Sensitivity Analysis of 2PRRR-1RRR Planar Parallel Manipulator

M.Mallesh[1], G. Satish Babu[2]

[1]Associate Professor, Department of Mechanical Engineering,
Vidya Jyothi Institute of Technology, Hyderabad.
[2]Professor (Retd.,), JNTUH College of Engineering, Hyderabad

Abstract: The 2PRRR-1RRR manipulator sensitivity indices are presented in this article. The sensitivity indices have position and orientation is conducted in mathematical software and the results are presented. The lower sensitivity indicates the better control of the manipulators. There are two possibilities for redundancy, kinematic and actuation redundancy. The 2PRRR- 1RRR planar mechanism is considered for analysis, the input and output relations are the model is mathematically modelled by the method proposed by S.Caro. The sensitivity indices derived for position and orientation and the results are plotted on the workspace.

Keywords: Planar Parallel Manipulators, Sensitivity Indices, 2PRRR-1RRR Mechanism, PPM with branches.

1 Introduction

"Kinematic redundancy" increases the mobility as well as degrees of freedom of actuated joint of the "parallel manipulators". "Kinematic redundancy" occurs when additional active joints or links are provided to the manipulators. Kinematically redundant parallel manipulators necessitate extra controlling parameters than necessary for a set of specified tasks. A manipulator with adequate "kinematic redundancy" can shun all interior singularities, has a more work volume, and enhanced manoeuvrability. "Kinematic redundancy" can also permit manipulators to deal with joint-jam breakdown. Higher stiffness, better accuracy as well as superior payload to weight ratio are the main advantages of "parallel manipulators" compare to "serial manipulators". "Parallel manipulators" are having large number of appliance fields such as space telescopes, fine positioning devises, fast packaging, high-speed machining, medical, and flight simulators etc. Most of the studies on parallel manipulators are on non redundant parallel manipulators. Redundant parallel manipulators are introduced to eliminate the some of the short comings of parallel manipulators. Redundancy is of two types: "Actuation redundancy" and "kinematic redundancy". Actuation redundancy is defined as replacing the excessive passive joints by active joints of manipulator. Actuation redundancy reduces the singularities in the manipulator workspace.

Stephane Caro et al [1-2], carried out "sensitivity analysis of 3RPR planar parallel manipulators". They proposed a method to compare 3RPR manipulators among to their dexterity, work volume and sensitivity. Stephane Caro et al [3-5], proposed a methodology for the multi-objective design optimisation of parallel kinematic manipulators. The dimensional synthesis of 3PRR planar parallel manipulator was considered. The approach adapted by Caro is aimed to minimise the mechanism of mass in motion as well as to maximise its usual shaped workspace. The constraints for the optimisation problem are resolute based on the "mechanism accuracy assembly" and condition number of the manipulator kinematic Jacobian matrix.

Wenger at al [6], introduced two corresponding methods to analyse the sensitivity of a three degree of freedom translational parallel kinematic machine with orthogonal linear joints, the ortho-glide. Nicolas Binaud et al [7], developed a methodology to obtain the sensitivity coefficients of the pose of the moving platform of the manipulators to variations in their geometric parameters and actuated variables. They have determined to aggregate sensitivity indices and compared 3RPR, 3RPR, 3RRR, 3RRR and 3PRR manipulators with regarded to their workspace size and sensitivity.

Jing wang and clement Gosselin [8], 2004, have presented three different parallel manipulators with one redundant degree of freedom for reducing singularities in the workspace. They have developed analytical expressions for singularity loci of type II and made a comparison between redundant and non-redundant manipulators. Sukham Lee and sungbok kim [9], 1993,have defined three orthogonal instantaneous motion spaces (IMSs) of a parallel manipulator and based on IMSs, they analysed the kinematic features such as deficiency, singularity and redundancy of platform type parallel manipulator system. They have also investigated the kinematic performance of parallel manipulators, such as dexterity and load capacity.

J.P Maerlet [10] et al developed geometrical algorithms for the determination of various workspaces of planar parallel manipulators. A detailed analysis on workspaces of planar parallel manipulators is carried out by them. They have determined the constant orientation works space, maximal workspace, inclusive maximal workspace, total orientation workspace and dextrous workspace of the 3RRR, 3PRR planar parallel manipulators.

Iman Ebrahimi et al [11] have studied a family of kinematically redundant planar parallel manipulator. They have analysed the 3PRRR manipulator for reachable workspace, dextrous workspace and kinematic singularities. Ma and Angeles [12] have studied the kinematic performance of planar manipulators by investigating the numerical conditioning of their Jacobian Matrix with respect to architectures. The dimensional in homogeneity of the Jacobian matrix is eliminated by introducing a characteristic link length. The optimum Architecture of the manipulator obtained by them resembles that used in flight simulators.

Han Sung Kim and Lung-Wen Tsai [13] presented a parallel manipulator to achieve translational motion of the moving platform. It was shown that the rotary actuation method results in many singular points within the workspace. Due to the orthogonal arrangement of the limbs, the Jacobian matrix is always isotropic and the manipulator behaves like a traditional X-Y-Z Cartesian machine. They have also suggested a method to maximize the stiffness, in order to minimize the deflection at the joints caused by the bending moment.

2 Mechanism Architecture

Planar Parallel mechanisms are designed to increase the redundancy by introducing two prismatic joints in two limbs. these joints are used to increase the mobility of mechanism, the mechanism has one moving platform and one fixed base platform, the radius of moving platform is taken as r and fixed platform as R, here, the motions in X, Y and Z are performed by three or more parallel axes that gives high stiffness and accuracy. In this work actuation redundancy is presented by introducing additional branch.

As shown in Fig. 1, two additional branches A_1B_1 and $A_2B_2$are introduced in the kinematic model. Addition of two actuated joints improves the performance of the mechanism and also increases the complexity of actuation of mechanism.

