model is smaller and rise time is very much less than higher order system hence the transient response is improved in reduced 2<sup>nd</sup> order system. Also, the reduced 2<sup>nd</sup> order system is a better system because of having smaller peak time and peak overshoot than higher order system. The unit step response for both the systems G(s) and G<sub>1</sub>(s) is shown below in Fig. (2) and (3) respectively,

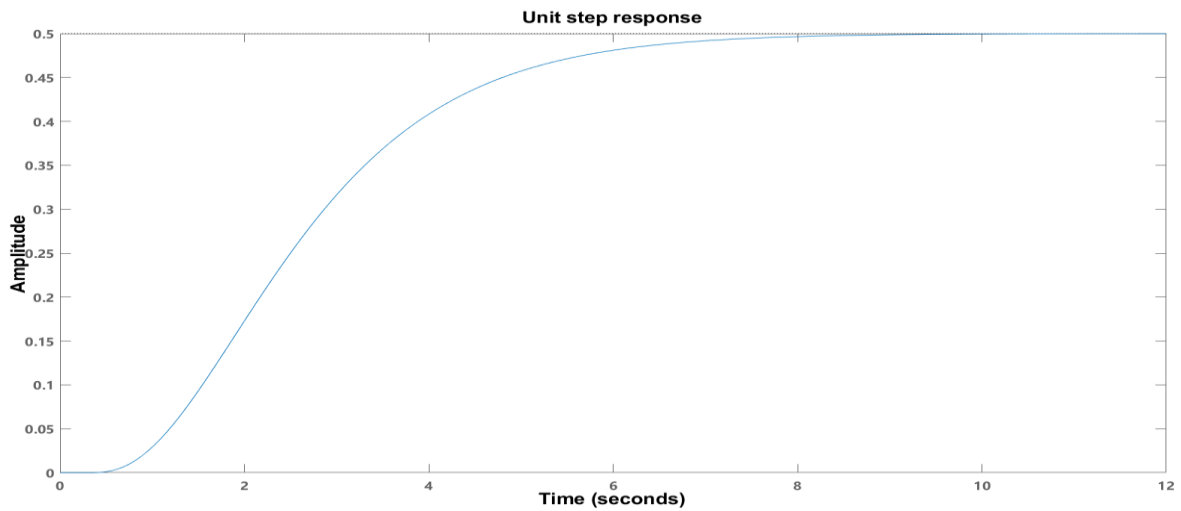


Fig. (2): Unit step response of the system  $G(s) = \frac{1}{(s+1)^2(s+2)} e^{-0.3s}$

