A New Class of continuous Sets In Nano Topological Spaces

Ms. F. Monisha[1], Dr. J. Arul Jesti[2]

[1]Research Scholar, Reg.No.20112212092001, Department of Mathematics, St. Mary's College (Autonomous),
[2]Assistant Professor,Department of Mathematics, St. Mary's College(Autonomous),
[1,2](Affiliated to Manonmaniam Sundaranar University, Abishekapatti, Tirunelveli)
Thoothukudi-1,TamilNadu, India

ABSTRACT: The aim of this paper is to introduce a new class of continuous sets, namely $N\delta g^{\wedge}$ -continuous and $N\delta g^{\wedge}$ - continuous sets in Nano topological spaces. Further we investigate fundamental properties are discussed. Additionally we relate with some other Nano topological spaces.

Keywords : Nano topological spaces, generalized closed sets, δg - closed sets, $\delta - closure$, $g^{\wedge}$ - open sets, α-continuous .

1 Introduction

The concept of continuity plays major role in general topology. In general, a continuous function is one, for which small changes in the input result in changes in the input result changes in the output. Many authors have studied different types of continuity. Topology, as a branch of mathematics, can be formally defined as " the study of qualitative" properties of certain objects (called topological spaces) that are invariant under certain kind of transformations (called continuous maps). Especially those properties that are invariant under a certain kind of equivalence. Further we have shown that composition of two $N\delta g^{\wedge}$ - continuous need not be $N\delta g^{\wedge}$ -continuous.

2 PRELIMINARIES

Throughout this chapter$(U, \tau_R(X))$ is a Nano topological space with respect to X where $X \subseteq U$, R is an equivalence relation on U, U/R denotes the family of equivalence classes of U by R. Here we recall the following known definitions and properties.

Definition 2.1[8] Let U be a non empty finite set of objects called the ***universe*** and R be an equivalence relation on U named as the indiscernibility relation. Then U is divided into disjoint equivalence classes. Elements belonging to the same equivalence class are said to be discernible with one another. The pair (U, R) is said to be the approximation space. Let $X \subseteq U$

1. The lower approximation of X with respect to R is the set of all objects which can be for certain classified as X with respect to R and it is denoted by$L_R(X)$. That is $L_R(X) = U_{x \in U}\{R(x) / R(x) \subseteq X\}$ where $R(x)$ denotes the equivalence class determined by X.
2. The upper approximation of X with respect to R is the set of all objects which can be possibly defined as X with respect to R and it is denoted by $U_R(X)$. That is UR(X) = $U_{x \in U}\{R(x) / R(x) \cap X \neq \phi\}$
3. The boundary region of X with respect to R is the set of all objects which can be neither as X nor as not X with respect to R and is denoted by $B_R(X)$. That is $B_R(X) = U_R(X) - L_R(X)$.

Proposition 2.2[2] If (U, R) is an approximation space and $X, Y \subseteq U$, then

1. $L_R(X) \subseteq X \subseteq U_R(X)$
2. $L_R(\emptyset)= U_R(\emptyset) = \emptyset$ and $L_R(U) = U_R(U) = U$
3. $U_R(X \cup Y) = U_R(X) \cup U_R(Y)$
4. $U_R(X \cap Y) \subseteq U_R(X) \cap U_R(Y)$
5. $L_R(X \cup Y) \supseteq L_R(X) \cup L_R(Y)$
6. $L_R(X \cap Y) = L_R(X) \cap L_R(Y)$
7. $L_R(X) \subseteq L_R(Y)$ and $U_R(X) \subseteq U_R(Y)$whenever $X \subseteq Y$
8. $U_R(X^c)= [L_R(X)]^c$ and $L_R(X^c) = [U_R(X)]^c$
9. URUR(X) = LRUR(X) = UR(X)
10. $L_R\, L_R(X) = UR\, L_R(X) = L_R(X)$

Definition 2.3[1] Let U be the universe, R be an ***equivalence relation*** on U and $\tau_R(X) = \{U, \emptyset, L_R(X), U_R(X), B_R(X)\}$ where X⊆U. Then by the proposition 2.2, $R(X)$ satisfies the following axioms:

1. U and $\emptyset \in \tau_R(X)$
2. The union of the elements of any subcollection of $(U, \tau_R(X))$ is in $(U, \tau_R(X))$.
3. The intersection of the elements of any finite subcollection of $(U, \tau_R(X))$ is in $(U, \tau_R(X))$.

That is $(U, \tau_R(X))$ is a topology on U called the Nano topology on U with respect to X. We call $(U, \tau_R(X))$ as the Nano topological space. The elements of $(U, \tau_R(X))$ are called as Nano open sets.

Remark 2.4[1] If $(U, \tau_R(X))$ is the Nano topology on U with respect to X, then the set $B = \{U, \emptyset, L_R(X), B_R(X)\}$ is the ***basis*** for $\tau_R(X)$.

Definition 2.5[1] If $(U, \tau_R(X))$ is a Nano topological space with respect to X and if $A \subseteq U$, then the ***Nano interior*** of A is defined as the union of all Nano-open subsets of A and is denoted by $Nint(A)$.That is, $Nint(A)$ is the largest Nano-open subset of A.

The ***Nano closure*** of A is defined as the intersection of all Nano -closed sets containing A and it is denoted by $Ncl(A)$. That is, $Ncl(A)$ is the smallest Nano-closed set containing A.

Definition 2.6[1,5] Let $(U, \tau_R(X))$ be a Nano topological space and $A \subseteq U$. Then A is said to be

(i) ***Nano pre-open*** if $A \subseteq Nint(Ncl(A))$
(ii) ***Nano semi-open*** if $A \subseteq Ncl(Nint(A))$
(iii) ***Nano α-open*** if $A \subseteq Nint(Ncl(Nint(A)))$

The complements of the above mentioned sets are called their respective ***Nano-closed*** sets.

Definition 2.7[7] Let $(U, \tau_R(X))$ be a Nano topological space. A subset A of $(U, \tau_R(X))$ is called ***Nano generalized-closed set*** (briefly $Ng -$ closed) if $Ncl(A) \subseteq V$ where $A \subseteq V$ and V is Nano-open.

The complement of Nano generalized -closed set is called as ***Nano generalized-open set***.

Definition 2.8[9] For every set $A \subseteq U$, the ***Nano generalized closure of A*** is defined as the intersection of all Ng- closed sets containing A and is denoted by $Ng - cl(A)$.

