Applications of Karry-Kalim-Adnan Transformations (KKAT) to Newton's Law of Cooling

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ABSTRACT: In this paper we have used the integral transform known as Karry-Kalim-Adnan Transformation (KKAT) to solve the problems of Newton's law of cooling.

KEYWORDS: Karry-Kalim-Adnan Transformation (KKAT), Newton's law of cooling.

I. INTRODUCTION:

Integral transformation is a very important tool to solve the differential equation for engineers and scientists. Integral transforms are widely used to determine the solution of differential equations with the initial values. Integral transforms have played a great role in engineering, chemistry, biology, astronomy, social sciences, radio physics, and nuclear science.
KKAT is closely related to Laplace transformation, Fourier transformation, Sumudu transformation, Elzaki transformation, Aboodh transformation, Mahgoub transformation, etc.
In general integral transformation, we take a set of functions of {f(x)} in the region $-\infty < a \le x \le b < \infty$, and then, we select a field function κ(x,β). So the integral transformation is defined by

$$T\{f(x)\} = F(\beta) = \int_a^b f(x)\kappa(x,\beta)dx$$

where the function $\kappa(x,\beta)$ is known as the kernel of the transformation T and β is a parameter. There are different kinds of transformations depending upon the kernel and the limits a and b.
Recently, integral transforms are one of the most useful and simple mathematical technique for obtaining the solutions of advance problems occurred in many fields like science, Engineering, technology, commerce and economics. To provide exact solution of problem without lengthy calculations is the important feature of integral transforms.
Due to this important feature of the integral transforms many researchers are attracted to this field and are engaged in introducing various integral transforms. Recently, Kushare and Patil [1] introduced new integral transform called as Kushare transform for solving differential equations in time domain. Further, Savita Khakale and Dinkar Patil [2] introduced Soham transform in November 2021. As researchers are interested in introducing the new integral transforms at the same time they are also interested in applying the transforms to various fields, various equations in different domain. In January 2022, Sanap and Patil [3] used Kushare transform for obtaining the solution of the problems on Newton's law of Cooling.
In April 2022 D. P. Patil, et al [4] solved the problems on growth and decay by using Kushare transform. D.P. Patil [5] also used Sawi transform in Bessel functions. Further, Patil [6] evaluate improper integrals by using Sawi transform of error functions. Laplace transforms and Shehu transforms are used in chemical sciences by Patil [7]. Dinkar Patil [8] used Sawi transform and its convolution theorem for solving wave equation. Using Mahgoub transform, parabolic boundary value problems are solved by D .P. Patil [9].
D .P. Patil [10] used double Laplace and double Sumudu transforms to obtain the solution of wave equation. Further Dr. Patil [11] also obtained dualities between double integral transforms. Kandalkar, Gatkal and Patil [12] solved the system of differential equations using Kushare transform. D. P. Patil [13] solved boundary value problems of the system of ordinary differential equations by using Aboodh and Mahgoub transforms. Double Mahgoub transformed is used by Patil [14] to solve parabolic boundary value problems.
Laplac, Sumudu , Aboodh , Elazki and Mahagoub transforms are compared and used it for solving boundary value problems by Dinkar Patil [15]. D. P. Patil et al [16] solved Volterra Integral equations of first kind by using Emad-Sara transform. Futher Patil with Tile and Shinde [17] used Anuj transform and solved Volterra integral equations for first kind. Rathi sisters and D. P. Patil [18] solved system of differential equations by using Soham transform. Vispute, Jadhav and Patil [19] used Emad Sara transform for solving telegraph equation. Kandalkar, Zankar and Patil [20] evaluate the improper integrals by using general integral transform of error function. Dinkar Patil, Prerana Thakare and Prajakta Patil[21] obtained the solution of parabolic boundary value problems by using double general integral transform. Dinkar Patil used Emad- Falih transform for solving problems based on Newton's law of cooling [22]. D. P. Patil et al [23] used Soham transform to obtain the solution of Newton's law of cooling. Dinkar Patil et al [24] used HY integral transform for handling growth and Decay problems, D. P. Patil et al used HY transform for Newton's law of cooling [25]. D. P. Patil et al [26] used Emad-Falih transform for general solution of telegraph equation. Dinkar Patil et al [27] introduced double kushare transform. Recently, D. P. Patil et al [28] solved population growth and decay problems by using Emad Sara transform. Alenzi transform is used in population growth and decay problems by patil et al [29]. Thete, et al [30] used Emad Falih transform for handling growth and decay problems. Nikam, Patil et al [31] used , Kushare transform of error functions in evaluating improper integrals. Wagh sisters and Patil used Kushare [32] and Soham [33] transform in chemical Sciences. Malpani, Shinde and Patil [34] used Convolution theorem for Kushare transform and applications in convolution type Volterra integral equations of first kind. Raundal and Patil [35] used double general integral transform for solving boundary value problems in partial differential equations. Rahane, Derle and Patil [36] eveloped generalized double rangaig integral transform. Kushare transform is used for solving Volterra Integro-Differential equations of first kind by Shinde, et al [38]. Kandekar et al [39] used new general

