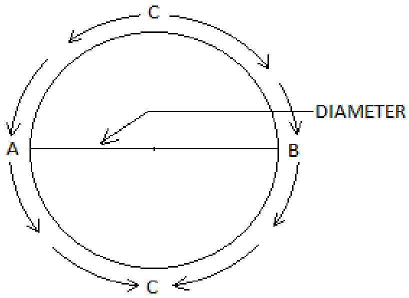
Approach to find exact value of PI (π)

Lokesh Dewangan
Mungeli (CG)

***Abstract-* This research study delves deep into the enigmatic realm of PI (π), a fundamental mathematical constant representing the relationship between a circle's circumference and diameter. While history has witnessed numerous approximations of PI, such as Archimedes' renowned 3.14159..., this research exposes the inherent shortcomings in these traditional values. By critically examining the extensive annals of mathematical history spanning nearly 4000 years, all prior attempts at determining PI are unequivocally dismissed. This study embarks on a rigorous mathematical calculation, employing a multifaceted approach to unravel the true essence of PI. Through meticulous processes and conclusive findings, it presents the definitive, 100% accurate value of PI.**



$$\text{PI } (\pi) = \frac{CIRCUMFERENCE}{DIAMETER}$$

***Keyword-* Exact value of PI, PI method circle, PI constant, Irrational number.**

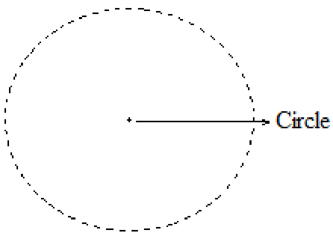
INTRODUCTION

PI, denoted as π, is a fundamental mathematical constant that plays a pivotal role in geometry and various branches of science and mathematics. It defines the ratio of a circle's circumference to its diameter and possesses a unique and intriguing property – its decimal representation extends infinitely without repeating, making it an irrational number. This peculiarity renders π impossible to express as a simple fraction, i.e., in the form p/q, where p and q are integers.

Mathematically, PI (π) is defined as:

π = Circumference of a Circular Structure / Diameter of the Circle

In its essence, a circle is a geometric shape composed of an infinite number of points in a plane, all equidistant from a central point known as the center. This geometric concept forms the basis for the calculation and understanding of π.



HISTORY OF PI (π)

This literature review commences by delving into the annals of mathematical history to explore the evolution of π approximations. It spans over four millennia, encompassing ancient civilizations, such as the Egyptians and Babylonians, who approximated π in various forms, to the remarkable contributions of ancient Greek mathematicians like Archimedes. Each civilization added a layer of understanding, albeit with varying degrees of accuracy.

Archimedes' method of inscribed and circumscribed polygons laid the foundation for a more precise π approximation, which remained unparalleled for centuries. However, these approximations, despite their significance, were inherently limited by the tools and techniques available in their respective eras.

The quest to understand the value of π dates back to ancient civilizations, each contributing to its approximation over the course of centuries.

I. Babylonians: The Babylonians were among the earliest to approximate π, arriving at an approximate value of 3 1/8, which is equivalent to 3.125.

II. Archimedes: In the third century BC, the renowned mathematician Archimedes made significant strides by calculating an approximation of π as 3.14159... His method involved inscribing and circumscribing polygons within and around a circle, progressively refining his approximation.

III. Egyptians: The Egyptians had their own method for approximating π, utilizing the formula $[8d/9]^2$ for the area of a circle, where 'd' represented the diameter. This yielded an approximate value of π as 3.16.

IV. China: Chinese mathematicians initially considered π as a whole number, 3. However, the mathematician Zu Chongzhi (429–501) made a notable contribution by offering a rational approximation of π as 355/113, demonstrating a keen understanding of its fractional nature.

V. Greek Symbol: The symbol for π, derived from the Greek letter π (pi), was popularized by the mathematician Euler in 1737 and has since become synonymous with this mathematical constant.

VI. Aryabhata's Verse: The Indian mathematician and astronomer Aryabhata provided a unique perspective on π in the form of a verse. According to his verse, π was calculated as follows: (100 + 4) × 8 + 62000 / 20000 = 3.1416. This representation offered a rational approximation and eventually became known as 22/7, a widely recognized but still approximate value of π.