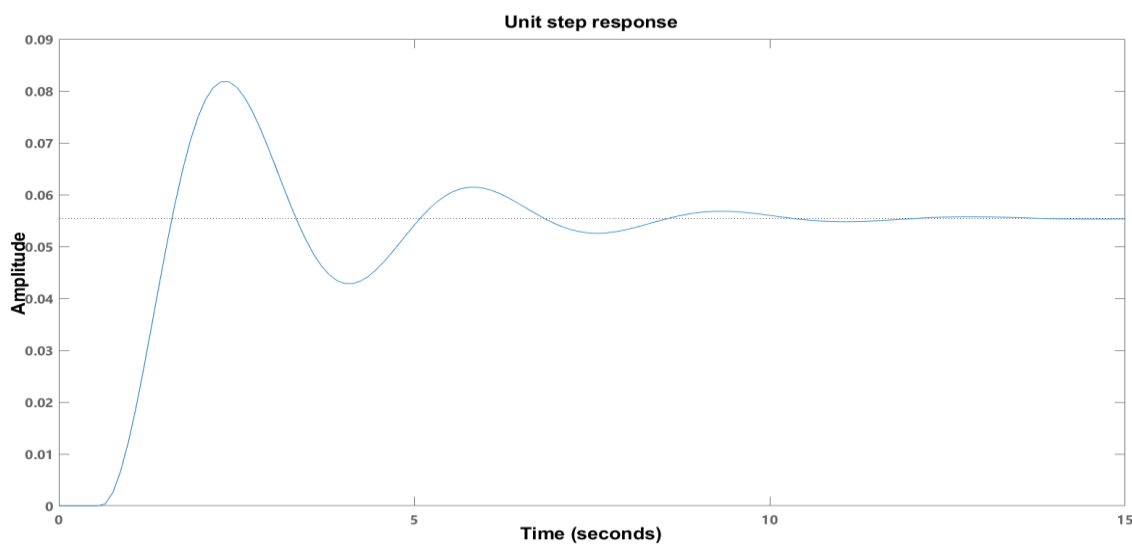


Fig. (3): Unit step response of the system,  $G_1(s) = \frac{1}{5.276s^2 + 4.469s + 18.040} e^{-0.595s}$

**Example 2:** Let us consider a system,

$$G(s) = \frac{1}{(s+2)^2(s+1)} e^{-0.5s}$$

Here two points are calculated by the method is  $w_b = 0.67$  and  $w_c = 1.57$ , and  $|G(jw_b)| = 0.1869$  and  $|G(jw_c)| = 0.0832$ .

The model of the process or the reduced order system is given by,

$$G_1(s) = \frac{1}{6.062s^2 + 1.078s + 3.043} e^{-0.09s}$$

And the PID parameter is calculated as,

$$K(s) = 4.786 + \frac{13.51}{s} + 26.94s$$

**Quantitative and comparative analysis**

The comparative analysis of both systems is shown below in table 2,

Table 2: Response values for unit step input

Parameters	Response values for Higher order system G(s)	Response values for reduced 2 <sup>nd</sup> order system G <sub>1</sub> (s)
Rise time	2.9139	1.6281
Transient Time	5.7812	4.431
Settling Time	5.7812	4.431
Overshoot (%)	10	6.93
Undershoot (%)	0	0
Peak	0.250	0.5486
Peak time	13.7695	4.434

From table, it is clear that the stability of the reduced 2<sup>nd</sup> order system is more because settling time of the system is smaller, and rise time of process model is smaller hence the transient response is improved also the percentage overshoot and peak time is smaller in reduced system, hence it's a better system than the original higher order system. The unit step response for both the system G(s) and G<sub>1</sub>(s) is shown below in Fig. (4) and Fig. (5) respectively,