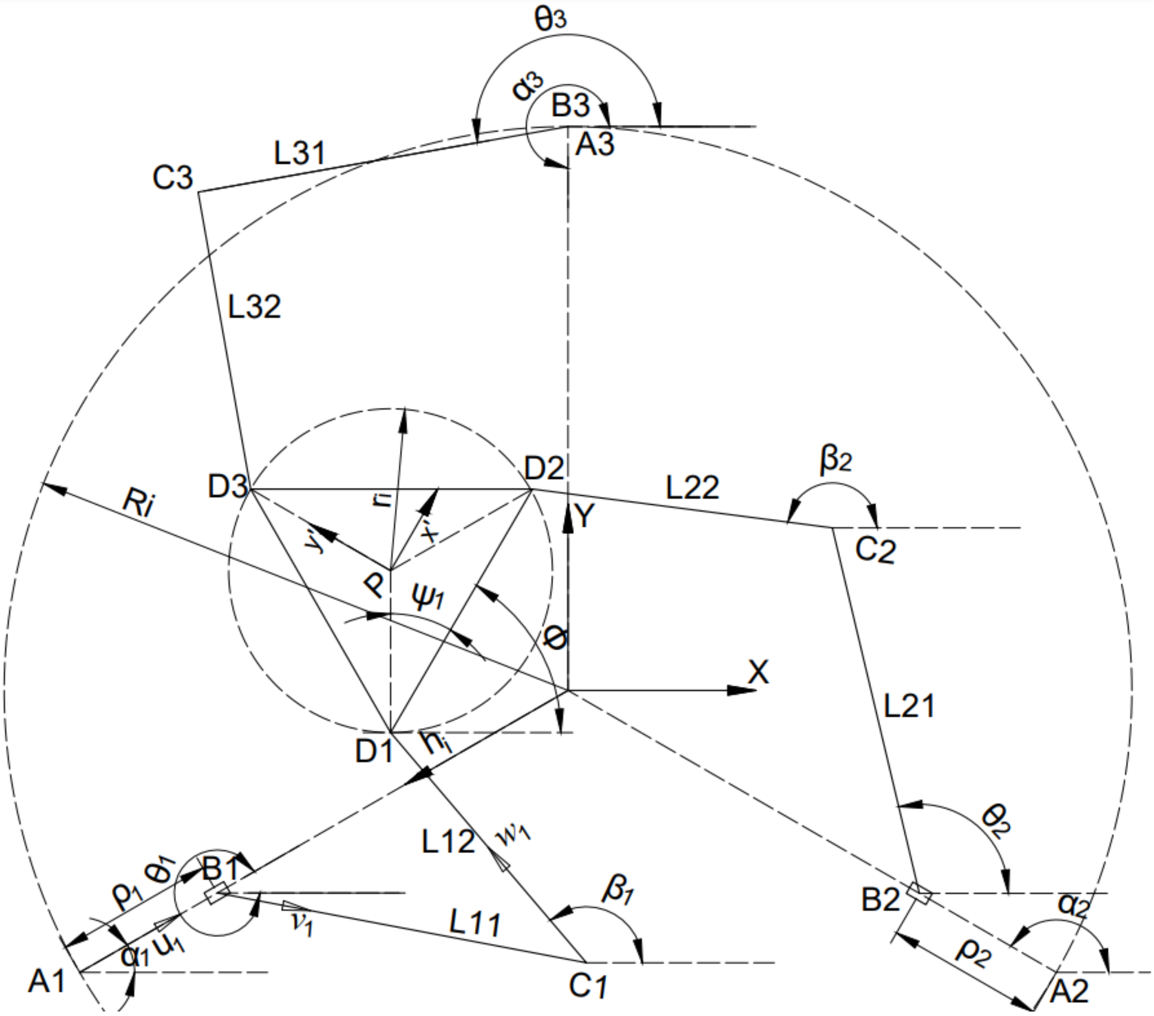

Fig.1 Kinematic Model of 2-PRRR – 1RRRPlanar Parallel manipulator

2.1 Input- Output relations of the 2-PRRR – 1RRR Planar Parallel manipulator

The input and output relations of limbs are derived from direct kinematic relations, the equations are as follows

$l_{i1}^2 = \left(x_p + rc_{(\emptyset+\varphi_i)} - Rc_{\tau_i} - \rho_i c_{\alpha_i} - l_{i1}c_{\theta_i}\right)^2 + \left(y_p + rs_{(\emptyset+\varphi_i)} - Rs_{\tau_i} - \rho_i s_{\alpha_i} - l_{i1}s_{\theta_i}\right)^2 (1)$

for i=1,2

$l_i^2 = \left(x_p + rc_{(\emptyset+\varphi_i)} - Rc_{\tau_i} - l_{i1}c_{\theta_i}\right)^2 + \left(y_p + rs_{(\emptyset+\varphi\psi_i)} - Rs_{\tau_i} - l_{i1}s_{\theta_i}\right)^2$------- (2)

for i=3

The input output relation (1) is expressed in the form of bi circular quadratic as follows

$x^2y^2 + a_{11}xy^2 + x^2 + a_{12}y^2 - a_{13}x - a_{14}xy - a_{15}y + a_{16} = 0$ --- (3)

The equation 3 can also be used for limb 3 by substituting the corresponding length is equals to zero.

where

x is the actuated prismatic joint displacement and y is the tan half angle of actuated revolute joint displacement.