Despite the efforts of these ancient civilizations and their various methods, it's important to note that all of these calculations yielded approximate values of π and not its actual, precise value. The search for the true value of π would continue throughout history, eventually leading to more sophisticated mathematical approaches and techniques.

ARCHIMEDES CALCULATION OF PI

Archimedes, one of the greatest mathematicians and scientists of antiquity, made significant contributions to the estimation of the mathematical constant π (pi) around the 3rd century BCE. His approach to calculating π is one of the most famous and innovative methods in the history of mathematics. He did not use modern mathematical notation or calculus, but his geometric method was groundbreaking and remarkably accurate for his time.

Archimedes' method involved inscribing and circumscribing polygons (polygons with a large number of sides) around a circle. By progressively increasing the number of sides of these polygons, he was able to approximate the value of π with increasing precision.

Here is a simplified outline of Archimedes' method:

Start with a Circle: Archimedes began with a circle, the object he wanted to measure.

Inscribe a Polygon: He inscribed a regular polygon inside the circle. In this context, a regular polygon is one where all sides and angles are equal.

Circumscribe a Polygon: He then circumscribed another regular polygon outside the circle, with the same number of sides as the inscribed polygon.

Comparison: Archimedes realized that the circumference of the circle must be larger than the perimeter of the inscribed polygon but smaller than the perimeter of the circumscribed polygon.

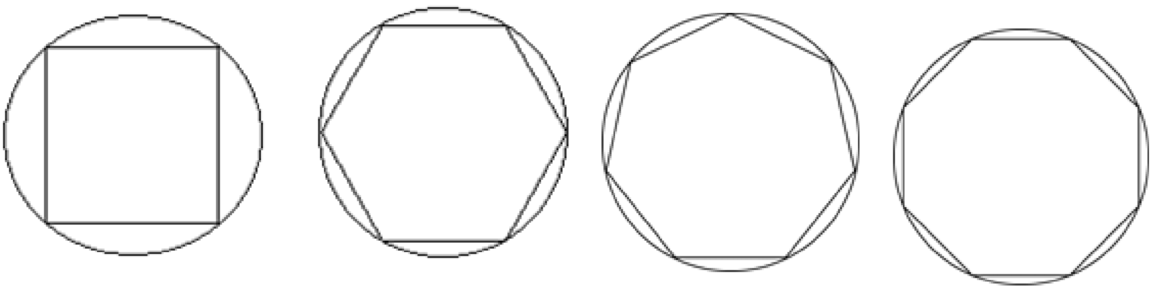
Refinement: To improve accuracy, Archimedes increased the number of sides of both the inscribed and circumscribed polygons. As he used polygons with more sides, the perimeters of these polygons became closer to the circumference of the circle.

Convergence: Archimedes continued this process, refining his approximations by using polygons with more and more sides. This allowed him to narrow down the range of possible values for π.

Limit Calculation: Although Archimedes didn't have the concept of limits as we do in modern calculus, his method essentially approached the limit as the number of sides of the polygons approached infinity. This limit became a very good approximation of π.

Archimedes didn't calculate π to a specific decimal place, but he demonstrated that π is greater than $3\frac{10}{71}$ (approximately 3.1408) and less than $3\frac{1}{7}$ (approximately 3.1429) . These bounds provided an impressive and accurate estimation of π given the mathematical tools available in his time.

Archimedes' method laid the foundation for later mathematicians to refine and improve π approximations. His work in this area remains a testament to the power of geometric and deductive reasoning in ancient mathematics.



NEW RESEARCH APPROACH FOR PI

First of all, will draw a circle of 3 cm circumference (Circle 1), whose radius (R_1 = ?) is not known. Now by increasing its radius to 1/2 centimeter, we will create another circle (Circle 2) So for circle 2-

R_2 = ½ cm

Circumference=$2\pi R_2$

$=2\times\pi\times1/2$

$= \pi$

Now again by increasing the same radius R_2 = 1/2 to R_3 = 3 cm, we will create a circle-3.