Definition 2.9[9] For every set $A \subseteq U$, the ***Nano generalized interior of A*** is defined as the union of all Ng- open sets contained in A and is denoted by $Ng - int(A)$.

Propsition 2.10[9] For any $A \subseteq U$ **,**

(i) $NgCl(A)$ is the smallest Ng closed set containing A.
(ii) A is Ng- closed if and only if $NgCl(A) = A$.
(iii) $A \subseteq NgCl(A) \subseteq Cl(A)$

Proposition 2.11[9] For any two subsets A and B of U,

(i) If $A \subseteq B$, then $NgCl(A) \subseteq NgCl(B)$
(ii) $NgCl(A \cap B) \subseteq NgCl(A) \cap NgCl(B)$

Definition 2.12 The nano δ – interior [13] of a subset A of X is the union of all regular open set of X contained in A and is denoted by $NInt_{\delta}(A)$. The subset A is called $N\delta$-open [13] if $A = NInt_{\delta}(A)$, i.e. a set is $N\delta$-open if it is the union of regular open sets. The complement of a $N\delta$-open is called $N\delta$-closed. Alternatively, a set $A \subseteq (U, \tau_R(X))$ is called $N\delta$-closed [13] if $A = Ncl_{\delta}(A)$, where $Ncl_{\delta}(A) = \{x \in X : int(cl(U) \cap A \neq \emptyset,\ U \in \tau_R(X)\ and\ x \in U\}$.

Definition 2.13[3] Let $(U, \tau_R(X))$ and $(V, \tau'_R(Y))$ be two Nano topological spaces. Then a mapping $f: (U, \tau_R(X)) \to (V, \tau'_R(Y))$ is Nano-continuous function on U if the inverse image of every Nano-open set in $(V, \tau'_R(Y))$ is Nano-open in $(U, \tau_R(X))$.

Definition 2.14[7] Let $(U, \tau_R(X))$ and $(V, \tau'_R(Y))$be two Nano topological spaces. Then a mapping $f: (U, \tau_R(X)) \to (V, \tau'_R(Y))$is **Nano generalized-continuous** function (shortly Ng-continuous) on U if the inverse image of every Nano-open set in $(V, \tau'_R(Y))$ is Nano generalized-open in $(U, \tau_R(X))$.

Definition 2.15[4] Let $(U, \tau_R(X))$ and $(V, \tau'_R(Y))$be two Nano topological spaces. Then a mapping $f: (U, \tau_R(X)) \to (V, \tau'_R(Y))$ is **Nano semi-continuous** function on U if the inverse image of every Nano-open set in $(V, \tau'_R(Y))$ is Nano semi-open in $(U, \tau_R(X))$.

Definition 2.16[12] Let $(U, \tau_R(X))$ and $(V, \tau'_R(Y))$be two Nano topological spaces. Then a mapping $f: (U, \tau_R(X)) \to (V, \tau'_R(Y))$is **Nano semi*-continuous** function on U if the inverse image of every Nano-open set in $(V, \tau'_R(Y))$ is Nano semi*-open in $(U, \tau_R(X))$.

Definition 2.17[4] Let $(U, \tau_R(X))$ and $(V, \tau'_R(Y))$be two Nano topological spaces. Then a mapping $f: (U, \tau_R(X)) \to (V, \tau'_R(Y))$ is **Nano α-continuous** function on U if the inverse image of every Nano-open set in $(V, \tau'_R(Y))$ is Nano α -open in $(U, \tau_R(X))$.

Definition 2.18[5] Let $(U, \tau_R(X))$ and $(V, \tau'_R(Y))$be two Nano topological spaces. Then a mapping $f: (U, \tau_R(X)) \to (V, \tau'_R(Y))$ is **Nano β-continuous** function on U if the inverse image of every Nano-open set in $(V, \tau'_R(Y))$ is Nano β -open in $(U, \tau_R(X))$.

Definition 2.19[3] Let $(U, \tau_R(X))$ and $(V, \tau'_R(Y))$ be two Nano topological spaces. Then a mapping $f: (U, \tau_R(X)) \to (V, \tau'_R(Y))$ is **Nano pre-open continuous** function on U if the inverse image of every Nano-open set in $(V, \tau'_R(Y))$ is Nano pre-open open in $(U, \tau_R(X))$.

3 $N\delta g^{\wedge}$ - CONTINUOUS FUNCTIONS

Definition 3.1 Let $(U, \tau_R(X))$and $(V, \tau'_R(Y))$be two Nano topological spaces. Then a mapping $f: (U, \tau_R(X)) \to (V, \tau'_R(Y))$ is $\boldsymbol{N\delta g^{\wedge}}$ –continuous function on U if the inverse image of every Nano-open set in $(V, \tau'_R(Y))$ is $\boldsymbol{N\delta g^{\wedge}}$ -open in $(U, \tau_R(X))$.

Example 3.2 Let $U = \{a, b, c, d\}$ with $U/R = \{\{a\}, \{b, c\}, \{d\}\}$ and $X = \{a, b\}$. Then $\tau_R(X)$=$\{U, \phi, \{a\}, \{b, c\}, \{a, b, c\}\}$. Then $N\delta g^{\wedge}$-open sets are $\{U, \phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}\}$. Let $V = \{x. y. z, w\}$ with $V/R' = \{\{x\}, \{y, w\}, \{z\}\}$ and $Y = \{x, y\}$. Then $\tau'_R(Y) = \{V, \phi, \{x\}, \{y, w\}, \{x, y, w\}\}$. Define $f: (U, \tau_R(X)) \to (V, \tau'_R(Y))$ as $f(a) = y$, $f(b) = z, f(c) = x, f(d) = w$ Then $f^{-1}(\{x\}) = \{c\}, f^{-1}(\{y, w\}) = \{a, d\}$,

f^{-1} $(\{x,y,w\}) = \{a,c,d\}$ and f^{-1} $(V) = U$. That is, the inverse image of every Nano-open set in V is $N\delta g^{\wedge}$ −open set in U. Therefore f is $N\delta g^{\wedge}$ -continuous.

Theorem 3.3 Every Nano-continuous function is $N\delta g^{\wedge}$ −continuous.

Proof : Let $f: (U,\tau_R(X)) \to (V,\tau'_R(Y))$ be Nano-continuous on $(U,\tau_R(X))$. Since f is Nano- continuous of $(U,\tau_R(X))$, the inverse image of every Nano open set in $(V,\tau'_R(Y))$ is Nano- open in $(U,\tau_R(X))$. But every Nano-open set is $N\delta g^{\wedge}$ −open set. Hence the inverse image of every Nano open set in $(V,\tau'_R(Y))$is $N\delta g^{\wedge}$ −open in $(U,\tau_R(X))$. Therefore f is $N\delta g^{\wedge}$ - continuous.