$x = \rho_i$, for i=1,2 and $x = 0$ for i=3 $y = \tan(\theta_i/2)$ for i=1,2 and 3

$a_{11} = 2l_{i2}c\alpha_i - 2Rc(\gamma_i - \alpha_i) - 2(x_p c\alpha_i + y_p s\alpha_i) + 2rc(\emptyset + \varphi_i - \alpha_i)$

$a_{12} = 2rl_{i2}[c(\emptyset + \varphi_i) + s(\emptyset + \varphi_i)] + 2l_{i2}x_p + 2Rl_{i2} + x_p^2 + y_p^2 + r^2 + R^2 + l_{i2}^2 - l_{i1}^2 + 2r[x_p c(\emptyset + \varphi_i) + y_p s(\emptyset + \varphi_i)] + 2rRc(\emptyset + \varphi_i - \gamma_i) + 2R(x_p c\gamma_i + y_p s\gamma_i)$

$a_{13} = 2l_{i2}c\alpha_i + 2Rc(\gamma_i - \alpha_i) + 2(x_p c\alpha_i + y_p s\alpha_i) - 2rc(\emptyset + \varphi_i - \alpha_i)$

$a_{14} = 4l_{i2}s\alpha_i$

$a_{15} = 2[2rl_{i2}s(\emptyset + \varphi_i) + 2y_p l_{i2} + 2Rl_{i2}s\gamma_i]$

$$a_{16} = -2rl_{i2}[c(\emptyset + \varphi_i) + s(\emptyset + \varphi_i)] - 2l_{i2}x_p - 2Rl_{i2} + x_p^2 + y_p^2 + r^2 + R^2 + l_{i2}^2 - l_{i1}^2 + 2r[x_p c(\emptyset + \varphi_i) + y_p s(\emptyset + \varphi_i)] + 2rRc(\emptyset + \varphi_i - \gamma_i) + 2R(x_p c\gamma_i + y_p s\gamma_i)$$

The constant design parameters of mechanism are as follows:
$\gamma_1 = 210, \gamma_2 = 330,\ \gamma_3 = 90;\ \alpha_1 = 30,\ \alpha_2 = 150,\ \alpha_3 = 270;\quad \varphi_1 = 30,\ \varphi_2 = 150,\ \varphi_2 = 90;$

3.0 Sensitivity coefficients

$$P = \begin{bmatrix} P_x \\ P_y \end{bmatrix} = a_i + (b_i - a_i) + (c_i - b_i) + (d_i - c_i) + (p_i - d_i),\ i = 1,2 \quad (1)$$
$$= b_i + (c_i - b_i) + (d_i - c_i) + (p_i - d_i),\ i = 3$$

$$P_x = R_i cos\alpha_i + \rho_i \cos\alpha_i + l_{i1}\cos\theta_i + l_{i2}\cos\beta_i + r_i\cos\gamma_i \text{ (i=1,2)}$$
$$= R_i cos\alpha_i + l_{i1}\cos\theta_i + l_{i2}\cos\beta_i + r_i\cos\gamma_i \text{ (i=3)}$$
$$P_y = R_i sin\alpha_i + \rho_i \sin\alpha_i + l_{i1}\sin\theta_i + l_{i2}\sin\beta_i + r_i\sin\gamma_i \text{ (i=1,2)}$$
$$= R_i sin\alpha_i + l_{i1}\sin\theta_i + l_{i2}\sin\beta_i + r_i\sin\gamma_i \text{ (i=3)}$$

a_i, b_i and c_i being the position vectors of points A_i, B_i and C_i expressed in Fig. Eq. (1) can also be written as,

$$P = R_i h_i + \rho_i u_i + l_{i1} v_i + l_{i2} w_i + r_i k_i \text{(i=1,2)} \quad (2)$$
$$= R_i h_i + d_i v_i + l_i w_i + r_i k_i \text{(i=3)}$$

with

$$h_i = \begin{bmatrix} -\cos\alpha_i \\ -\sin\alpha_i \end{bmatrix},\ u_i = \begin{bmatrix} \cos\alpha_i \\ \sin\alpha_i \end{bmatrix},\ v_i = \begin{bmatrix} \cos\theta_i \\ \sin\theta_i \end{bmatrix},\ w_i = \begin{bmatrix} \cos\beta_i \\ \sin\beta_i \end{bmatrix},$$

$$k_i = \begin{bmatrix} cos(\emptyset + \varphi_i + \pi) \\ sin(\emptyset + \varphi_i + \pi) \end{bmatrix}$$

Where R_i is the distance between points O and A_i, ρ_i is the variable distance between points A_i and B_i, l_{i1} is the variable distance between points B_i and C_i, l_{i2} is the variable distance between points C_i and D_i, r is the distance between points D_i and P.

$$\delta p = \delta R_i h_i + R_i \delta\alpha_i E h_i + \delta\rho_i u_i + \rho_i \delta\theta_i E u_i + \delta l_{i1} v_i + l_{i1}\delta\theta_i E v_i + \delta l_{i2} w_i + l_{i2}\delta\beta_i E v_i + \delta r_i k_i + r_i(\delta\emptyset + \delta\varphi_i)E k_i \quad \text{(i=1,2)} \quad (3)$$

$$= \delta R_i h_i + R_i \delta\alpha_i E h_i + \delta l_{i1} v_i + l_{i1}\delta\theta_i E v_i + \delta l_{i2} w_i + l_{i2}\delta\beta_i E v_i + \delta r_i k_i + r_i(\delta\emptyset + \delta\varphi_i)E k_i \quad \text{(i=3)}$$

With matrix E defined as

$$E=\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \quad (4)$$

δp and $\delta\emptyset$ are the moving platform position and moving platform orientation errors. Likewise δR_i, $\delta\alpha_i, \delta\rho_i, \delta l_{i1}, \delta l_{i2}, \delta r_i$ *and* $\delta\beta_i$ denote the variations in $R_i, \alpha_i, \rho_i, l_{i1},\ l_{i1}, r$ and β_i respectively.

$$l_{i2} w_i^T \delta p = l_{i2}\delta R_i w_i^T h_i + l_{i2} R_i \delta\alpha_i w_i^T E h_i + l_{i2}\delta\rho_i w_i^T u_i + l_{i2}\rho_i\delta\alpha_i w_i^T E u_i + l_{i2}\delta l_{i1} w_i^T v_i + l_{i2} l_{i1}\delta\theta_i w_i^T E v_i + l_{i2}\delta l_{i2} + l_i \delta r_i w_i^T k_i + l_{i2} r_i(\delta\emptyset + \delta\varphi_i) w_i^T E k_i \quad \text{(i=1,2)}$$
$$= l_{i2}\delta R_i w_i^T h_i + l_{i2} R_i \delta\alpha_i w_i^T E h_i + l_{i2}\delta l_{i1} w_i^T v_i + l_{i2} l_{i1}\delta\theta_i w_i^T E v_i + l_{i2}\delta l_{i2} + l_i\delta r_i w_i^T k_i + l_{i2} r_i(\delta\emptyset + \delta\varphi_i) w_i^T E k_i \quad \text{(i=3)} \quad (5)$$

