

Self-heating of composites under cyclic loading

¹Hadjov K., ²Slavov S., ³Dontchev D.

University of Chemical Technologies and Metallurgy
 dep. Applied Mechanics
 blv. Kl. Ohridski 8, 1756 Sofia, Bulgaria.

Abstract- In this paper the authors on the basis of the general heat equation have obtain a differential equation describing the temperature enhancement (self-heating) by cycling of nonlinear elastoviscous composite materials as a function of imposed strain amplitude and frequency. One assumes homogeneous temperature field. Thermo-elastic and thermo-damage connectivity have been neglected. It is acceptable, due to the small cross section of the samples and the relatively small deformations. Natural and forced convection to describe the heat losses in the surrounding have been take into account. The dissipation energy has been divided into heat loss and damage development energies in order to take into account the damage by cycling loading. Some comparisons with experimental results for glass fiber composite (30% glass fibers) when take into account the heat losses in the surrounding confirm the applicability of the proposed approach. The surface temperature of the samples was measured with laser infrared thermograph.

Key words: heat equation, forced convection, cycling, composites, nonlinear viscoelasticity.

1. INTRODUCTION

The study of the cyclic loading of a nonlinear viscoelastic material involves a hysteresis curve in stress-strain coordinates [1]. Such a curve shows that some of the strain energy is not recovered but dissipated during the cycling. A large part of the mechanical energy when cycling viscoelastic solids is converted into heat [2]. The fraction of viscous work converted into heat has been shown to be strain and stress dependent [1,2]. When no heat is released from the structure, such as under adiabatic conditions, the temperature of the material increases, sometimes by noticeable levels [1]. This effect is evident at high loading rates, it can also be significant when energy is continuously supplied to the structure, as in the case of cyclic loading. Many problems related to cyclic loading at small deformations in the linear case have been solved and discussed in [2,3] using the complex moduli. The problem is defined on a fixed domain (the initial configuration of the body) and thermal diffusion is independent of temperature. Heat losses in the surrounding have been take into account in the case of natural and forced convection. In order to avoid an equilibrium loss we used here as imposed loads - strain pulsations, due to the cross-sectional length ratio of our specimens.

2.CONSTITUTIVE EQUATIONS

Assuming similarity of the isochrones stress relaxation curves, which for fiber reinforced composite materials is acceptable [4], let introduce the following nonlinear integral equation to describe the mechanical behavior of a GFC [4,5]

$$\sigma(t) = \phi(\varepsilon(t)) - \int_0^t R(t, \tau) \phi(\varepsilon(\tau)) d\tau. \quad (1)$$

Here $\sigma(t)$ is the stress as a function of time t , $\varepsilon(t)$ is the imposed strain, $R(t, \tau)$ is the relaxation kernel, which can be found from stress relaxation tests, $\phi(\varepsilon(t))$ is the instantaneous nonlinear stress-strain curve. To well describe this curve one can apply the Ogden relation [6], also valid for larger nonlinear deformations

$$\phi(\varepsilon) = \sum_{i=1}^3 \mu_i (\lambda(\varepsilon)^{\kappa_i - 1} - \lambda(\varepsilon)^{-\frac{\kappa_i}{2} - 1}), \quad (2)$$

here $\mu_1, \mu_2, \mu_3, \kappa_1, \kappa_2, \kappa_3$ are parameters obtained from instantaneous stress-strain test and the stretch λ is related with the engineering strain as follows $\lambda(\varepsilon) = 1 + \varepsilon$.