Remark 3.4 The converse of the above theorem is not true as seen from the following example.

Example 3.5 Let $U = \{a,b,c,d\}$ with $U/R = \{\{a\},\{b,c\},\{d\}\}$ and $X = \{a,b\}$. Then $\tau_R(X)$=$\{U ,\phi ,\{a\},\{b,c\},\{a,b,c\}\}$. Then $\boldsymbol{N\delta g^{\wedge}}$ -open sets are $\{U ,\phi ,\{a\},\{b\},\{c\},\{a,b\},\{a,c\},\{a,d\},\{b,c\},\{b,d\},\{c,d\},\{a,b,c\},\{a,b,d\},\{a,c,d\},\{b,c,d\}\}$. Let $V = \{x.y.z,w\}$ with $V/R' = \{\{x\},\{y,w\},\{z\}\}$ and $Y = \{x,y\}$. Then $\tau'_R(Y) = \{V,\phi ,\{x\},\{y,w\},\{x,y,w\}\}$. Define $f: (U,\tau_R(X)) \to (V,\tau'_R(Y))$ as $f(a) = x$, $f(b) = y, f(c) = z, f(d) = w$ .Then f^{-1} $(\{y,w\}) = \{b,d\}$ which is not Nano-open set in $(U,\tau_R(X))$. Hence f is not Nano-continuous.

Theorem 3.6 Every Nano α −continuous function is Nano $N\delta g^{\wedge}$ -continuous.

Proof : Let $f: (U,\tau_R(X)) \to (V,\tau'_R(Y))$ be Nano α -continuous on $(U,\tau_R(X))$. Let C be Nano open in V. Since f is Nano α-continuous of $(U,\tau_R(X))$,the inverse image of every Nano-open set in $(V,\tau'_R(Y))$ is Nano α-open in $(U,\tau_R(X))$. Hence f^{-1} (C) is $N\delta g^{\wedge}$ -open in $(U,\tau_R(X))$. But every Nano α-open set is $N\delta g^{\wedge}$ -open set. Therefore f^{-1} (C) is $N\delta g^{\wedge}$ -open in $(U,\tau_R(X))$.Hence the inverse image of every Nano-open set in $(V,\tau'_R(Y))$ is $N\delta g^{\wedge}$ −open in $(U,\tau_R(X))$.Therefore f is $N\delta g^{\wedge}$ -continuous.

Remark 3.7 The converse of the above theorem is not true as seen from the following example.

Example 3.8 Let $U = \{a,b,c,d\}$ with $U/R = \{\{a\},\{b,c\},\{d\}\}$ and $X = \{a,b\}$. Then $\tau_R(X) = \{U,\phi,\{a\},\{b,c\},\{a,b,c\}\} = N\alpha O$. Then $N\delta g^{\wedge}$−open sets are $\{U,\phi,\{a\},\{b\},\{c\},\{a,b\},\{a,c\},\{a,d\},\{b,c\},\{b,d\},\{c,d\},\{a,b,c\},\{a,b,d\},\{a,c,d\},\{b,c,d\}\}$. Let $V = \{x.y.z,w\}$ with $V/R' = \{\{x\},\{y,w\},\{z\}\}$ and $Y = \{x,y\}$. Then $\tau'_R(Y) = \{V,\phi ,\{x\},\{y,w\},\{x,y,w\}\}$. Define $f: (U,\tau_R(X)) \to (V,\tau'_R(Y))$ as $f(a) = x$, $f(b) = y$, $f(c) = z$, $f(d) = w$. Then f^{-1} $(\{x,y,w\}) = \{a,b,d\}$ which is not Nano α-open set in $(U,\tau_R(X))$.Hence f is not Nano α-continuous.

Theorem 3.9 Every Nano generalized−continuous function is $N\delta g^{\wedge}$ -continuous.

Proof : Let $f: (U,\tau_R(X)) \to (V,\tau'_R(Y))$ be Nano generalized continuous on $(U,\tau_R(X))$.Since f is Nano generalized-continuous of $(U,\tau_R(X))$,the inverse image of every Nano generalized-open set in $(V,\tau'_R(Y))$ is Nano generalized-open set in $(U,\tau_R(X))$. But every Nano generalized-open set is $N\delta g^{\wedge}$ −open set. Hence the inverse image of every Nano-open set in $(V,\tau'_R(Y))$ is $N\delta g^{\wedge}$ - open in $(U,\tau_R(X))$.Therefore f is $N\delta g^{\wedge}$ -continuous.

Remark 3.10 The converse of the above theorem is not true as seen from the following example.

Example 3.11 Let $U = \{a,b,c,d\}$ with $U/R = \{\{a\},\{b,c\},\{d\}\}$ and $X = \{a,b\}$. Then $\tau_R(X) = \{U ,\phi,\{a\},\{b,c\},\{a,b,c\}\}$. Then $N\delta g^{\wedge}$-open sets are $\{U ,\phi ,\{a\},\{b\},\{c\},\{a,b\},\{a,c\},\{a,d\},\{b,c\},\{b,d\},\{c,d\},\{a,b,c\},\{a,b,d\},\{a,c,d\},\{b,c,d\}\}$. Let $V = \{x,y,z,w\}$ with $V/R' = \{\{x\},\{y,w\},\{z\}\}$ and $Y = \{x,y\}$. Then $\tau'_R(Y)$ $\{V,\phi ,\{x\},\{y,w\},\{x,y,w\}\}$. Define $f: (U,\tau_R(X)) \to (V,\tau'_R(Y))$ as f $(a)y$, f $(b) = z, f$ $(c) = x, f$ $(d) = w$. Then f^{-1} $(\{x\}) = \{c\}, f^{-1}(\{y,w\}) = \{a,d\}$, f^{-1} $(\{x,y,w\})$ $= \{a,c,d\}$ and f^{-1} $(V) = U$. Here f^{-1} $(\{x,y,w\}) = \{a,c,d\}$ which is not Nano generalized-open set in $(U,\tau_R(X))$. Hence f is not Nano generalized-continuous.

Theorem 3.12 Every Nano semi-continuous function is $\boldsymbol{N\delta g^{\wedge}}$ -continuous.