Eq. (5) can now be cast in vector form:

$$A\begin{bmatrix} \delta\emptyset \\ \delta p \end{bmatrix} = H_R \begin{bmatrix} \delta R_1 \\ \delta R_2 \\ \delta R_3 \end{bmatrix} + H_\alpha \begin{bmatrix} \delta\alpha_1 \\ \delta\alpha_2 \\ \delta\alpha_3 \end{bmatrix} + B \begin{bmatrix} \delta\rho_1 + \delta\theta_1 \\ \delta\rho_2 + \delta\theta_2 \\ \delta\theta_3 \end{bmatrix} + H_{li1} \begin{bmatrix} \delta l_{i1} \\ \delta l_{i2} \\ \delta l_{i3} \end{bmatrix} + H_{li2} \begin{bmatrix} \delta l_{21} \\ \delta l_{22} \\ \delta l_{23} \end{bmatrix} + H_\gamma \begin{bmatrix} \delta\gamma_1 \\ \delta\gamma_2 \\ \delta\gamma_3 \end{bmatrix} + H_r \begin{bmatrix} \delta r_1 \\ \delta r_2 \\ \delta r_3 \end{bmatrix} \quad (6)$$

with

$$A = \begin{bmatrix} m_1 & l_{12} w_1^T \\ m_2 & l_{22} w_2^T \\ m_3 & l_{32} w_3^T \end{bmatrix}, \quad (6a)$$

$$B = diag[l_{12} w_2^T (u_1 + v_1) \quad l_{22} w_1^T (u_2 + v_2) \quad l_{32} w_1^T (v_3)] \quad (6b)$$

$$H_R = diag[l_{12} w_1^T h_1 l_{22} w_2^T h_2 l_{32} w_3^T h_3] \quad (6c)$$

$$H_{li1} = diag[l_{11} l_{21} l_{31}] \quad (6d)$$

$$H_{li2} = diag[l_{12} l_{22} l_{32}] \quad (6e)$$

$$H_r = diag[l_{12} w_1^T k_1 l_{22} w_2^T k_2 l_{32} w_3^T k_3] \quad (6f)$$

$$H_\gamma = diag[l_{12} r_1 w_1^T E k_1 l_{22} r_2 w_2^T E k_2 l_{32} r_3 w_3^T E k_3] \quad (6g)$$

$$H_\alpha = diag[l_{12} R_1 W_1^T E h_1 + l_{12} \rho_1 W_1^T E u_1 l_{22} R_2 W_2^T E h_2 + l_{22} \rho_2 W_2^T E u_2 l_{32} R_3 W_3^T E h_3] \quad (6h)$$

And

$$m_i = -l_{i2} r_i w_i^T E k_i, i = 1,..,3 \quad (7)$$

It is clear that A and B are the direct and the inverse Jacobian matrices of the manipulator, respectively. Assuming that A is nonsingular, i.e., the manipulator does not meet any second type singularity [10 –13], we obtain upon multiplication of Eq. (6) by A^{-1}:

$$\begin{bmatrix} \delta \emptyset \\ \delta p \end{bmatrix} = J_R \begin{bmatrix} \delta R_1 \\ \delta R_2 \\ \delta R_3 \end{bmatrix} + J_\alpha \begin{bmatrix} \delta \alpha_1 \\ \delta \alpha_2 \\ \delta \alpha_3 \end{bmatrix} + J \begin{bmatrix} \delta \rho_1 + \delta \theta_1 \\ \delta \rho_2 + \delta \theta_2 \\ \delta \theta_3 \end{bmatrix} + J_{l_{i1}} \begin{bmatrix} \delta l_{11} \\ \delta l_{12} \\ \delta l_{13} \end{bmatrix} + J_{l_{i2}} \begin{bmatrix} \delta l_{21} \\ \delta l_{22} \\ \delta l_{23} \end{bmatrix} + J_r \begin{bmatrix} \delta r_1 \\ \delta r_2 \\ \delta r_3 \end{bmatrix} + J_\gamma \begin{bmatrix} \delta \gamma_1 \\ \delta \gamma_2 \\ \delta \gamma_3 \end{bmatrix} \quad (8)$$

With

$$J = A^{-1} B \quad (8a)$$
$$J_R = A^{-1} H_R \quad (8b)$$
$$J_\alpha = A^{-1} H_\alpha \quad (8c)$$
$$J_{l1i} = A^{-1} H_{l1i} \quad (8d)$$
$$J_{l2i} = A^{-1} H_{l2i} \quad (8e)$$
$$J_r = A^{-1} H_r \quad (8f)$$
$$J_\gamma = A^{-1} H_\gamma \quad (8g)$$

And

$$A^{-1} = \frac{1}{det(A)} \begin{bmatrix} w_1 & w_2 & w_3 \\ x_1 & x_2 & x_3 \end{bmatrix} \quad (9)$$

$$\omega_i = l_j l_k (w_j \times w_k)^T k \quad (9a)$$

$$x_i = E(m_j l_k w_k - m_k l_j w_j) \quad (9b)$$

$$det(A) = \sum_{i=1}^{3} m_i \omega_i \quad (9c)$$

$j = (i + 1)$ Modulo 3;
$k = (i + 2)$ Modulo 3; i =1, 2, 3.