The solution of Equation 1 can be represented as follows [4,5]

$$\phi(\varepsilon(t)) = \sigma(t) + \int_0^t K(t, \tau) \sigma(\tau) d\tau. \quad (3)$$

To obtain the strain creep curve (nonlinear creep) one should use the inverse function $\psi(\varepsilon(t)) = \phi^{-1}(\varepsilon(t))$. The above-mentioned integral equations of Volterra have been longtime employed to describe the viscoelastic behavior of polymers, glass fiber composites and other materials [4,5,7]. Due to the extremely high strain (stress) rate at the beginning in creep as well as in stress relaxation conditions one needs to introduce singular kernels. In order to increase the creep or stress relaxation time interval and thus well describe the experimental data from the beginning to the end in the case of large time interval, we have proposed in our previous work [8] to involve in the nonlinear hereditary theory a sum of singular kernels as follows

$$R(t) = \sum_{i=1}^N R_i(t), \quad \text{with} \quad R_i(t) = A_i \frac{e^{-\beta_i t}}{t^{\alpha_i}}. \quad (4)$$

In this case the resolving kernel is [4,5]

$$K(t) = \sum_{i=1}^N K_i(t), \quad \text{with} \quad K_i(t) = \frac{e^{-\beta_i t}}{t} \sum_{n=1}^{\infty} A_i \Gamma(\alpha_i)^n t^{\alpha_i n} / \Gamma(\alpha_i n). \quad (5)$$

Note. Practically one need to stop the summation in Equations 4 and 5 after the 3-th member $N = 3$.

3. DIFFERENTIAL EQUATION OF SELFHEATING

The heat equation, in the general case, can be written as follows [1,3,9,10]

$$\rho C_p \frac{\partial T}{\partial t} = \sigma_{ij}^v \dot{\varepsilon}_{ij}^v - A_i \dot{V}_i + k \nabla^2 T + T \left(\frac{\partial \sigma_{ij}}{\partial T} \dot{\varepsilon}_{ij}^e + \frac{\partial A_i}{\partial T} \dot{V}_i \right), \quad [W/m^3], \quad (6)$$

where ρ - the density, C_p - the thermal capacity, T - the temperature, $\sigma_{ij}^v, \varepsilon_{ij}^v$ - the components of the tensor of the inelastic part of the stress and deformation, $A_i \dot{V}_i$ - the non-recoverable energy (internal variables) associated with restructuring and damage of the material, k - coefficient of thermal conductivity, ∇ - nabla operator, $\frac{\partial \sigma_{ij}}{\partial T} \dot{\varepsilon}_{ij}^e + \frac{\partial A_i}{\partial T} \dot{V}_i$ - the energy of the thermomechanical connectivity - the first term expresses the thermo-elastic connectivity, the second the thermo-damage one. This connectivity should be takes into account in the cases of large strain amplitudes (over 5%). In our case the strain amplitude is 0.35%. Thus, this connectivity will be neglected in the future and we obtain the following simplified heat equation:

$$\rho C_p \frac{\partial T}{\partial t} = \sigma_{ij}^v \dot{\varepsilon}_{ij}^v - A_i \dot{V}_i + k \nabla^2 T, \quad [W/m^3]. \quad (7)$$

Important note: During cycling, the inelastic deformation and the corresponding stresses, which for viscoelastic materials are the viscous ones, can be obtained from the full tensors minus their elastic part. Thus, to the viscous strains we have $\varepsilon_{ij}^v = \varepsilon_{ij} - \varepsilon_{ij}^e$ and to the viscous stresses $\sigma_{ij}^v = \sigma_{ij} - \sigma_{ij}^e$. If we have imposed strains $\varepsilon_{ij,imp}^v = \varepsilon_{ij,imp}$ i.e. viscous strains are equal to those applied. If we have imposed stresses, then the viscous stresses are equal to the applied (imposed) stresses $\sigma_{ij,imp}^v = \sigma_{ij,imp}$. Thus, instead to the viscous power per unit volume $\sigma_{ij}^v \dot{\varepsilon}_{ij}^v$, if imposing strains, we should write $\sigma_{ij}^v \dot{\varepsilon}_{ij}$. In the future for brevity to the viscous power per unit volume we will write $\sigma_{ij} \dot{\varepsilon}_{ij}$. Our imposed strains are not very high (see the experimental results bellow) and thus, we exclude plastic deformations.