integral equation to solve Abel's integral equations. Pardeshi, Shaikh and Patil[40] used Kharrat Toma transform for solving population growth and decay problems.

II. PRELIMINARY:

2.1. DEFINITION OF KKAT[37]:

A transformation defined for function of exponential order from set S

$S = \{ (x) : \exists P;\ a_1, a_2 > 0, |f(x)| < Pe^{|x|/a_i}\},\ x \epsilon (=1)^i \times [0,\infty)$,

where constant P is a finite number and a_1, a_2 may be finite or infinite.

KKAT is represented by operator K(.) and is defined by

$$K\{f(x)\} = \frac{1}{\beta}\int_0^\infty f(\alpha x)e^{-\beta x}dx,\ x \geq 0;\ \alpha, \beta \in [a_1, a_2],$$

where α and β are constants and α, β ≠ 0.x K{$f(x)$} can also be written in the form

$$K\{f(x)\} = \frac{1}{\alpha\beta}\int_0^\infty f(x)e^{-(\beta/\alpha)x}dx = F\left(\frac{\beta}{\alpha}\right).$$

Also

$$f(x) = K^{-1}\left\{F\left(\frac{\beta}{\alpha}\right)\right\}.$$

2.2. KKAT OF SOME IMPORTANT FUNCTIONS[37]:

Sr. No.	$f(x)$	$F(\beta/\alpha)$
1	1	$1/\beta^2$
2	x^n	$\frac{n!.\alpha^n}{\beta^{n+2}}$
3	$e^{\lambda x}$	$1/\beta(\beta - \lambda\alpha)$
4	$sin\ \lambda x$	$\lambda\alpha/\beta(\beta^2 + \lambda^2\alpha^2)$
5	$cos\ \lambda x$	$\frac{1}{\beta^2 + \lambda^2\alpha^2}$
6	$f'(x)$	$\frac{\beta}{\alpha} K\{f(x)\} - \frac{f(0)}{\alpha\beta} K$
7	$f''(x)$	$(\beta^2/\alpha^2)\ K\{f(x)\} - f'(0)/\alpha\beta - f(0)/\alpha^2$

2.3. LINEARITY PROPERTY OF KKAT[37]:

If $G_1(\beta/\alpha)$, $G_2(\beta/\alpha)$ are KKAT of $g_1(x)$, $g_2(x)$ ϵ A, then KKAT of $ag_1(x) + bg_2(x)$ is $aG_1(\beta/\alpha) + bG_2(\beta/\alpha)$, where a, b are constant.

Proof : By definition of KKAT, we get

$$K\{f(x)\} = \frac{1}{\alpha\beta}e^{-(\beta/\alpha)x}g(x)dx$$

$$K\big(ag_1(x) + bg_2(x)\big) = \frac{1}{\alpha\beta}\int_0^\infty e^{-\left(\frac{\beta}{\alpha}\right)x}(ag_1(x) + bg_2(x))dx$$

$$= a\frac{1}{\alpha\beta}\int_0^\infty e^{-\left(\frac{\beta}{\alpha}x\right)}g_1(x)\,dx + b\frac{1}{\alpha\beta}\int_0^\infty e^{-\left(\frac{\beta}{\alpha}\right)x}g_2(x)\,dx$$

Hence, we obtain

$$K\big(ag_1(x) + bg_2(x)\big) = aG_1\left(\frac{\beta}{\alpha}\right) + bG_2\left(\frac{\beta}{\alpha}\right)$$

2.4. INVERSE OF KKAT[37]:

If $F\left(\frac{\beta}{\alpha}\right)$ be the KKAT of $f(x)$, then $f(x)$ is called the inverse KKAT of $F\left(\frac{\beta}{\alpha}\right)$. The inverse KKAT is expressed in the following equation,

$$K^{-1}\left\{F\left(\frac{\beta}{\alpha}\right)\right\} = f(x)$$

III. Applications:

In this section we use KKAT to to solve the problems based on Newton's law of cooling.