$$\begin{bmatrix} \delta a_{ix} \\ \delta a_{iy} \end{bmatrix} = \begin{bmatrix} 0 & -\rho_i \sin \alpha_i \\ 0 & \rho_i \cos \alpha_i \end{bmatrix} \begin{bmatrix} \delta \rho_i \\ \delta \alpha_i \end{bmatrix} \text{ (i=1,2)} \quad (10)$$

$$\begin{bmatrix} \delta b_{ix} \\ \delta b_{iy} \end{bmatrix} = \begin{bmatrix} cos\beta_i & -r_i \sin \beta_i \\ sin\beta_i & r_i \cos \beta_i \end{bmatrix} \begin{bmatrix} \delta r_i \\ \delta \beta_i \end{bmatrix} \text{ (i=1,2,3)} \quad (11)$$

$$\begin{bmatrix} \delta c_{ix} \\ \delta c_{iy} \end{bmatrix} = \begin{bmatrix} cos\alpha_i & -R_i \sin \alpha_i \\ sin\alpha_i & R_i \cos \alpha_i \end{bmatrix} \begin{bmatrix} \delta R_i \\ \delta \alpha_i \end{bmatrix} \text{ (i=1,2,3)} \quad (12)$$

$$\begin{bmatrix} \delta d_{ix} \\ \delta d_{iy} \end{bmatrix} = \begin{bmatrix} cos\Theta_i & 0 \\ sin\Theta_i & 0 \end{bmatrix} \begin{bmatrix} \delta l_{i1} \\ \delta \Theta_i \end{bmatrix} \text{ (i=1,2,3)} \quad (13)$$

$$\begin{bmatrix}\delta\emptyset\\ \delta p\end{bmatrix}=J_A\begin{bmatrix}\delta a_{1x}\\ \delta a_{1y}\\ \delta a_{2x}\\ \delta a_{2y}\\ 0\\ 0\end{bmatrix}+J\begin{bmatrix}\delta\rho_1\\ \delta\Theta_1\\ \delta\rho_2\\ \delta\Theta_2\\ 0\\ \delta\Theta_3\end{bmatrix}+J_D\begin{bmatrix}\delta d_{1x}\\ \delta d_{1y}\\ \delta d_{2x}\\ \delta d_{2y}\\ \delta d_{3x}\\ \delta d_{3y}\end{bmatrix}+J_B\begin{bmatrix}\delta b_{1x}\\ \delta b_{1y}\\ \delta b_{2x}\\ \delta b_{2y}\\ \delta b_{3x}\\ \delta b_{3y}\end{bmatrix}+J_{(li1+li2)}\begin{bmatrix}\delta l_{11}\\ \delta l_{21}\\ \delta l_{12}\\ \delta l_{22}\\ \delta l_{13}\\ \delta l_{23}\end{bmatrix} \quad (14)$$

$$J_A=[J_{A1}J_{A2}\quad 0] \quad (14a)$$

$$J_B=[J_{B1}J_{B2}J_{B3}] \quad (14b)$$

$$J_D=[J_{D1}J_{D2}J_{D3}] \quad (14d)$$

$$J=[J_1J_2J_3] \quad (14e)$$

$$J_{li1}=[J_{l11}J_{l12}J_{l13}]$$
$$J_{li2}=[J_{l21}J_{l22}J_{l23}] \quad (14f)$$

$$J_{Ai}=\begin{bmatrix}J_{Ai\emptyset}\\ J_{Aip}\end{bmatrix}, i=1,2 \quad (14g)$$

$$J_{Bi}=\begin{bmatrix}J_{Bi\emptyset}\\ J_{Bip}\end{bmatrix}, i=1,2,3 \quad (14h)$$

$$J_{Ci}=\begin{bmatrix}J_{Ci\emptyset}\\ J_{Cip}\end{bmatrix}, i=1,2,3 \quad (14i)$$

$$J_{Di}=\begin{bmatrix}J_{Di\emptyset}\\ J_{Dip}\end{bmatrix}, i=1,2,3 \quad (14j)$$

$$J_{i}=\begin{bmatrix}J_{i\emptyset}\\ J_{ip}\end{bmatrix}, i=1,2,3 \quad (14k)$$

$$J_{li1}=\begin{bmatrix}J_{li1\emptyset}\\ J_{li1p}\end{bmatrix}, i=1,2,3 \quad (14l)$$

$$J_{li2}=\begin{bmatrix}J_{li2\emptyset}\\ J_{li2p}\end{bmatrix}, i=1,2,3 \quad (15)$$

With

$$J_{Ai\emptyset}=\frac{1}{det(A)}[\omega_i o_i\quad \omega_i p_i] \quad (16)$$

$$J_{Bi\emptyset}=\frac{1}{det(A)}[\omega_i q_i\quad \omega_i r_i] \quad (17)$$

$$J_{Ci\emptyset}=\frac{1}{det(A)}[\omega_i s_i\quad \omega_i t_i] \quad (18)$$

$$J_{Di\emptyset}=\frac{1}{det(A)}[\omega_i e_i\quad \omega_i f_i] \quad (19)$$

$$J_{i\emptyset}=\frac{\omega_i l_{i2} w_i^T v_i}{det(A)} \quad (20)$$

$$J_{(li2)\emptyset}=\frac{\omega_i l_{i2}}{det(A)} \quad (21)$$

$$J_{Aip}=\frac{1}{det(A)}\begin{bmatrix}o_i x_i^T i & p_i x_i^T i\\ o_i x_i^T j & p_i x_i^T j\end{bmatrix} \quad (22)$$

$$J_{Bip}=\frac{1}{det(A)}\begin{bmatrix}q_i x_i^T i & r_i x_i^T i\\ q_i x_i^T j & r_i x_i^T j\end{bmatrix} \quad (22)$$

$$J_{Cip} = \frac{1}{det(A)}\begin{bmatrix} s_i x_i^T i & t_i x_i^T i \\ s_i x_i^T j & t_i x_i^T j \end{bmatrix} \quad (23)$$

$$J_{Dip} = \frac{1}{det(A)}\begin{bmatrix} e_i x_i^T i & f_i x_i^T i \\ e_i x_i^T j & f_i x_i^T j \end{bmatrix} \quad (24)$$

$$J_{ip} = \frac{1}{det(A)}\begin{bmatrix} l_i w_i^T i v_i x_i^T i \\ l_i w_i^T i v_i x_i^T j \end{bmatrix} \quad (25)$$