If the process is adiabatic or if a thin sample (as in our case) is introduced, the temperature distribution in sample space is constant, or in other words, the temperature field is homogeneous. In this case, we obtain an ordinary differential equation describing the heat enhancement

$$\sigma_{ij} \dot{\varepsilon}_{ij} = \rho C_p \dot{T} + A_i \dot{V}_i, \quad [W/m^3]. \quad (8)$$

The damage enhancement due to deformations can be expressed as follows [10]

$$A_i \dot{V}_i = d(t) \sigma_{ij} \dot{\varepsilon}_{ij}, \quad (9)$$

Where $\sigma_{ij}^e \dot{\epsilon}_{ij}^e$ represents the elastic recoverable power and $d(t)$ is the relative damage. The so-called anisotropic damage is assumed here. In this case, the heat Equation takes the form:

$$\sigma_{ij} \dot{\epsilon}_{ij} = \rho C_p \dot{T} + \sigma_{ij} \dot{\epsilon}_{ij} d(t) . \quad (10)$$

This expression can be used in the case of an arbitrary stress and strain states. In order to obtain relations for different stress and strain states we will decompose the stress and strain tensors on deviatoric and spheric (bulk) parts

$$\sigma_{ij} \dot{\epsilon}_{ij} = s_{ij} \dot{e}_{ij} + \sigma_V \dot{\epsilon}_{kk} \delta_{ij} + s_{ij} \dot{e}_{kk} + \sigma_V \dot{e}_{ij} . \quad (11)$$

Here $\sigma_{ij}; \epsilon_{ij}; s_{ij}; e_{ij}; \sigma_V; \epsilon_{kk}$ are respectively the components of the stress and strain tensors, their deviatoric and spheric parts and velocities (with point over). δ_{ij} is the Kronecker symbol.

We will simplify this expression, bearing in mind that elastomers are incompressible, i.e. their volumetric part and respective velocity are equal to zero: $\epsilon_{kk} = 0, \dot{\epsilon}_{kk} = 0$, after that we arrive to

$$\sigma_{ij} \dot{\epsilon}_{ij} = s_{ij} \dot{e}_{ij} + \sigma_V \dot{e}_{ij} . \quad (12)$$

Concerning the one dimensional case of pure traction from Equation 12 we have

$$\sigma_{ij} \dot{\epsilon}_{ij} = \frac{2}{3} \sigma_{11} \dot{\epsilon}_{11} + \sigma_{11} \frac{1}{3} \dot{\epsilon}_{11} = \sigma_{11} \dot{\epsilon}_{11} . \quad (13)$$

Here σ_{11} and ϵ_{11} are the normal stress and the respective strain by traction, which we will henceforth denote as σ and ϵ respectively.

Equation 10 in the one-dimensional case (traction pulsations or sinusoidal loading), taking into account the above *important note*, can be represented as follows:

$$\sigma \dot{\epsilon} = \rho C_p \dot{T} + \sigma \dot{\epsilon} d(t) . \quad (14)$$

4. DAMAGE ACCUMULATION

In Equation 14 we need to obtain the damage accumulation $d(t)$. The number of cycles and time are related as follows:

$$t = 2\pi N / \omega , \quad (15)$$

where ω is the angular frequency of the applied cycling load.

In the case of imposed positive sinusoidal strains (pulsations), we have

$$\epsilon_{imp}(t) = \epsilon_o + \epsilon_o \sin(\omega t - \pi/2) . \quad (16)$$

In this case the stress response can be obtained using Equations 1, 2 and 16.