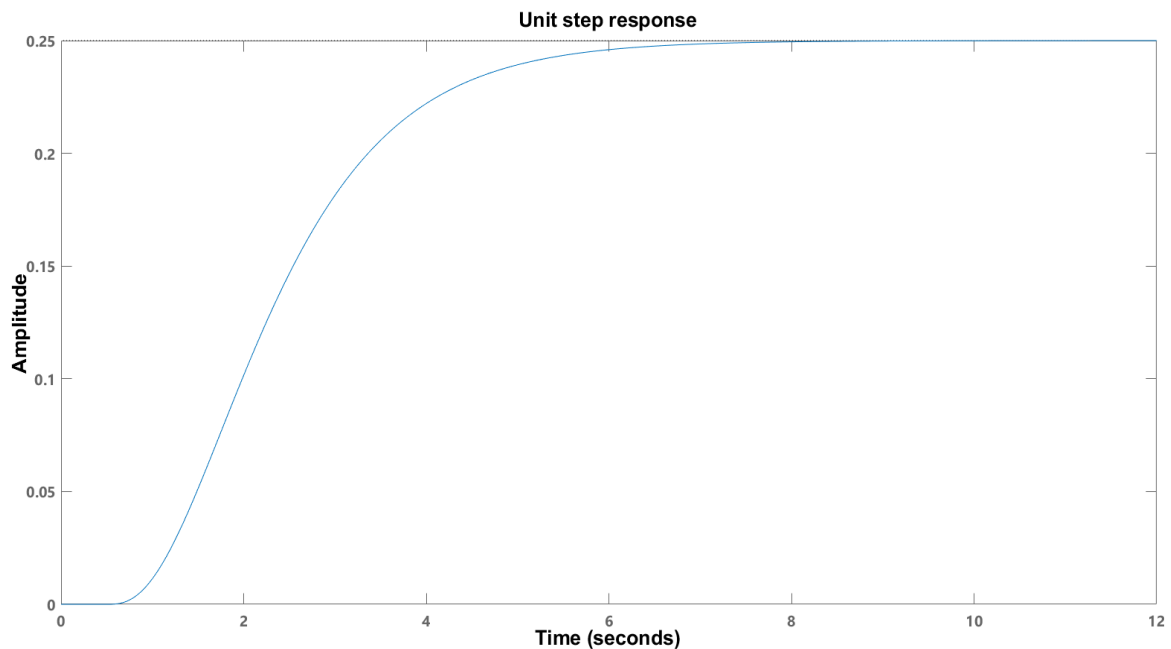


Fig. (4): Unit step response of the system  $G(s) = \frac{1}{(s+2)^2(s+1)} e^{-0.5s}$

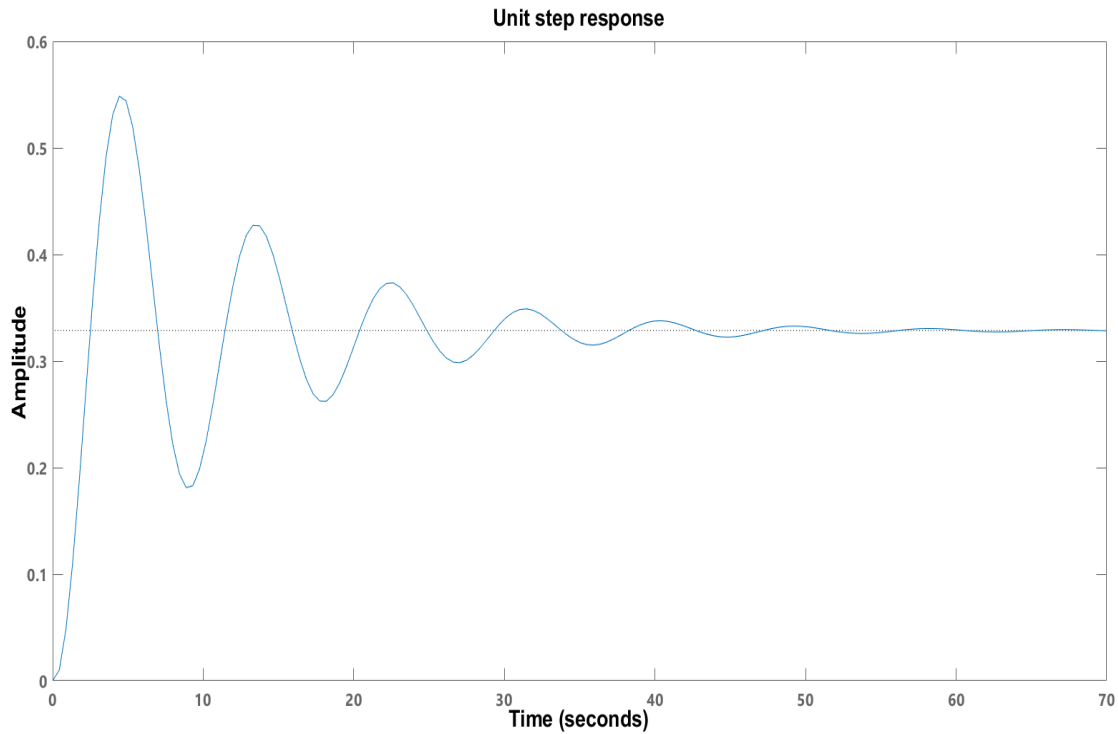


Fig. (5): Unit step response of the system  $G_1(s) = \frac{1}{6.062 s^2 + 1.078 s + 3.043} e^{-0.09s}$