$$J_{(li1+li2)p} = \frac{1}{det(A)}\begin{bmatrix} l_i x_i^T i \\ l_i x_i^T j \end{bmatrix} \quad (26)$$

o_i, p_i, q_i, r_i, s_i, t_i, e_i, and f_i taking the form:

$$o_i = l_{i2} w_i^T i \quad (26a)$$

$$p_i = l_{i2} w_i^T j \quad (26b)$$

$$q_i = -l_{i2} w_i^T E u_i \sin\alpha_i \quad (26c)$$

$$r_i = l_{i2} w_i^T E u_i \cos\alpha_i \quad (26d)$$

$$s_i = -l_{i2} w_i^T v_i \sin\alpha_i \quad (26e)$$

$$t_i = l_{i2} w_i^T v_i \cos\alpha_i \quad (26f)$$

$$e_i = l_{i2} w_i^T k_i \cos\beta_i - l_{i2} w_i^T k_i \sin\beta_i \quad (26g)$$

$$f_i = l_{i2} w_i^T k_i \sin\beta_i + l_{i2} w_i^T k_i \cos\beta_i \quad (26h)$$

$J_{Ai\emptyset}, J_{Bi\emptyset}, J_{Ci\emptyset}, J_{Di\emptyset}$ and $J_{(li1+li2)\emptyset}$ contain the sensitivity coefficients of the MP orientation to variations in the Cartesian coordinates of points.
$J_{Aip}, J_{Bip}, J_{Cip}, J_{Dip}$ and $J_{(li1+li2)p}$ contain the sensitivity coefficients of the MP position to variations in the Cartesian coordinates of points

3.1 Two aggregate sensitivity indices

The current section intends to find the indices so as to compare distinct PPMs with respect to sensitivity of the moving platform's pose to differences in their geometric parameters.

$$\begin{bmatrix} \delta\emptyset \\ \delta P \end{bmatrix} = [J_{SM2} V_{M2}] \quad (27)$$

$$\boldsymbol{J_{S_{M2}}} = [\boldsymbol{J_{A_{M2}}} \quad \boldsymbol{J_{B_{M2}}} \quad \boldsymbol{J_{l_{i1M}} J_{l_{i2M}}} \quad \boldsymbol{J_{D_{M2}}}] \quad (28)$$

$\boldsymbol{J_{C_{M2}}}$ is not required as there is a variation in the li1 and li2 taken for consideration

$$\boldsymbol{V_{VM2}} = [\boldsymbol{\delta_{ai}} \quad \boldsymbol{\delta_{bi}} \quad \boldsymbol{\delta_{l_{i2M}}} \quad \boldsymbol{\delta_{D_{M2}}}] \quad (29)$$

with

$$\boldsymbol{\delta b_i} = [\boldsymbol{\delta a_{1x}} \quad \boldsymbol{\delta a_{1y}} \quad \boldsymbol{\delta a_{2x}} \quad \boldsymbol{\delta a_{2y}} \quad \boldsymbol{\delta a_{3x}} \quad \boldsymbol{\delta a_{3y}}] \quad (29a)$$

$$\boldsymbol{\delta b_i} = [\boldsymbol{\delta b_{1x}} \quad \boldsymbol{\delta b_{1y}} \quad \boldsymbol{\delta b_{2x}} \quad \boldsymbol{\delta b_{2y}} \quad \boldsymbol{\delta b_{3x}} \quad \boldsymbol{\delta b_{3y}}] \quad (29b)$$

$$\boldsymbol{\delta \rho_i} = [\boldsymbol{\delta \rho_1} \quad \boldsymbol{\delta \rho_2} \quad \boldsymbol{0}] \quad (29c)$$

$$\boldsymbol{\delta d_i} = [\boldsymbol{\delta d_{1x}} \quad \boldsymbol{\delta d_{1y}} \quad \boldsymbol{\delta d_{2x}} \quad \boldsymbol{\delta d_{2y}} \quad \boldsymbol{\delta d_{3x}} \quad \boldsymbol{\delta d_{3y}}] \quad (29d)$$

The 3 $\times n_M$ matrices Js_M comprises of two blocks, $js_{M\phi}$ and Js_{Mp}, i.e.,

$$J_{S_M} = \begin{bmatrix} J_{S_{M\emptyset}} \\ J_{S_{Mp}} \end{bmatrix} \quad (30)$$

$$J_{S_{M\emptyset}} = [J_{1\emptyset} J_{2\emptyset}\ J_{B1\emptyset} J_{B2\emptyset} J_{B3\emptyset} J_{D1\emptyset} J_{D2\emptyset} J_{D3\emptyset}] \quad (31)$$

$$J_{S_{Mp}} = [J_{1p} J_{2p}\ J_{B1p} J_{B2p} J_{B3p} J_{D1p} J_{D2p} J_{D3p}] \quad (32)$$

$v_{\emptyset M}$ which is the aggregate sensitivity index of the moving platform orientation to changes in its geometric parameters can be expressed as:

$$v_{\emptyset M} = \frac{\|J_{S_{M\emptyset}}\|_2}{n_V} \quad (33)$$

Likewise, an aggregate sensitivity index v_{pM} of the moving platform position to variations in its geometric parameters is defined as:

$$v_{pM} = \frac{\|J_{S_{Mp}}\|_2}{n_N} \quad (34)$$

3.1.2 Workspace of Manipulator

Regular workspace of robot manipulator is defined as the set of points that can be reached by its end effector. Put in other words, the workspace of a robot is the space in which the mechanism is working

Reachable Position Workspace: The reachable workspace, as defined by as the set of points that can be reached by a reference point on a manipulator with at least one orientation and does not include singular points where the manipulator loses one or more degrees of freedom

Full Orientation Workspace: The fully dexterous workspace or the full orientation workspace is defined as a space in which a point is approached in all directions. The dexterous workspace is a subset of the reachable workspace.

Operating Volume: The operating volume is the total volume of space that the manipulator and its links occupy while reaching every point in the workspace.