In Equation 16 ϕ is the phase angle shift, ϵ_o, σ_o are the imposed strain and stress amplitude and ω is the angular frequency related with the imposed period T as follows

$$T = 2\pi / \omega . \quad (17)$$

On the other hand, to the stored energy per cycle in the more general case (nonlinearity) as a function of the strain amplitude ϵ_o and the angular frequency ω in the case of pulsations, we can respectively write [10, 11]

$$U(\epsilon_o) = \int \sigma d\epsilon = \int_{(n-1)T}^{(n-0.5)T} \sigma(t, \epsilon_o) \dot{\epsilon}(t, \epsilon_o) dt = \int_{(n-1)2\pi/\omega}^{(2n-1)\pi/\omega} \sigma(t, \epsilon_o) \dot{\epsilon}(t, \epsilon_o) dt . \quad (18)$$

Here $n = 1, 2 \dots N$, where N is the final cycle number. Here we have used Equation (17).

It is evident that these energies are cycle number dependent. But this dependence is not strong. To derive the lower and upper limits in (18) we have used Equation 17.

To obtain the damage $d(n)$ as a function of the cycle number n we need to define the damage for the n -th cycle as in [11,12,13,14]

$$dn(n) = 1 - \frac{U(n)}{U_{in}} , \quad (19)$$

where $U(n)$ and U_{in} are the stored energies for the n -th cycle and the initial one. Then the relative damage accumulation per cycle is $d(n) = [d(n+1)-d(n)]/din$. After summation of the relative damage accumulation per cycle we obtain the damage as a function of the cycle numbers

$$d(N) = \sum_{n=1}^N dn(n) . \quad (20)$$

To obtain the damage enhancement $d(t)$ as a function of time in Equation 14, we need to take into account Equations (19,20) and transform the cycle numbers in time unities. To do this we should inverse Equation 15 and express the cycle number as

$$N = t \omega / 2\pi , \quad (21)$$

5. HEAT LOSSES IN THE SURROUNDING

Concerning the heat losses in the surrounding we need to distinguish two cases. Natural convection – the surrounding fluid (mostly air) remains stationary relative to the sample and forced convection – flow around the sample with constant velocity.

5.1. NATURAL CONVECTION

Inserting the damage growth according to Equations 19 and 20 into the heat Equation 14, to the heat generation by cycling we obtain an ordinary differential Equation 21 which does not account the energy losses to the surroundings. In order to take into account these losses we will use the Newton's cooling equation. In the one-dimensional case we have [9,10]

$$k \frac{dT}{dx} + h(T - T_{\infty}) = 0, \quad \text{for } x = 0 \quad \text{and} \quad x = \delta, \quad (22)$$

where T_{∞} is the ambient temperature coincident with the initial one T_0 ; k, h are the coefficients of thermal conductivity and heat transfer respectively; x is the coordinate of an arbitrary point measured on the left side of the specimen and δ is the thickness of the specimen. In the case of a thin specimen or an adiabatic process, the right-hand side of Equation (15) divided by the specimen height H (to obtain the energy lost per unit volume) must be added to the heat Equation (9). In this way, we arrive at the ordinary differential Equation, taking into account the losses in the surroundings

$$\sigma \dot{\varepsilon} = \rho C_p \dot{T} + \sigma \dot{\varepsilon} d(t) + \frac{h}{H} (T - T_0) \quad . \quad [W/m^3] \quad (23)$$

This equation concerns the so called natural convection.

5.1 FORCED CONVECTION

We will note that the transfer of heat to the environment by means of Newton's equation represents the so-called natural convection. For this we will need to find the heat transfer coefficient in the case of forced convection. We will follow the derivations in [15], where the expression for the heat transfer coefficient is given

$$h_f = \frac{k}{H} Nu. \quad (24)$$

In this equation k is the coefficient of the thermal conductivity [$W/m \text{ degK}$], H – the length (high) of the specimen [m], Nu – the Nusselt number – without dimension. To this number we have

$$Nu = 0.102 Re^{0.675} Pr^{1/3}, \quad (25)$$

where Re is the Reynolds number, Pr - the Prandtl's one. Both are dimensionless. To the Reynolds number we have

$$Re = \frac{vH}{\nu} \quad (26)$$

In this Equation 26 ν – is the cinematic viscosity [m^2/s], v is the velocity of the surrounding fluid [m/s] (ordinary the air).