Example 1: A heated metal beam cools at the rate of 30°C per minute when its temperature is 50°C. Find the time taken to cool at 36°C if the temperature of the surroundings is 27°C.

Solution: Assuming that a heated metal beam obeys Newton's law of cooling, we have

$$\frac{df}{dx} = -C(f - 27)\ ,\ f(0) = 50,\ f'(0) = 3$$

First, we will find the value of C by using initial condition,

$$-3 = -C(50 - 27)$$

$$\Rightarrow -3 = -23C$$

$$\Rightarrow C = 0.13$$

$$\therefore \frac{df}{dx} = -0.13(f - 27) \qquad \ldots.(1)$$

Applying KKAT to equation (1),

$$K\{f'(x)\} = -0.13\, K\{f(x)\} + 0.13 \text{ x } 27\, K\{1\}$$

$$\frac{\beta}{\alpha} K\{f(x)\} - \frac{f(o)}{\alpha\beta} = -0.13\, K\{f(x)\} + 0.13 \text{ x } 27 \text{ x } \frac{1}{\beta^2}$$

$$K\{f(x)\}\left[\frac{\beta}{\alpha} + 0.13\right] = \frac{50}{\alpha\beta} + \frac{0.13 \text{ x } 27}{\beta^2}$$

$$K\{f(x)\}\left[\frac{\beta+0.13\alpha}{\alpha}\right] = \frac{50}{\alpha\beta} + \frac{0.13 \text{ x } 27}{\beta^2}$$

$$K\{f(x)\} = \frac{50}{\alpha\beta}\left[\frac{\alpha}{\beta+0.13\alpha}\right] + \frac{0.13 \text{ x } 27}{\beta^2}\left[\frac{\alpha}{\beta+0.13\alpha}\right]$$

$$= 50\left[\frac{1}{\beta(\beta+0.13\alpha)}\right] + \frac{27}{\beta^2}\left[\frac{0.13\alpha}{\beta+0.13\alpha}\right]$$

$$= 50\left[\frac{1}{\beta(\beta+0.13\alpha)}\right] + \frac{27}{\beta^2}\left[\frac{\beta+0.13\alpha-\beta}{\beta+0.13\alpha}\right]$$

$$= 50\left[\frac{1}{\beta(\beta+0.13\alpha)}\right] + 27\left[\frac{\beta+0.13\alpha}{\beta^2(\beta+0.13\alpha)} - \frac{\beta}{\beta^2(\beta+0.13\alpha)}\right]$$

$$= 50\left[\frac{1}{\beta(\beta+0.13\alpha)}\right] + 27\left[\frac{1}{\beta^2} - \frac{1}{\beta(\beta+0.13\alpha)}\right]$$

$$= 50\left[\frac{1}{\beta(\beta+0.13\alpha)}\right] + 27\left[\frac{1}{\beta^2}\right] - 27\left[\frac{1}{\beta(\beta+0.13\alpha)}\right]$$

$$= 27\left[\frac{1}{\beta^2}\right] + (50-27)\left[\frac{1}{\beta(\beta+0.13\alpha)}\right]$$

$$K\{f(x)\} = 27\left[\frac{1}{\beta^2}\right] + 23\left[\frac{1}{\beta(\beta+0.13\alpha)}\right] \qquad \ldots (2)$$

Now, apply inverse of KKAT to equation (2), we get

$$f(x) = 27K^{-1}\left\{\left(\frac{1}{\beta^2}\right)\right\} + 23K^{-1}\left\{\frac{1}{\beta(\beta+0.13\alpha)}\right\}$$

$$= 27(1) + 23e^{-0.13x}$$

$$f(x) = 27 + 23e^{-0.13x}$$

Putting f = 36 in above equation, we get

$$36 = 27 + 23e^{-0.13x}$$

$$23e^{-0.13x} = 9$$

$$e^{-0.13x} = 0.3913$$

$$e^{0.13x} = 2.5556$$

$$0.13x = 0.9383$$

$$x = 7.2176 \text{ min.}$$

∴ A heated metal beam will take 7.22 minutes for cooling to a temperature of 36°C.

