A Study on Graph Labelling and Its Applications

[1]Likesy Saviyo, [2]Josna John, [3]Dona Jose

[1]Degree Student,[2]Degree Student,[3]Assistant Professor
[1]Department of Computer Science, Kerala, India, Computer Science, Kerala,

Abstract: In the field of mathematics Graph Theory plays vital role in numerous ways. Among these, one of the important areas in graph theory is Graph Labelling. It is used in many applications like coding theory, x-ray crystallography, radar, astronomy, circuit design, communication network addressing, data base management etc. This paper addresses an idea of labelling graphs in heterogeneous fields to some extent but mainly focuses on the communication networks. Communication network has two types 'Wireless communication' and 'wired communication'. This paper also explored a part of labelling graphs in expanding the utility of process in communication networks. This paper give an overview of how the method of graph labelling be applied to social networks, network security, channel assignment process, network addressing. An overview and new inventiveness has been proposed here.

Keywords: labelling, graceful labelling, graph colouring, crystallography

I. INTRODUTION

The field of Graph Theory plays a vital role in various fields. Graph Labelling is the assignment of integer's from 1 to n for vertex, edges and both of the graphs respectively. One of the important area in graph theory is Graph Labelling which is used in many applications like coding theory, radar, astronomy, circuit design, missile guidance, communication network addressing, x-ray crystallography, data base management. Here we would like to enhance the graph labelling applications in the field of computer science. This paper gives an overview of labelling of graphs in heterogeneous fields to some extent, but mainly focuses on important major areas of computer science like data mining, image processing, cryptography, software testing, communication networks etc….These are various subjects in engineering studies and these are more expertly used in various sectors like government sectors, corporate sectors. Various papers based on graph theory and graph labelling applications have been studied and we explore the usage of Graph Labelling in several areas like data mining, communication networks, image processing, cryptosystem, computer science applications and an overview has been proposed here.[1]

II. GRAPH LABELING

In the mathematical application of graph theory, graph labelling is the process of assigning labels to a graph's edges and vertices, which are often represented by integers.

Consider graph G = first (V, E). A graph with such a function specified is referred to as a vertexlabelled graph. A vertex labelling is a function of V to a collection of labels. An edge-labelled graph is one that has edges that are labelled as a function of E and a set of labels. It is referred to as a weighted graph when the edge labels are individuals from an ordered set. The phrase "labelled graph" often refers to a vertex-labelled graph with different labels when used without qualifier. A graph of this type can be equivalently labelled by the numbers 1...., |V|, where |V| is the graph's vertex count.[2]

According to the definition above, a graph is a finite undirected simple graph. The concept of labelling, however, is applicable to all graph extensions and generalisations. Consider labelled multigraphs, for instance, as they are practical to use in formal language theory and automata theory.

III. SPECIAL CASES

i. Graceful Labelling

When a graph's edges are labelled from 1 to |E| and its vertices are labelled from 0 to |E|, the graph is said to be gracious. The positive difference between the two vertices occurring with any edge e serves as its label. In other words, E will be labelled |i j| if it is incident with the vertices I and j. The only way a graph G = (V, E) can be elegant is if and only if an injection exists that causes a bijection from E to the positive integers up to |E|.

ii. Edge-Graceful Labelling

An edge-graceful labelling on a simple graph without loops or multiple edges on p vertices and q edges is a labelling of the edges by distinct integers in {1, …, q} such that the labelling on the vertices induced by labelling a vertex with the sum of the incident edges taken modulo p assigns all values from 0 to p − 1 to the vertices. A graph G is said to be "edge-graceful" if it admits an edge-graceful labelling.

iii. Harmonious Labelling

On a graph G, a "harmonious labelling" is an injection from the vertices to the group of integers modulo k, where k is the number of edges in G, which results in a bijection between the edges of G and the numbers modulo k by assuming that the edge label for an edge (x, y) is the sum of the labels of the two vertices x, y. (mod k). A graph with a harmonic labelling is known as a "harmonious graph." If only one vertex label can be reused, it is hypothesised that all trees will coexist peacefully. [3]

iv. Graph Colouring

A subtype of graph labelling is graph colouring. While edge colouring assigns several labels to neighbouring edges, vertex colouring assigns various labels to adjacent vertices.