Proof : Let f: $(U,\tau_R(X)) \to (V,\tau'_R(Y))$ be Nano semi-continuous on $(U,\tau_R(X))$.Since f is Nano semi-continuous of $(U,\tau_R(X))$,the inverse image of every Nano semi-open set in $(V,\tau'_R(Y))$ is Nano semi-open in $(U,\tau_R(X))$. But every Nano semi-open set is $\boldsymbol{N\delta g^{\wedge}}$ -open set. Hence the inverse image of every Nano semi-open set in $(V,\tau'_R(Y))$ is $\boldsymbol{N\delta g^{\wedge}}$ −open in $(U,\tau_R(X))$.Therefore f is $\boldsymbol{N\delta g^{\wedge}}$ −continuous.

Remark 3.13 The converse of the above theorem is not true as seen from the following example.

Example 3.14 Let $U = \{a,b,c,d\}$ with $U/R = \{\{a\},\{b,c\},\{d\}\}$ and $X = \{a,b\}$. Then $\tau_R(X) = \{U ,\phi ,\{a\},\{b,c\},\{a,b,c\}\}$. Then $\boldsymbol{N\delta g^{\wedge}}$-open sets are $\{U ,\phi ,\{a\},\{b\},\{c\},\{a,b\},\{a,c\},\{a,d\},\{b,c\},\{b,d\},\{c,d\},\{a,b,c\},\{a,b,d\},\{a,c,d\},\{b,c,d\}\}$. Here Nano semi-open sets are $\{U,\phi,\{a\},\{a,c\},\{b,d\},\{a,b,d\}$. Let $V = \{x.y.z,w\}$ with $\frac{V'}{R} = \{\{x\},\{y,w\},\{z\}\}$ $and\ Y = \{x,y\}.Then\ \tau'_R(Y) = \{V,\phi ,\{x\},\{y,w\},\{x,y,w\}\}$. Define $f: (U,\tau_R(X)) \to (V,\tau'_R(Y))$ as f $(a) = y, f(b) = z, f(c) = x, f(d) = w$. Then $f^{-1}(\{x,y,w\}) = \{a,c,d\}$ which is not Nano semi-open set in $(U,\tau_R(X))$. Hence f is not Nano semi-continuous.

Theorem 3.15 Every Nano $semi^*$-continuous function is $\boldsymbol{N\delta g^{\wedge}}$ −continuous.

Proof : Let $f: (U,\tau_R(X)) \to (V,\tau'_R(Y))$ be Nano $semi^*$-continuous on $(U,\tau_R(X))$.Since f is Nano $semi^*$-continuous of $(U,\tau_R(X))$,the inverse image of every Nano $semi^*$-open set in $(V,\tau'_R(Y))$is Nano $semi^*$-open in $(U,\tau_R(X))$. But every Nano $semi^*$-open set is $\boldsymbol{N\delta g^{\wedge}}$ −open set. Hence the inverse image of every Nano $semi^*$-open set in $(V,\tau'_R(Y))$ is $\boldsymbol{N\delta g^{\wedge}}$ −open in $(U,\tau_R(X))$.Therefore f is $\boldsymbol{N\delta g^{\wedge}}$ -continuous.

Remark 3.16 The converse of the above theorem is not true as seen from the following example .

Example 3.17 Let $U = \{a,b,c,d\}$ with $U/R = \{\{a\},\{b,c\},\{d\}\}$ and $X = \{a,b\}$. Then $\tau_R(X) = \{U,\phi,\{a\},\{b,c\},\{a,b,c\}\}$. Then $\boldsymbol{N\delta g^{\wedge}}$ - open sets are $\{ U,\phi,\{a\},\{b\},\{c\},\{a,b\},\{a,c\},\{a,d\},\{b,c\},\{b,d\},\{c,d\},\{a,b,c\},\{a,b,d\},\{a,c,d\},\{b,c,d\}\}$. Here Nano $semi^*$- open sets are $\{U,\phi,\{a\},\{a,c\},\{b,d\},\{a,b,d\}$. Let $V = \{x.y.z,w\}$ with $V/R' = \{\{x\},\{y,w\},\{z\}\}$ and $Y = \{x,y\}$.

Then $\tau_R'(Y) = \{V, \phi, \{x\}, \{y,w\}, \{x,y,w\}\}$. Define $f: (U, \tau_R(X) \to (V, \tau_R'(Y))$ as $f(a) = y$, $f(b) = z$, $f(c) = x$, $f(d) = w$. Then $f^{-1}(\{x\}) = \{c\}$ which is not Nano $semi^*$- open set in $(U, \tau_R(X))$. Hence f is not Nano $semi^*$- continuous.

Theorem 3.18 Every Nano β-continuous function is $N\delta g^\wedge$ −continuous.

Proof : Let $f: (U, \tau_R(X)) \to (V, \tau_R'(Y))$ be Nano β −continuous on $(U, \tau_R(X))$. Since f is Nano β -continuous of $(U, \tau_R(X))$, the inverse image of every Nano β -open set in $(V, \tau_R'(Y))$ is Nano β-open set in $(U, \tau_R(X))$. But every Nano β-open set is $N\delta g^\wedge$ −open set. Hence the inverse image of every Nano-open set in $(V, \tau_R'(Y))$ is $N\delta g^\wedge$ −open in $(U, \tau_R(X))$. Therefore f is $N\delta g^\wedge$ -continuous.

Remark 3.19 The converse of the above theorem is not true as seen from the following example

Example 3.20 Let $U = \{a,b,c,d\}$ with $U/R = \{\{a\},\{b,c,d\}\}$ and $X = \{a,b\}$. Then $\tau_R(X) = \{U, \phi, \{a\}, \{b,c,d\}\}$. Then $N\delta g^\wedge$−open sets are $\{ U, \phi, \{a\}, \{b\}, \{c\}, \{d\}, \{a,b\}, \{a,c\}, \{a,d\}, \{b,c\}, \{b,d\}, \{c,d\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}, \{b,c,d$ Here Nano β -open sets are $\{U, \phi, \{b\}, \{b,c\}, \{b,d\}, \{a,b\}, \{a,b,c\}, \{a,b,d\}, \{b,c,d\}\}$. Let $V = \{x.y.z,w\}$ with $V/R' = \{\{x\}, \{w,z\}, \{y\}$ and $Y = \{y\}$. Then $\tau_R'(Y) = \{V, \phi, \{y\}\}$. Define $f: (U, \tau_R(X)) \to (V, \tau_R'(Y))$ as $f(a) = y, f(b) = x, f(c) = z, f(d) = w$. Then $f^{-1}(\{y\}) = \{a\}$, and $f^{-1}(\{v\}) = U$. Here $f^{-1}(\{y\}) = \{a\}$ which is not Nano β-open set in $(U, \tau_R(X))$. Hence f is not Nano β continuous.