Example 2: You take an ice-cream out of the freezer, kept at -18°C. Outside it is 32°C. After one minute, the ice-cream has warmed to -8°C. What is the temperature of the ice-cream after 5 minutes?

Solution: By Newton's law of cooling,

$$\frac{df}{dx} = -C(f-32)\,,\ f(o) = -18$$

$$\frac{df}{dx} = -Cf(x) + \text{C x } 32 \qquad \ldots (3)$$

First, we have to find the value of C.

Applying KKAT to equation (2),

$$K\{f'(x)\} = -C\, K\{f(x)\} + C\text{x } 32\, K\{1\}$$

$$\frac{\beta}{\alpha} K\{f(x)\} - \frac{f(o)}{\alpha\beta} = -C\, K\{f(x)\} + 32 \text{ x C x } \frac{1}{\beta^2}$$

$$K\{f(x)\}\left[\frac{\beta}{\alpha} + C\right] = \frac{f(0)}{\alpha\beta} + \frac{32\,C}{\beta^2}$$

$$K\{f(x)\}\left[\frac{\beta+C\alpha}{\alpha}\right] = \frac{-18}{\alpha\beta} + \frac{32C}{\beta^2}$$

$$K\{f(x)\} = \frac{-18}{\alpha\beta}\left[\frac{\alpha}{\beta+C\alpha}\right] + \frac{32C}{\beta^2}\left[\frac{\alpha}{\beta+C\alpha}\right]$$

$$= -18\left[\frac{1}{\beta(\beta+C\alpha)}\right] + \frac{32}{\beta^2}\left[\frac{C\alpha}{\beta+0.13\alpha}\right]$$

$$= -18\left[\frac{1}{\beta(\beta+C\alpha)}\right] + \frac{32}{\beta^2}\left[\frac{\beta+C\alpha-\beta}{\beta+C\alpha}\right]$$

$$= -18\left[\frac{1}{\beta(\beta+0.13\alpha)}\right] + 32\left[\frac{\beta+C\alpha}{\beta^2(\beta+C\alpha)} - \frac{\beta}{\beta^2(\beta+C\alpha)}\right]$$

$$= -18\left[\frac{1}{\beta(\beta+C\alpha)}\right] + 32\left[\frac{1}{\beta^2} - \frac{1}{\beta(\beta+C\alpha)}\right]$$

$$= -18\left[\frac{1}{\beta(\beta+0.13\alpha)}\right] + 32\left[\frac{1}{\beta^2}\right] - 32\left[\frac{1}{\beta(\beta+C\alpha)}\right]$$

$$= 32\left[\frac{1}{\beta^2}\right] + (-18-32)\left[\frac{1}{\beta(\beta+C\alpha)}\right]$$

$$K\{f(x)\} = 32\left[\frac{1}{\beta^2}\right] - 50\left[\frac{1}{\beta(\beta+C\alpha)}\right] \qquad \ldots (4)$$

Now, apply inverse of KKAT to equation (4), we get

$$f(x) = 32K^{-1}\left\{\left(\frac{1}{\beta^2}\right)\right\} - 50K^{-1}\left\{\frac{1}{\beta(\beta+C\alpha)}\right\}$$
$$= 32(1) - 50e^{-Cx}$$
$$f(x) = 32 - 50e^{-Cx}$$

We have given that, after one minute, the ice-cream has warmed to -8°C.

$$\therefore f = -8 \text{ at } x = 1$$
$$\therefore -8 = 32 - 50\, e^{-C(1)}$$
$$\therefore -8 = 32 - 50\, e^{-C}$$
$$\therefore 50\, e^{-C} = 40$$
$$\therefore e^{-C} = \frac{4}{5} \Rightarrow e^{C} = \frac{5}{4}$$
$$\Rightarrow C = ln\left(\frac{5}{4}\right) = 0.231$$
$$\Rightarrow f(x) = 32 - 50e^{-0.2231x}$$

Hence, when x = 5, we have,

$$f(5) = 32 - 50e^{-0.2231(5)}$$
$$f(5) = 32 - 50e^{-1.1155}$$
$$= 32 - 50 \text{ x } 0.3278$$
$$f(5) = 15.6124°C$$

∴ The temperature of ice-cream after 5 minutes is 15.6124°C.