For Circle-3 –

R_3 =3 cm

Circumference=2π R_3

$=2\times\pi\times3$

$= 6\pi$

Now repeating the same process, we will create a circle with radius R_3 = 3 cm increased to $R_4= 3\times\sqrt{360}$ cm. (Circle 4)

So for circle 4

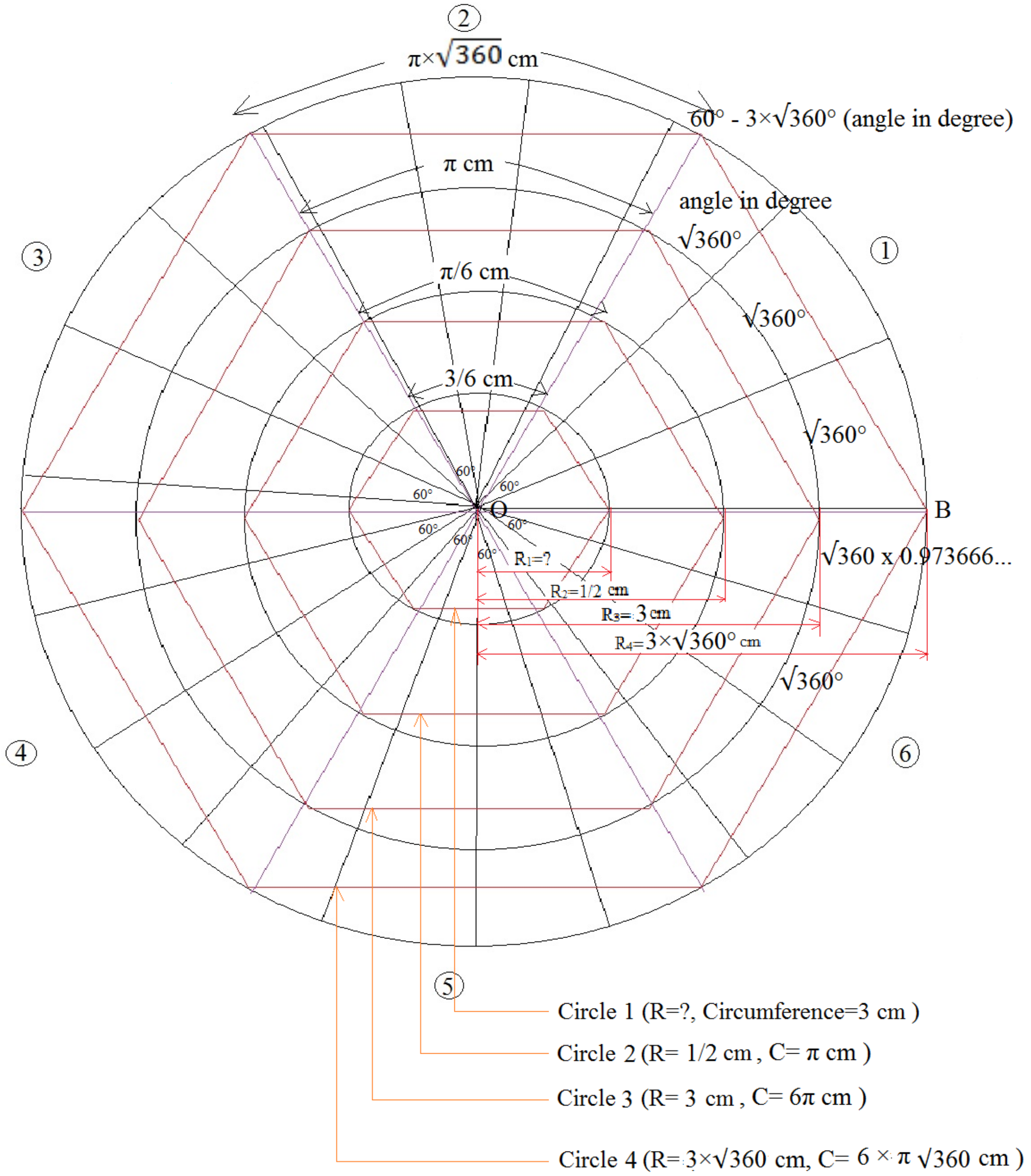
R= $3\times\sqrt{360}$ =$\sqrt{3240}$ =56.9209979…cm

Circumference=2π R_4

$=2\times\pi\times3\times\sqrt{360}$

$= 6\pi\sqrt{360}$

Now in the next step for the process, everyone will create an equilateral triangle inside the circle with its side equal to its radius (R). Since one angle of an equilateral triangle is 60° degrees, hence 6 such triangles will be formed inside each circle [$\frac{360°}{60°} = 6$], which will divide the circumference of that circle into 6 equal parts. And each part of all the circles will represent an angle of 60° degrees at the center.