Theorem 3.21 Every Nano pre-continuous function is $N\delta g^\wedge$ −continuous.

Proof : Let $f: (U, \tau_R(X)) \to (V, \tau_R'(Y))$ be Nano pre-continuous on $(U, \tau_R(X))$. Since f is Nano pre-continuous of $(U, \tau_R(X))$, the inverse image of every Nano pre-open set in $(V, \tau_R'(Y))$ is Nano pre-open in $(U, \tau_R(X))$. But every Nano pre-open set is $N\delta g^\wedge$ −open set. Hence the inverse image of every Nano pre-open set in $(V, \tau_R'(Y))$ is $N\delta g^\wedge$ − open in $(U, \tau_R(X))$. Therefore f is $N\delta g^\wedge$-continuous.

Remark 3.22 The converse of the above theorem is not true as seen from the following example

Example 3.23 Let $U = \{a,b,c,d\}$ with $U/R = \{\{a\},\{b,c\},\{d\}\}$ and $X = \{a,b\}$. Then $\tau_R(X) = \{U, \phi, \{a\}, \{b,c\}, \{a,b,c\}\}$. Then $\boldsymbol{N\delta g}^\wedge$ -open sets are $\{ U, \phi, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{a,d\}, \{b,c\}, \{b,d\}, \{c,d\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}, \{b,c,d\}\}$. Here Nano pre-open sets are $\{U, \phi, \{a\}, \{b\}, \{d\}, \{a,b\}, \{a,d\}, \{b,d\}, \{a,b,c\}, \{a,c,d\}, \{a,b,d\}$. Let $V = \{x.y.z,w\}$ with $V/R' = \{\{x\}, \{y,w\}, \{z\}\}$ and $Y = \{x,y\}$. Then $\tau_R'(Y) = \{V, \phi, \{x\}, \{y,w\}, \{x,y,w\}\}$. Define $f: (U, \tau_R(X)) \to (V, \tau_R'(Y))$ as $f(a) = y$, $f(b) = z, f(c) = x, f(d) = w$. Then $f^{-1}(\{x\}) = \{c\}$ which is not Nano pre-open set in $(U, \tau_R(X))$. Hence f is not Nano pre-continuous.

Theorem 3.24 A function $f: (U, \tau_R(X)) \to (V, \tau_R'(Y))$ be $N\delta g^\wedge$ −continuous if and only if the inverse image of every Nano-closed set in V is $N\delta g^\wedge$ -closed in U .

Proof : Let $f: (U, \tau_R(X)) \to (V, \tau_R'(Y))$ be $N\delta g^\wedge$ −continuous function and F be Nano-closed in $(V, \tau_R'(Y))$. That is, $V - F$ is Nano-open in $(V, \tau_R'(Y))$. Since f is $N\delta g^\wedge$ -continuous, the inverse image of every Nano-open set in $(V, \tau_R'(Y))$ is $N\delta g^\wedge$ -open in $(U, \tau_R(X))$. Hence $f^{-1}(V - F)$ is $N\delta g^\wedge$ - open in $(U, \tau_R(X))$. That is, $f^{-1}(V - F) = f^{-1}(V) - f^{-1}(F) = U - f^{-1}(F)$ is $N\delta g^\wedge$ -open in $(U, \tau_R(X))$. Therefore, $f^{-1}(F)$ is $N\delta g^\wedge$− closed in $(U, \tau_R(X))$. Conversely, let the inverse image of every Nano-closed set in $(V, \tau_R'(Y))$ is $N\delta g^\wedge$ -closed in $(U, \tau_R(X))$. Let G be Nano-open in $(V, \tau_R'(Y))$. Then $V - G$ is Nano-closed in $(V, \tau_R'(Y))$. Hence $f^{-1}(V - G)$is $N\delta g^\wedge$ − closed in $(U, \tau_R(X))$. That is, $U - f^{-1}(G)$ is $N\delta g^\wedge$ -closed in $(U, \tau_R(X))$. Therefore, $f^{-1}(G)$ is $N\delta g^{\wedge\wedge}$ −open in $(U, \tau_R(X))$.Thus, the inverse image of every Nano-open set in $(V, \tau_R'(Y))$ is $N\delta g^\wedge$-open in $(U, \tau_R(X))$.That is, f is $N\delta g^\wedge$ −continuous on $(U, \tau_R(X))$.

Theorem 3.25 A function $f: (U, \tau_R(X)) \to (V, \tau_R'(Y))$ is $N\delta g^\wedge$ -continuous if and only if $f(N\delta g^\wedge * Cl(A)) \subseteq NCl(f(A))$ for every subset A of U.

Proof : Let f be a $N\delta g^\wedge$ -continuous and $A \subseteq U$. Then $f(A) \subseteq V$. Since f is $N\delta g^\wedge$ −continuous and $NCl(f(A))$ is Nano-closed in V, $f^{-1}(NCl(f(A)))$ is $N\delta g^\wedge$ -closed in U. Since $f(A) \subseteq NCl(f(A))$, $f^{-1}(f(A)) \subseteq f^{-1}(NCl(f(A)))$, then $N\delta g^\wedge Cl(A) \subseteq N\delta g^\wedge Cl[f^{-1}(NCl(f(A)))] = f^{-1}(NCl(f(A)))$. Thus $N\delta g^\wedge Cl(A) \subseteq f^{-1}(NCl(f(A)))$. Therefore $f(N\delta g^\wedge Cl(A)) \subseteq NCl(f(A))$ for every subset A of U. Conversely, let $f(N\delta g^\wedge Cl(A)) \subseteq NCl(f(A))$ for every subset A of U. If F is Nano-closed in V, since $f^{-1}(F) \subseteq U$, $f(N\delta g^\wedge Cl(f^{-1}(F))) \subseteq NCl(f(f^{-1}(F))) = NCl(F)$. That is, $N\delta g^\wedge Cl(f^{-1}(F)) \subseteq f^{-1}(NCl(F)) = f^{-1}(F)$, since F is Nano-closed. Thus $N\delta g^\wedge Cl(f^{-1}(F)) \subseteq f^{-1}(F)$. But $f^{-1}(F) \subseteq N\delta g^\wedge Cl(f^{-1}(F))$. Thus $N\delta g^\wedge Cl(f^{-1}(F)) = f^{-1}(F)$. Therfore $f^{-1}(F)$ is $N\delta g^\wedge$ −closed in U for every Nano-closed set F in V. That is, f is $N\delta g^\wedge$ −continuous.