Example 3: If some water at 10°C is put into a freezer at -10°C, it takes an hour to reach 0°C. If water is put into the freezer and then an hour later the water's temperature is measured at 10°C, what was the original temperature of the water?

Solution: Here, we are given the system's parameter at two times and we are asked to find the system's parameter at a third time.

By Newton's law of cooling,

$$\frac{df}{dx} = -C(f - (-10)),\ f(o) = 10$$
$$\frac{df}{dx} = -Cf(x) - C \text{ x } 10 \qquad \text{.... (5)}$$

First, we have to find the value of C.

Applying KKAT to equation (5),

$$K\{f'(x)\} = -C\, K\{f(x)\} - C\text{x } 10\, K\{1\}$$
$$\frac{\beta}{\alpha}K\{f(x)\} - \frac{f(o)}{\alpha\beta} = -C\, K\{f(x)\} - 10 \text{ x C x } \frac{1}{\beta^2}$$
$$K\{f(x)\}\left[\frac{\beta}{\alpha} + C\right] = \frac{f(0)}{\alpha\beta} - \frac{10\,C}{\beta^2}$$
$$K\{f(x)\}\left[\frac{\beta+C\alpha}{\alpha}\right] = \frac{10}{\alpha\beta} - \frac{10C}{\beta^2}$$
$$K\{f(x)\} = \frac{10}{\alpha\beta}\left[\frac{\alpha}{\beta+C\alpha}\right] - \frac{10C}{\beta^2}\left[\frac{\alpha}{\beta+C\alpha}\right]$$
$$= 10\left[\frac{1}{\beta(\beta+C\alpha)}\right] - \frac{10}{\beta^2}\left[\frac{C\alpha}{\beta+0.13\alpha}\right]$$
$$= 10\left[\frac{1}{\beta(\beta+C\alpha)}\right] - \frac{10}{\beta^2}\left[\frac{\beta+C\alpha-\beta}{\beta+C\alpha}\right]$$
$$= 10\left[\frac{1}{\beta(\beta+0.13\alpha)}\right] - 10\left[\frac{\beta+C\alpha}{\beta^2(\beta+C\alpha)} - \frac{\beta}{\beta^2(\beta+C\alpha)}\right]$$
$$= 10\left[\frac{1}{\beta(\beta+C\alpha)}\right] - 10\left[\frac{1}{\beta^2} - \frac{1}{\beta(\beta+C\alpha)}\right]$$
$$= 10\left[\frac{1}{\beta(\beta+0.13\alpha)}\right] - 10\left[\frac{1}{\beta^2}\right] + 10\left[\frac{1}{\beta(\beta+C\alpha)}\right]$$
$$= -10\left[\frac{1}{\beta^2}\right] + (10+10)\left[\frac{1}{\beta(\beta+C\alpha)}\right]$$
$$K\{f(x)\} = -10\left[\frac{1}{\beta^2}\right] + 20\left[\frac{1}{\beta(\beta+C\alpha)}\right] \qquad \text{.... (6)}$$

Now, apply inverse of KKAT to equation (4), we get

$$f(x) = -10K^{-1}\left\{\left(\frac{1}{\beta^2}\right)\right\} + 20K^{-1}\left\{\frac{1}{\beta(\beta+C\alpha)}\right\}$$
$$= -10(1) + 20e^{-Cx}$$
$$f(x) = -10 + 20e^{-Cx}$$

An hour later, temperature of water is 0°C.

$$\therefore f(1) = 0 \text{ when } x = 1$$
$$0 = -10 + 20e^{-C(1)}$$
$$\Rightarrow 20e^{-C} = 10$$
$$\Rightarrow e^{-C} = \frac{1}{2}$$
$$\Rightarrow -C = ln\left(\frac{1}{2}\right)$$
$$\Rightarrow C = ln\,(2)$$

$$\therefore f(x) = -10 + 20e^{-ln\,(2)\,x}$$

We want to find the temperature of the water an hour before its temperature is 10°C, i.e., at x = -1.

$$\therefore f(-1) = -10 + 20e^{-ln\,(2)\,(-1)}$$
$$= -10 + 20e^{ln\,(2)}$$
$$= -10 + 20 \text{ x } 2$$
$$\therefore f(-1) = 30°C$$

Which is the temperature of water an hour before it is measured at 10°C.