In its most basic form, vertex colouring is a technique for colouring a graph's vertices so that no two neighbouring vertices share the same colour. In a similar way, a planar graph's face colouring applies a colour to each face or region so that no two faces that share a boundary have the same colour. This prevents adjacent edges from having the same colour.[4]

v. Lucky Labelling

A lucky labelling of a graph G is the assignment of positive integers to its vertices in such a way that, if
S(v) is the total of the labels on v's neighbours, then S is G's vertex colouring. The smallest k such that G has a lucky labelling with the integers "1,..., k" is the "lucky number" for G.

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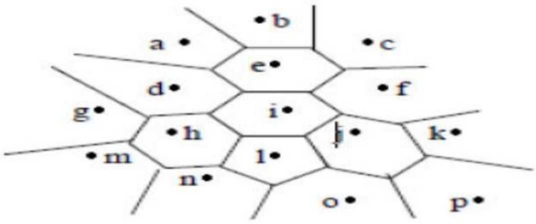
IV. APPLICATION

1. Analysing Communication Efficiency in sensor networks with Voronoi Graph

The applications for sensor networks are numerous, tracking of moving objects, gathering environmental data, providing medical treatment, etc.,

To analyse the communication effectiveness, a graph model of the sensor network is used. In this case, the sensor network is modelled using a Voronoi graph. Voronoi graphs are built in the shape of polygons on a plane, with the nodes acting as sensors and the edges of the polygons serving as the sensing range of each sensor.



Voronoi graph

2. Reducing the Complexity of Algorithms in Compression Networks

Here, a sizable network is taken into account. In all ways, dealing with such vast networks is a difficult undertaking. This creates a graphic description of the enormous graphs that appear in the area of enterprise IP networks. The domain contains many enormous constructed graphs. This is accomplished by downsizing the original graph using its structural properties, which have clear definitions in our field. When presented with such a data set, the smaller, resulting graph can then be viewed and understood by a human. To distinguish it from algorithmic graph compression, when a graph is compressed to lessen the time or space complexity of a graph algorithm, this method is referred to as graph compression.

3. Graph labelling in X-Ray crystallography

The most effective approach, known as X-Ray diffraction, can be used to characterise the various structural characteristics of crystalline solids. During this process, an X-Ray radiation beam or X-Ray tube particles hit with a crystal and cause diffraction of the rays into different directions. In some instances, more than one structure provides comparable diffraction data throughout this process. This X-ray crystallography problem can be expressed mathematically as the decision to label all relevant graphs with a predetermined set of arc labels.

4. Graph Labelling in Communication Relevant to Networks

Problems with Mobile Networks (MANETS) can be solved via graph labelling. Graph models can be used to analyse issues like connection, scalability, routing, network modelling, and simulation that need to be taken into account. Algorithms can be used to analyse the problems and express graphs as matrices. A random graph can be used to mimic node density, mobility among nodes, link generation between nodes, and packet routing. When these networks are modelled based on graph theoretical concepts, the congestion in MANETs can be analysed using a variety of algorithms.

5. Automatic Routing with labelling

By linking nodes in a certain topology, a static network can be represented as a particular type of graph, and labelling can be used to automatically route data in a network. The graph can have a fixed network represented by a cycle, path, circuit, or walk. Every network is labelled using a constant, which enables routing to find the next node in the network automatically.

V. ADVANTAGES AND DISADVANTAGES

Advantages

- Multiple classes of data in one chart
- Easy understanding
- Offers easy calculations of data accuracy
- Requires little explanation

Disadvantages

- Doesn't reveal exact values
- Multiple graphs are needed for time-lapse data
- Key assumptions, causes, effect, and patterns are not revealed
- Manipulated easily, causing false impressions or interpretations[6]

VI . CONCLUSION

This paper's primary goal is to investigate the function of graph labelling in the communication sector. Graph labelling is a useful tool that simplifies tasks in a number of networking-related domains. The main purpose of the overview is to introduce the concept of Graph Labelling. Researchers may learn more about graph labelling and its uses in the communication industry, as well as gain insight into potential directions for their own study.[7]

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