Remark 3.26 If $f: (U, \tau_R(X)) \to (V, \tau_R'(Y))$ be $N\delta g^\wedge$ −continuous then $f(N\delta g^\wedge Cl(A))$ is not necessarily equal to $NCl(f(A))$ for every subset A of U.

Example 3.27 Let $U = \{a,b,c,d\}$ with $U/R = \{\{a,b\},\{c\},\{d\}\}$ and $X = \{a,c\}$. Then $\tau_R(X) = \{U, \phi, \{a\}, \{b\}, \{a,b,c\}\}$. Then $N\delta g^\wedge$-open sets $\{ U, \phi, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{a,d\}, \{b,c\}, \{b,d\}, \{c,d\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}, \{b,c,d\}\}$. Let $V = \{x.y.z,w\}$ with $V/R' = \{\{x\}, \{y,z,w\}\}$ and $Y = \{x,y\}$. Then $\tau_R(Y) = \{V, \phi, \{x\}, \{y,z,w\}\}$. Define $f: (U, \tau_R(X)) \to (V, \tau_R'(Y))$ as $f(a) = y$, $f(b) = z$, $f(c) = x, f(d) = w$. Then $f^{-1}(\{x\}) = \{c\}$, $f^{-1}(\{y,z,w\}) = \{a,b,d\}$ and $f^{-1}(V) = U$. That is, the inverse image of every Nano-open set in V is the $N\delta g^\wedge$ -open set in U. Therefore f is $N\delta g^\wedge$-continuous. Let $A = \{b,c,d\} \subseteq V$. Then $f(N\delta g^\wedge Cl(A)) = f(\{b,c,d\}) = \{x,z,w\}$. But $NCl(f(A)) = NCl(\{x,z,w\}) = V$. Thus $f(N\delta g^\wedge Cl(A)) \neq NCl(f(A))$. That is, equality does not hold in the previous theorem when f is $N\delta g^\wedge$ −continuous.

Theorem 3.28 Let $(U, \tau_R(X))$ *and* $(V, \tau_R'(Y))$ be two Nano topological spaces where $X \subseteq U$ and $Y \subseteq V$. Then $\tau_R'(Y) = \{V, \phi, LR'(Y), UR'(Y), BR'(Y)\}$ and its basis is given by $BR' = \{V, LR'(Y), BR'(Y)\}$. A function $f: (U, \tau_R(X)) \to (V, \tau_R'(Y))$ be $N\delta g^\wedge$ −continuous if and only if the inverse image of every member of BR' is $N\delta g^\wedge$ in U.

Proof : Let f be a $N\delta g^{\wedge}$−continuous on U. Let $B \in BR'$. Then B is Nano-open in V. That is, $B \in \tau_R'(Y)$. Since $N\delta g^{\wedge}$ -continuous, $f^{-1}(B) \in \tau_R(X)$. That is, inverse image of every member of BR' is $N\delta g^{\wedge}$ in U. Conversely, let inverse image of every member of BR' is $N\delta g^{\wedge}$ in U. Let G be a Nano-open in V. Then $G = \cup\{B: B \in B_1\}$, where $B_1 \subseteq BR'$. Then $f^{-1}(G) = f^{-1}(\cup\{B: B \in B_1\}) = \cup\{f^{-1}(B): B \in B_1\}$, where $f^{-1}(B)$ is $N\delta g^{\wedge}$ in U and hence their union, which is $f^{-1}(G)$ is $N\delta g^{\wedge}$ in U. Thus f is $N\delta g^{\wedge}$ -continuous on U.

Theorem 3.29 A function $f: (U, \tau_R(X)) \to (V, \tau_R'(Y))$ is $N\delta g^{\wedge}$ -continuous if and only if $f^{-1}(NInt(B)) \subseteq N\delta g^{\wedge} Int(f^{-1}(B))$ for every subset B of $(V, \tau_R'(Y))$.

Proof : Let $f: (U, \tau_R(X)) \to (V, \tau_R'(Y))$ be $N\delta g^{\wedge}$ −continuous. By the given hypothesis $B \subseteq V$. Then, $NInt(B)$ is Nano-open in V. As f is $N\delta g^{\wedge}$ −continuous, $f^{-1}(NInt(B))$ is $N\delta g^{\wedge}$ - open in U. Hence it follows that $N\delta g^{\wedge} Int(f^{-1}(NInt(B))) = f^{-1}(NInt(B))$. Also, for $B \subseteq V, NInt(B) \subseteq B$ always. Then $f^{-1}(NInt(B)) \subseteq f^{-1}(B)$. Since f is $N\delta g^{\wedge}$−continuous, it follows that $N\delta g^{\wedge} Int(f^{-1}(NInt(B))) \subseteq N\delta g^{\wedge} Int\, f^{-1}(B)$, hence $f^{-1}(NInt\, B) \subseteq N\delta g^{\wedge} Int(f^{-1}(B))$. Conversely, let $f^{-1}(NInt\, B) \subseteq N\delta g^{\wedge} Int(f^{-1}(B))$ for every subset B of V. Let B be Nano-open in V and hence $NInt(B) = B$, Given $f^{-1}(NInt(B)) \subseteq N\delta g^{\wedge} Int(f^{-1}(B))$, that is $f^{-1}(B) \subseteq N\delta g^{\wedge} Int(f^{-1}(B))$. Also $N\delta g^{\wedge} Int(f^{-1}(B)) \subseteq f^{-1}(B)$. Hence it follows that $f^{-1}(B) = N\delta g^{\wedge} Int(f^{-1}(B))$ which implies that $f^{-1}(B)$ is $N\delta g^{\wedge}$−open in U for every subset B of V. Therefore, $f: (U, \tau_R(X)) \to (V, \tau_R'(Y))$ is $N\delta g^{\wedge}$ − continuous.

Theorem 3.30 A function $f: (U, \tau_R(X)) \to (V, \tau_R'(Y))$ is $N\delta g^{\wedge}$−continuous if and only if $N\delta g^{\wedge} Cl(f^{-1}(B)) \subseteq f^{-1}(NCl(B))$ for every subset B of $(V, \tau_R'(Y))$.