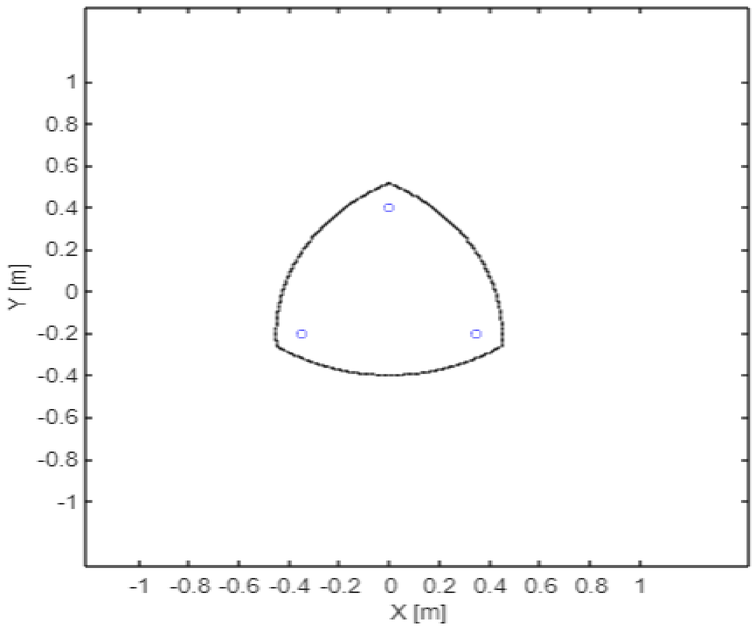

Fig 2 Workspace of 2PRRR 1RRR Manipulator

3.2 Sensitivity Analysis

Planar parallel manipulators are suitable for high-speed applications, therefore the sensitivity of a manipulator is one of the important performance indices and it should be considered in the design. Based on the position Jacobian (J_x) and orientation Jacobian (J_q) the sensitivity index is derived.

The manipulator configuration and variations in the geometric parameters alter the manipulator's pose errors. In order to analyse the influence of the manipulator configuration on those errors. The sensitivity index S_q of the moving platform due to orientation is given as

$$S_q = \frac{\|J_q\|}{\mathrm{N}}$$

The sensitivity index S_x of the moving platform due position is given as [9]:

$$S_x = \frac{\|J_x\|}{\mathrm{N}}$$

(Where, N is number of geometric variables)

4.0 Results and Discussion

To demonstrate the approach, geometric parameters, link lengths (l_{i2}, l_{i3}) as 0.2 m and prismatic link length (l_{i1}) as 0.1m are considered. The fixed and mobile platforms are treated as equilateral triangles of sizes 0.5 m and 0.2 m respectively. The non-dimensional parameters can be obtained by dividing each variable by R. From Eq. (5) l_{i1}=0.3636, l_{i2} = l_{i3} = 0.7273, h_b=1.8182, h_m = 0.7273, ϕ = 0° and the number of geometric variables (N) (l_{i1}, l_{i2}, l_{i3}, h_b, θ_i, β_i, ϕ_i) The iso-contour lines based on orientation sensitivity index is spread uniformly in the entire workspace of this manipulator. The values are ranging from 0.039 to 0.021 shown in Fig. 2. shows with different colours ranging from maximum to minimum.

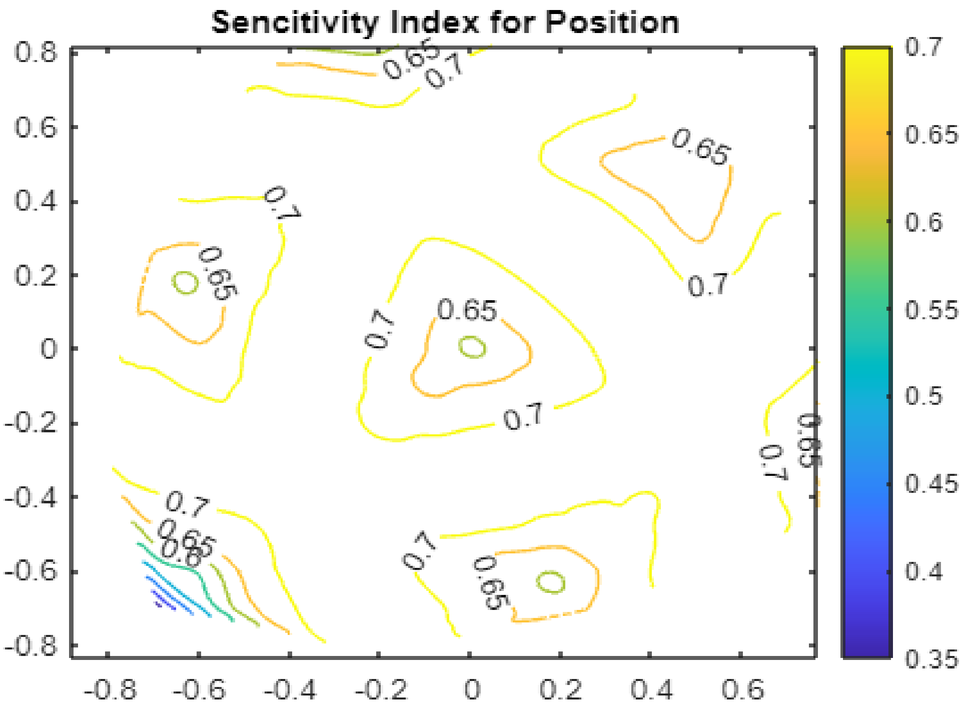

Fig. 3. 2PRRR – 1RRR Sensitivity Indices for position
(X and Y axis values indicates CG of MP reach)

The position sensitivity index iso-contour lines are spread in the workspace and are confining to the central region and boundaries for the 2PRRR – 1RRR manipulator. The values are ranging from 0.014 to 0.048 shown in Fig. 3. When compared to orientation the positioning sensitivity index values are better for the same manipulator.