Example 4: You can find the temperature inside your refrigerator without putting a thermometer inside. Take a can of soda from the refrigerator, let it warm for half an hour, then record its temperature. Let it warm for another half an hour and record its temperature again. Suppose that the readings are $f\left(\frac{1}{2}\right)$ = 45°F and $f(1)$ = 55°F. Assuming that the room temperature is 70°C, what is the temperature inside the refrigerator?

Solution: Taking the time one half hour after the soda was removed from the refrigerator to be the "zero time" (and starting the given information in a appropriate way), we have, by Newton's law of cooling,

$$\frac{df}{dx} = -C(f-70)\ ,\ f(o) = 45,\ f(1/2) = 55$$
$$\frac{df}{dx} = -Cf(x) + \text{C x } 70 \qquad \text{.... (7)}$$

First, we have to find the value of C.

Applying KKAT to equation (7),

$$K\{f'(x)\} = -C\ K\{f(x)\} + C\text{x } 70\ K\{1\}$$
$$\frac{\beta}{\alpha}\ K\{f(x)\} - \frac{f(o)}{\alpha\beta} = -C\ K\{f(x)\} + 70 \text{ x C x } \frac{1}{\beta^2}$$
$$K\{f(x)\}\left[\frac{\beta}{\alpha} + C\right] = \frac{f(0)}{\alpha\beta} + \frac{70\,C}{\beta^2}$$
$$K\{f(x)\}\left[\frac{\beta+C\alpha}{\alpha}\right] = \frac{45}{\alpha\beta} + \frac{70C}{\beta^2}$$
$$K\{f(x)\} = \frac{45}{\alpha\beta}\left[\frac{\alpha}{\beta+C\alpha}\right] + \frac{70C}{\beta^2}\left[\frac{\alpha}{\beta+C\alpha}\right]$$
$$= 45\left[\frac{1}{\beta(\beta+C\alpha)}\right] + \frac{70}{\beta^2}\left[\frac{C\alpha}{\beta+0.13\alpha}\right]$$
$$= 45\left[\frac{1}{\beta(\beta+C\alpha)}\right] + \frac{70}{\beta^2}\left[\frac{\beta+C\alpha-\beta}{\beta+C\alpha}\right]$$
$$= 45\left[\frac{1}{\beta(\beta+0.13\alpha)}\right] + 70\left[\frac{\beta+C\alpha}{\beta^2(\beta+C\alpha)} - \frac{\beta}{\beta^2(\beta+C\alpha)}\right]$$
$$= 45\left[\frac{1}{\beta(\beta+C\alpha)}\right] + 70\left[\frac{1}{\beta^2} - \frac{1}{\beta(\beta+C\alpha)}\right]$$
$$= 45\left[\frac{1}{\beta(\beta+0.13\alpha)}\right] + 70\left[\frac{1}{\beta^2}\right] - 70\left[\frac{1}{\beta(\beta+C\alpha)}\right]$$
$$= 70\left[\frac{1}{\beta^2}\right] + (45-70)\left[\frac{1}{\beta(\beta+C\alpha)}\right]$$
$$K\{f(x)\} = 70\left[\frac{1}{\beta^2}\right] - 25\left[\frac{1}{\beta(\beta+C\alpha)}\right] \qquad \text{.... (8)}$$

Now, apply inverse of KKAT to equation (8), we get

$$f(x) = 70K^{-1}\left\{\left(\frac{1}{\beta^2}\right)\right\} - 25K^{-1}\left\{\frac{1}{\beta(\beta+C\alpha)}\right\}$$
$$= 70(1) - 25e^{-Cx}$$
$$f(x) = 70 - 25e^{-Cx}$$

At x = ½, $f(x)$ = 55.

$$\therefore 55 = 70 - 25e^{-C\left(\frac{1}{2}\right)}$$
$$\Rightarrow e^{-C/2} = \frac{3}{5}$$
$$\frac{-C}{2} = ln\left(\frac{3}{5}\right)$$
$$\Rightarrow \frac{C}{2} = ln\left(\frac{5}{3}\right) = 0.5108$$

We want to find temperature at x = -1/2,

$$f(-1/2) = 70 - 25e^{-C\left(-\frac{1}{2}\right)}$$
$$= 70 - 25e^{C/2}$$
$$= 70 - 25e^{ln\left(\frac{5}{3}\right)}$$
$$= 70 - 25 \text{ x } \frac{5}{3}$$
$$f(-1/2) = 28.3°F.$$

5. Conclusion: We successfully solved problems based on Newton's law of cooling by using KKAT.

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