Proof : Let $B \subseteq V$ and $f: (U, \tau_R(X)) \to (V, \tau_R'(Y))$ be $N\delta g^{\wedge}$ -continuous. Then $NCl(B)$ is Nano-closed in $(V, \tau_R'(Y))$ and hence $f^{-1}(NCl(B))$ is $N\delta g^{\wedge}$ -closed in $(U, \tau_R(X))$. Therefore, $N\delta g^{\wedge} Cl(f^{-1}(NCl(B))) = f^{-1}(NCl(B))$. Since $B \subseteq NCl(B)$, then $f^{-1}(B) \subseteq f^{-1}(NCl(B))$, that is $N\delta g^{\wedge} Cl(f^{-1}(B)) \subseteq N\delta g^{\wedge} Cl(f^{-1}(NCl(B))) = f^{-1}(NCl(B))$. Hence $N\delta g^{\wedge} Cl(f^{-1}(B)) \subseteq f^{-1}(NCl(B))$. Conversely, let $N\delta g^{\wedge} Cl(f^{-1}(B)) \subseteq f^{-1}(NCl(B))$ for every subset $B \subseteq V$. Now, let B be a Nano-closed set in $(V, \tau_R'(Y))$, then $NCl(B) = B$. By the given hypothesis, $N\delta g^{\wedge} Cl(f^{-1}(B)) \subseteq f^{-1}(NCl(B))$ and hence $N\delta g^{\wedge} Cl(f^{-1}(B)) \subseteq f^{-1}(B)$. But we also have $f^{-1}(B) \subseteq N\delta g^{\wedge} Cl(f^{-1}(B))$ and hence $N\delta g^{\wedge} Cl(f^{-1}(B)) = f^{-1}(B)$. Thus $f^{-1}(B)$ is $N\delta g^{\wedge}$ -closed set in $(U, \tau_R(X))$ for every Nano-closed set B in $(V, \tau_R'(Y))$. Hence $f: (U, \tau R(X)) \to (V, \tau R'(Y))$ is $N\delta g^{\wedge}$ −continuous.

Example 3.31 Let $U = \{a, b, c, d\}$ with $U/R = \{\{a\}, \{b, c\}, \{d\}\}$ and $X = \{a, b\}$. Then $\tau_R(X) = \{U, \emptyset, \{a\}, \{b, c\}, \{a, b, c\}\}$ and hence the Nano-closed sets in U are $\{U, \emptyset, \{d\}, \{a, d\}, \{b, c, d\}\}$. The $N\delta g^{\wedge}$−open sets are $\{U, \emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, c\}, \{a, b. d\}, \{a, c, d\}, \{b, c, d\}\}$. Let $V = \{x, y, z, w\}$ with $V/R' = \{\{x\}, \{y, z, w\}\}$ and $Y = \{x, y\}$. Then $\tau_R'(Y) = \{V, \emptyset, \{x\}, \{y, z, w\}\}$ and hence Nano-closed sets in V are $\{V, \emptyset, \{x\}, \{y, z, w\}\}$. Define $f: (U, \tau_R(X)) \to (V, \tau_R'(Y))$ as $f(a) = y, f(b) = z, f(c) = x, f(d) = w$. Then f is $N\delta g^{\wedge}$ -continuous on U, since inverse image of every Nano-open set in V is $N\delta g^{\wedge}$ −open in U . Let $B = \{x, z\} \subset V$. Then $N\delta g^{\wedge} Cl(f^{-1}(B)) = N\delta g^{\wedge} Cl(f^{-1}(x, z)) = N\delta g^{\wedge} Cl(\{b, c\}) = \{b, c\}$ and $f^{-1}(NCl(B)) = f^{-1}(NCl(\{x, z\})) = f^{-1}(V) = U$. Thus, $N\delta g^{\wedge} Cl(f^{-1}(B)) \neq f^{-1}(NCl(B))$. Also $f^{-1}(NInt\, B) = f^{-1}(NInt\{x, z\}) = f^{-1}(z) = \{a\}$ and $N\delta g^{\wedge} Int(f^{-1}(B) = N\delta g^{\wedge} Int(f^{-1}(\{x, z\})) = N\delta g^{\wedge} Int(\{a, b\}) = \{a, b\}$. That is, $f^{-1}(NInt\, B) \neq N\delta g^{\wedge} Int(f^{-1}(B))$. Thus, equality does not hold in theorem 3.2.29 and theorem 3.2.30 when f is $N\delta g^{\wedge}$-continuous.

Theorem 3.32 Let $(U, \tau_R(X))$ and $(V, \tau_R'(Y))$ be two Nano Topological space with respect to $X \subseteq U$ and $Y \subseteq V$ respectively. Then for any function $f: (U, \tau_R(X)) \to (V, \tau_R'(Y))$, the following are equivalent

(i) f is $N\delta g^{\wedge}$ −continuous.
(ii) The inverse image of every Nano-closed set in V is $N\delta g^{\wedge}$−closed in $(U, \tau_R(X))$.
(iii) $f(N\delta g^{\wedge} Cl(A)) \subseteq NCl(f(A))$ for every subset A of $(U, \tau_R(X))$.
(iv) The inverse image of every member of BR' is $N\delta g^{\wedge}$ − open in $(U, \tau_R(X))$.
(v) $f^{-1}(NInt(B)) \subseteq N\delta g^{\wedge} Int(f^{-1}(B))$ for every subset B of $(V, \tau_R'(Y))$.
(vi) $N\delta g^{\wedge} Cl(f^{-1}(B)) \subseteq f^{-1}(NCl(B))$ for every subset B of $(V, \tau_R'(Y))$.

Proof : The proof of this theorem follows from 3.2.24 to 3.2.30.

Theorem 3.33 If a map $f: (U, \tau_R(X)) \to (V, \tau_R'(Y))))$ be $N\delta g^{\wedge}$ − continuous and $g: (V, \tau_R'(Y) \to (W, \tau_R''(Z))$ is Nano-continuous, then $(g \circ f)$ is $N\delta g^{\wedge}$-continuous.