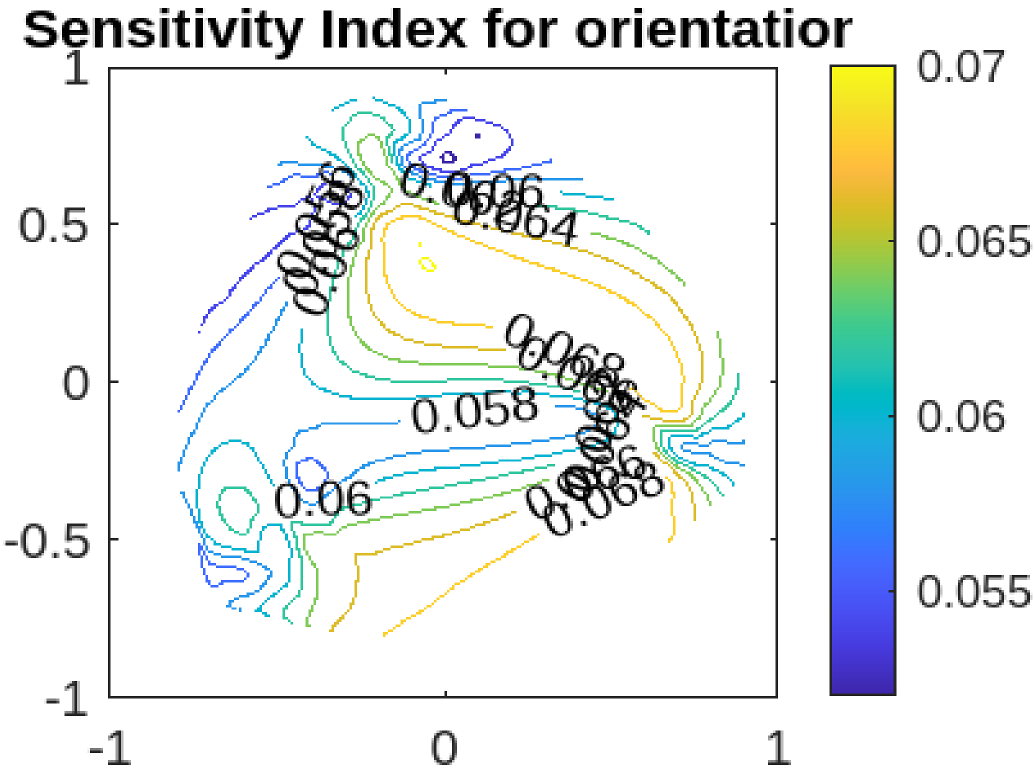

Fig. 4, 2PRRR – 1RRR Sensitivity indices for orientation
(X and Y axis values indicates CG of MP reach)

5.0 Conclusions

This paper investigates the performance based on sensitivity performance evaluation of Planar Parallel Manipulators by introducing two additional branches. The new proposed structure becomes kinematically redundant. The results clearly depict that the planar parallel manipulator having branches additionally has relatively superior sensitivity performance in orientation as well as in position. The kinematically redundant manipulator with additional branches is suitable for precise positioning applications. While selecting the manipulator the designer should also consider other dexterity indices for best performance.

References

1. Stephane Caro. “Sensitivity Analysis of 3-RPR Planar Parallel Manipulators,” ASME the Journal of Mechanical Design, Vol. 131, 121005 pp 1-13, Dec 2009.
2. Stephane Caro D. Chablat, R. Ur-Rehman, and P. Wenger. “Multi-objective Design Optimization of 3–PRR Planar Parallel Manipulators,” A. Bernard (ed.), Global Product Development, Springer-Verlag Berlin Heidelberg, 2011.
3. Stephane Caro, Philippe Wenger, Fouad Bennis and Damien Chablat. “Sensitivity Analysis of the Orthoglide: A Three-DOF Translational Parallel Kinematic Machine”, Journal of Mechanical Design, Transactions of the ASME, Vol. 298, pp 392-402, March 2006.
4. Saheb, S. H. (2017). Design and analysis of light weight agriculture robot. Global Journals of Research in Engineering, 17(6), 23-39.
5. Caro, S., Wenger, P., Bennis, F., and Chablat, D. (June 8, 2005). "Sensitivity Analysis of the Orthoglide: A Three-DOF Translational Parallel Kinematic Machine." ASME. J. Mech. Des. March 2006; Vol.128(2): pp. 392–402.
6. Wenger, P., Gosselin, C., and Maillé, B., 1999, “A Comparative Study of Serial and Parallel Mechanism Topologies for Machine Tool,” Int. Workshop on Parallel Kinematic Machines, Milan, Italy, pp. 23–35.
7. Nicolas Binaud, Stéphane Caro and Philippe Wenger, “Sensitivity comparison of planar parallel manipulators”, Mechanism and Machine Theory, Vol 45, pp 1477–1490, 2010.
8. Saheb, S. H., & Satish Babu, G. (2023). Robustness Indices of 3R and 4R Planar Serial Manipulators with Fixed Actuation Scheme. In Computer Communication, Networking and IoT (pp. 105-115). Springer, Singapore.
9. Sukham Lee and sungbok kim, “Kinematic Analysis of Generalized Parallel Manipulator Systems”, Proceedings of the 32nd conference on Decision and control, San Antonlo, Texas, Dec 1993.
10. Saheb, S. H., & Babu, G. S. (2021). Kinematic Analysis of 3RRR Planar Parallel Manipulator by Using Adaptive Neuro Fuzzy Inference System (ANFIS). Solid State Technology, 64(2), 01-12.
11. Iman Ebrahimi, Juan A. Carretero and Roger Boudreau, “ A Family of Kinematically Redundant Planar Parallel Manipulators”, Journal of Mechanical Design, Transactions of the ASME, Vol. 130,pp.405-413, 2008.
12. O. Ma and J. Angeles. “Optimum Architecture and Design of Platform Manipulators,” IEEE Proc. International Conference on Advanced Robotics, pp.1130-1135 (1991).
13. Han Sung Kim and Lung-Wen Tsai. “Design Optimization of a Cartesian Parallel Manipulator,” ASME the Journal of Mechanical Design, Vol. 125, pp 43-56, March 2003.