**Example 3:** Let us consider a higher order system,

$$G(s) = \frac{1}{(s^2 + s + 1)(s + 1)} e^{-0.3s}$$

Here two points are calculated by the method is  $w_b = 2.60$  and  $w_c = 1.20$ , and  $|G(jw_b)| = 0.1014$  and  $|G(jw_c)| = 0.501$ . The model of the process or the reduced order system is given by,

$$G_1(s) = \frac{1}{1.272 s^2 + 0.389 s + 3.773} e^{-0.197s}$$

And the PID parameter is calculated as,

$$K(s) = 0.374 + \frac{3.637}{s} + 1.226s$$

**Quantitative and comparative analysis**

The comparative analysis of both system is shown below in table 3,

Table 3: Response values for unit step input

Parameters	Response values for Higher order system $G(s)$	Response values for reduced 2 <sup>nd</sup> order system $G_1(s)$
Rise time	2.2910	0.6496
Transient Time	6.9379	4.418
Settling Time	6.9379	4.418
Overshoot (%)	8.1432	7.52
Undershoot (%)	0	0
Peak	1.081	0.465
Peak time	5.249	2.0

From table, it is clear that the stability of the reduced 2<sup>nd</sup> order system is more because settling time of the system is smaller, and rise time of process model is smaller hence the transient response is improved also for a better system we always want lower percentage overshoot. Her for 2<sup>nd</sup> order system, the percentage overshoot and peak time is smaller hence it's a better system than the original higher order system. The unit step response for both the system  $G(s)$  and  $\square_1(s)$  is shown below in Fig. (6) and Fig. (7) respectively,



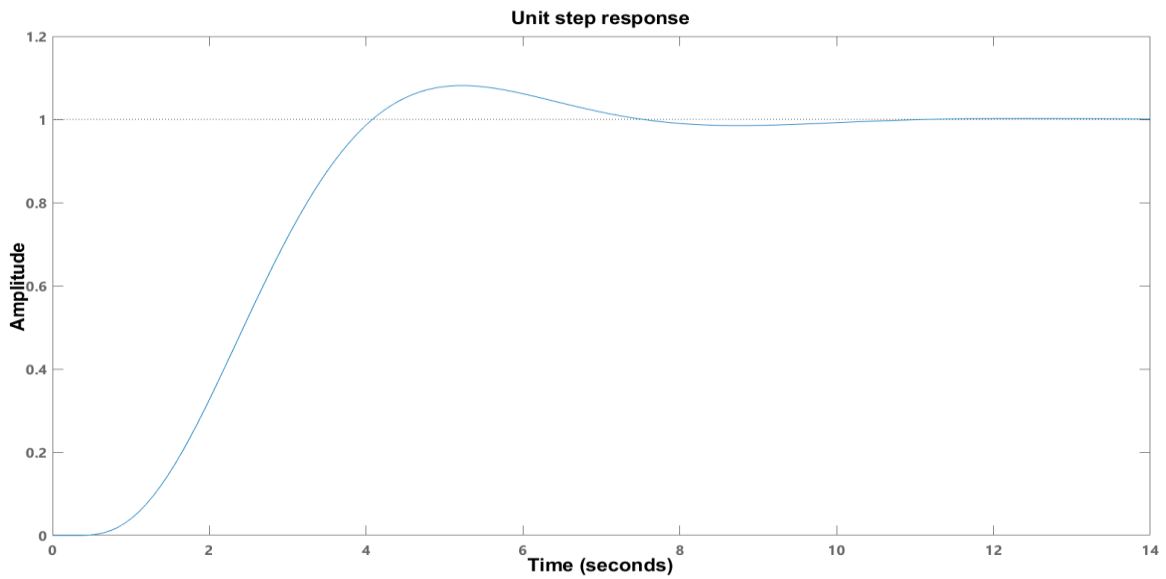


Fig. (6): Unit step response of the system  $G(s) = \frac{1}{(s^2 + s + 1)(s + 1)} e^{-0.3s}$