Now according to the figure, we will calculate the ratio of 1 part of the circumference of the circle and its radius in all the circles.

Circle 2

$$\frac{part\ of\ circumference}{radius} = \frac{\pi/6}{1/2} = \frac{\pi}{3}$$

Circle 3

$$\frac{6\pi/6}{3} = \frac{\pi}{3}$$

Circle 4

$$\frac{\pi\sqrt{360}}{3\times\sqrt{360}} = \frac{\pi}{3}$$

It is clear from the above analysis that, dividing the circumference of any circle into 6 equal parts, the ratio of one part of its circumference and the radius of that circle will be pi / 3, which will be a constant ratio for all circles.

- Because the ratio of a part of the circle to the radius is π/3. Therefore, for the proper calculation of π, a circle of radius R = 3 cm with a side length of π cm (circle 3) will be compared with a circle of circumference 3 cm (circle 1).
- Circle 1 (C = 3 cm) is drawn to calculate the angle subtended by an arc of radius equal to the circumference of a circle with R = 3 cm at the center of the circle (i.e. 1 radian angle).
- Now in this we will divide the circle (i.e. circle 1, 2, 3, 4) in the ratio of the central angle to the square root of 360 degrees i.e. √360.

By the above process, the circumference of all the circles will be divided into 18.973666... Units with respect to the angle √360= 18.973666... Degrees. (As per picture)

Calculation of the PI (π) and radian

Comparative analysis of circle -1 and circle -3

On dividing all the circles in the ratio of √360 degrees

Circumference of circle -1 = 3/√360 cm

=0.158113883…

= √0.025 cm

That is, one unit of central angle in circle 1 i.e. √360 degree angle represents √0.025 cm arc on the circumference of the circle.

Then

For any circle, we have obtained the ratio π/3, one-sixth of its circumference and radius. It also represent the ratio of unit of an angle in center, present by PI arch and arch of radius.

Hence, the angle subtended at the center by the portion of pi in circle 3 is-

60°=3.16227766… unit of √360

$$= \frac{3.16227766...}{3} = 1.05409255\ldots unit$$

Therefore, in any circle, the angle subtended at the center by one sixth of the circumference of the circle will 1.054092255... times greater than the angle subtended by an arch equal to its radius..

Therefore in the complete circle

$1.05409255\ldots \times 6$

= 6.32455532…

Therefore, in any circle, the angle at the center (360 degrees) made by its circumference will be 6.32455532... = √40 times the angle made by the arc equal to its radius.

Hence 1 radian angle = interior angle at the center subtended by an arc equal to the radius

$$= \frac{360}{\sqrt{40}} = 56.9209979\ldots degree$$

$$= \sqrt{3240}$$

Calculation of PI (π)

As a result of the above calculation, we can see that in any circle the arc internalized on the circumference by one unit of the central angle i.e. √360 degree will be √40 times the arc made by the same angle on a circle with circumference equal to the radius of that circle.

Or

Compared to the radius of any circle, its circumference will be √40=6.32455532... times.

Circumference = R x √40

Hence the arc subtended by √360 in circle 3

= √0.025×√40

= 1 cm

Total length of circumference =

√360×1 = √360 =18.973666… cm

Calculation of π

$$6\pi = 18.973666/6$$

π = 3.16227766...

= √10

Calculation of PI (π) in circle – 2

R=1/2

Circumference = R ×√40

= ½ ×√40

= 3.16227766...

=√10

π=√10

Conclusion

In conclusion, the results presented demonstrate a fundamental relationship between a circle's circumference and its radius. For any circle, the ratio of one-sixth of its circumference and its radius R has been presented as π /3, which is a constant ratio for all circles. And the exact value of PI (π) has been presented, which is π=√10 = 3. 16227766…, and radian = √3240= 56.9209979… degrees. These results affirm the enduring significance of π in geometry and trigonometry and highlight its practical applications in various mathematical contexts.

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