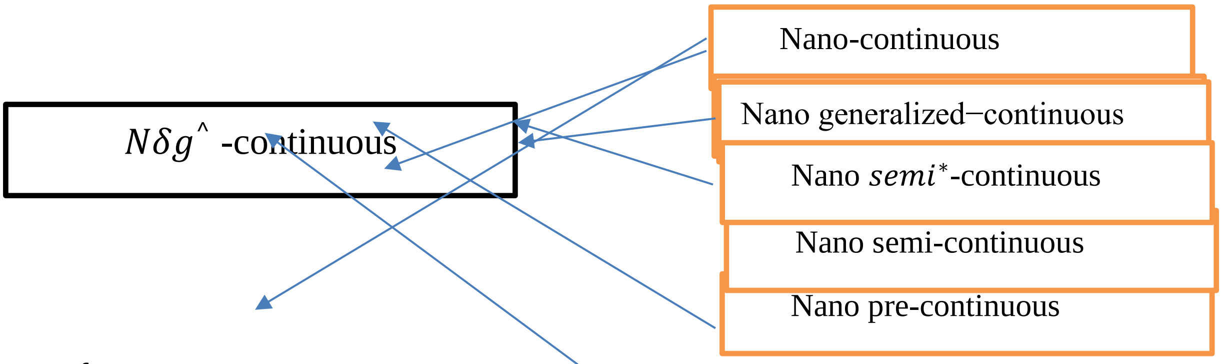
Proof : Let G be Nano-open set in W. Since g is Nano-continuous $g^{-1}(G)$ is Nano-open in V and we know that f is $N\delta g^{\wedge}$ -continuous then, $(g \circ f)^{-1}(G) = f^{-1}(g^{-1}(G))$ is $N\delta g^{\wedge}$ -open in U. Therefore, $(g \circ f)$ is $N\delta g^{\wedge}$ -continuous.

Remark 3.34 Composition of two $N\delta g^{\wedge}$ -continuous maps need not be $N\delta g^{\wedge}$ −continuous maps.

Example 3.35 Let $U = \{a, b, c, d\}$ with $U/R = \{\{a\}, \{b, c\}, \{d\}\}$ and $X = \{a, b\}$. Then $\tau_R(X) = \{U, \phi, \{a\}, \{b, c\}, \{a, b, c\}\}$. Then $N\delta g^{\wedge}$ -open sets are $\{U, \phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}\}$. Let $V = \{x. y. z, w\}$ with $V/R' = \{\{x\}, \{y, w, z\}\}$ and $Y = \{x, y\}$. Then $\tau_R'(Y) = \{V, \phi, \{x\}, \{y\}, \{z\}, \{w\}, \{x, y\}, \{x. z\}, \{x, w\}, \{y, z\}, \{y, w\}, \{z, w\}, \{x, y, z\}, \{x, y, w\}, \{x, z, w\}, \{y, z, w\}\}$. Define $f: (U, \tau_R(X)) \to (V, \tau_R'(Y))$ as $f(a) = y, f(b) = z, f(c) = x, f(d) = w$. Then $f^{-1}(\{x\}) = \{c\}$, $f^{-1}(\{y, z, w\}) = \{a, b, d\}$ and $f^{-1}(V) = U$. That is, the inverse image of every Nano-open set in V is the $N\delta g^{\wedge}$ -open set in U. Therefore f is $N\delta g^{\wedge}$-continuous. Let $W = \{a, b, c, d\}$ with $W/R'' = \{\{a\}, \{b, d\}, \{c\}\}$ and $Z = \{a, b\}$. Then $\tau_R''(Z) = \{W, \phi, \{a\}, \{b, d\}, \{a, b, d\}\}$.
Define $g: (V, \tau_R'(Y) \to (W, \tau_R''(Z))$ as $g(x) = b, g(y) = c, g(z) = d, g(w) = a$. Then $g^{-1}(\{b, d\}) = \{x, z\}$ $g^{-1}(\{a\}) = \{w\}$, $g^{-1}(\{a, b, d\}) = \{x, z, w\}$ and $f^{-1}(W) = V$. That is, the inverse image of every Nano-open set in W is the $N\delta g^{\wedge}$ -open set in V.

Therefore f is $N\delta g^{\wedge}$-continuous. But $(g \circ f)^{-1}(\{a\}) = f^{-1}(g^{-1}(\{a\})) = f^{-1}(\{w\}) = \{d\}$ is not $N\delta g^{\wedge}-$open in U. Therefore, Composition of two $N\delta g^{\wedge}$ $-$continuous maps need not be $N\delta g^{\wedge}$-continuous maps.

Diagram 3.36 The following diagram shows the relationship between $N\delta g^{\wedge}$ -continuous and other Nano- continuous that are studied in this section.



References

1. Thivagar, M.L., Richard, c: On nano forms of weekly open sets. Int.J. Math. Stat: Inven. 1(1),31-37 (2013
2. I.L Reilly and V. amanamurthy, On α- sets in topological spaces, Tamkang J. Math, 16 (1985), 7-11.
3. J Dontchev and M Ganster, On δ-generalized closed sets and T3/4-spaces, Mem.Fac.Sci.Kochi Univ.Ser.A, Math., 17(1996),15-31.
4. S.P Arya and T Nour, Characterizations of S-normal spaces, Indian J.Pure.Appl.MAth.,21(8)(1990), 717-719.
5. P Bhattacharya and B.K Lahiri, Semi-generalized closed sets in topology, Indian J.Math., 29(1987), 375-382.
6. H Maki, R Devi and K Balachandran, Generalized α $-$closed sets in topology, Bull.Fukuoka Uni.Ed part III, 42(1993), 13-21.
7. Bhuvanesshwari k . Mythili gnanapriya k. Nano Generalized closed sets, International journal of scientific and Research publication, 2014, 14(5) : 1-3.
8. awlak, Z : Rough sets. Int.J. Comput. Inform.sci.11(5),341-356(1982)
9. K.Bhuvaneswari and k. Mythili Gnanapriya, "On Nano generelized continuous function in nano topological space international journal of mathematical archive, 2015, 6(6) :182- 186.
10. N Levine, Semi-open sets and semi-continuity in topological spaces Amer Math. Monthly, 70(1963), 36-41.
11. Thivagar, M.L., Richard, C.: On nano continuity Math. Theory Model.7, 32-37 (2013)
12. M. Hosny, Nano $\delta\beta$-opensets and $\delta\beta$- continuity, J.Egypt. Math. Soc.26(2), 365 – 375(2018).
13. Nasef, A.A., Aggour, A.l., Darwesh, S.M. On some classes of nearly open sets in nano topological space J. Egypt. Math. SOC 24 (4), 585 – 589 (2016)
14. Bhuvanesshwari k . Mythili gnanapriya k. Nano Generalized closed sets, International journal of scientific and Research publication, 2014, 14(5) : 1-3.
15. H Maki, R Devi and K Balachandran, Associated topologies of generalized α-closed sets and α-generalized closed sets, Mem. Fac. Sci.Kochi Univ. Ser. A. Math., 15(1994), 57-63.