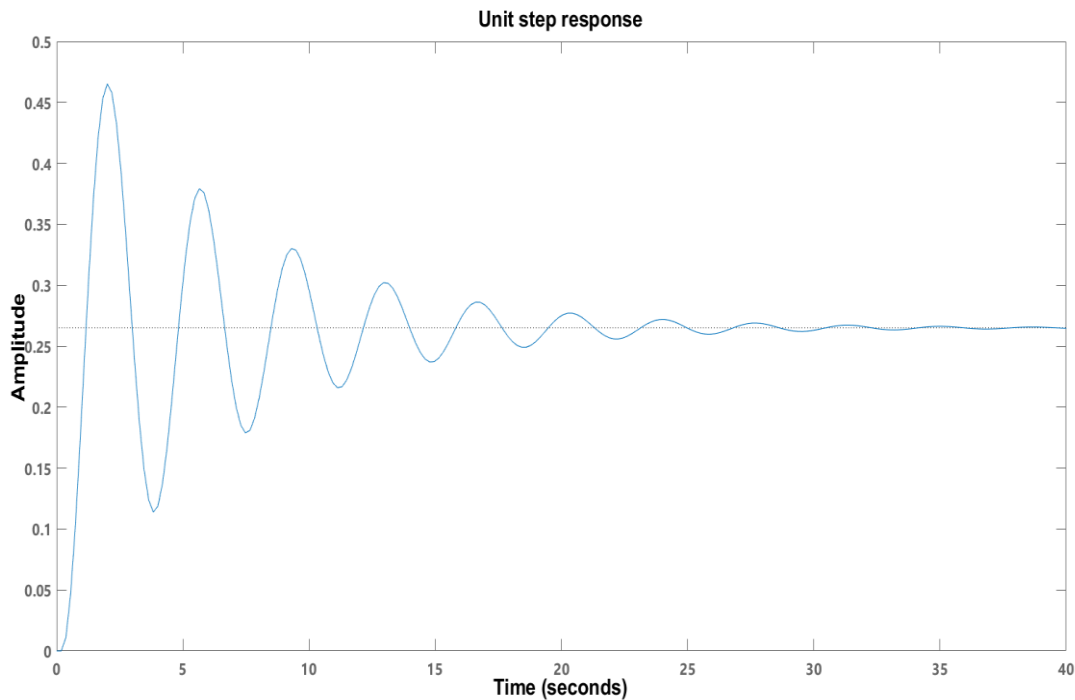


Fig. (7): Unit step response of the system,  $G_1(s) = \frac{1}{1.272s^2 + 0.389s + 3.773} e^{-0.197s}$

**VII ACKNOWLEDGEMENT:**

In this research work, the focus has been given towards the applications of tuning of PID controllers for the higher order system to reduce the system order. In today’s process control industries, the major attention is given to the development of those controllers which are most suitable and the best for application-based processes. There are several methods of tuning of PID controllers for FOPDT (First order plus dead time) systems, but the biggest problem of FOPDT tuning is that it is unable to generate peaks for monotonic systems. So, here tuning of PID controllers for SOPDT systems is done to find out the controller parameters.

In this research work Root -Locus technique is used for modelling of SOPDT systems and PID tuning is done. In this method second order system by using angle conditions in which two angle conditions are divided into four parts so that four variables (a, b, c and delay  $\tau_0$ ) are calculated. Here in this method for the Tuning process, if model poles are monotonic (all poles lie in the negative half of s-plane and hence the system is stable) then un-cancelled dynamics do not produce any oscillations. The Root-Locus Technique for tuning of PID Controllers provides a satisfactory result over older Ziegler-Nichols Method used for

FOPDT systems. The results shows that the Root-Locus method improves the speed and improves the transient response and increase the stability of the system as well.

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