Using this way from Equations 24, 25, 26 to the coefficient of the heat transfer coefficient we arrive to

$$h_f = \frac{k}{H} [0.102 (\frac{vH}{\nu})^{0.675} Pr^{1/3}], [W/m^2 deg K] \quad (27)$$

If we replace in Equation (23) the coefficient of the natural convection with this of the forced one, using Equation (27) we will obtain a differential equation taking into account this effect

$$\dot{T}(t) + \frac{h_f}{H\rho c_p} T(t) = \frac{1-d(t)}{\rho c_p} \sigma \dot{\varepsilon} + \frac{h_f}{H\rho c_p} T_o \quad (28)$$

To the viscous stress response, according to the *important note* make above, we can write $\sigma(t) = \int_0^t R(t, \tau) \phi(\varepsilon(\tau)) d\tau$. This expression should be introduced in Equation 28, respectively in Equations 21,23. Note that these particular cases can be obtained from Equation 28 putting $d(t) = 0$ (no damage), $h_f = h$ (natural convection) and $h_f = 0$ (no losses in the surrounding).

6. EXPERIMENTAL RESULTS AND COMPARISONS

For this purpose, we will apply the solutions according to the differential equations of self-heating (Equations 21,23,28) for GFC with 30% glass fibers. The computational procedures in the Mathcad software environment require joint solution of the equations in the following order:

1. Determination of the nonlinearity parameters in the Ogden Equation 2
2. Determination of the relaxation kernel parameters in the nonlinear hereditary Equation 1 with relaxation kernel consisting of sum of singular kernels - Equations 4
3. Determination of the damage parameters - Equations 19,20
4. Solution of the differential equations of self-heating according to Equations 21,23,28 concerning three characteristic cases, namely without taking into account the damage and heat losses in the surrounding, taking into account the damage and natural convection and finally taking into account damage and forced convection (heat losses due to air blowing of the sample with constant air velocity).

In the case of imposed strains (controlled deformation) we should replace ε with the imposed strain law $\varepsilon_{imp}(t)$ – see Equation 16.

For our FGC with 30% glass fibers, the following experimental results were obtained:
Parameters of the Ogden equation:

$$\mu_1 = 9.93; \quad \mu_2 = -39.9; \quad \mu_3 = 1.2; \quad k_1 = 29.95; \quad k_2 = 59.81; \quad k_3 = 5.4.$$

Parameters of the relaxations kernel

$$A_1 = 0.0001; \quad a_1 = 0.8; \quad b_1 = 0.01; \quad A_2 = 0.015; \quad a_2 = 0.94; \quad b_2 = 0.31; \quad A_3 = 0.013; \quad a_3 = 0.2; \quad b_3 = 0.01.$$

The relative stress relaxation curve according to these parameters are illustrated in our previous work in the same journal [17].

Figure 1 below illustrates the increase in damage with increasing time on the basis of Equations 19,20.

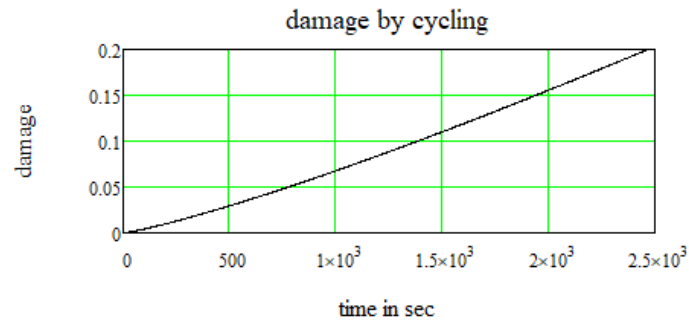


Fig.1. GFC. Growth of damage with time.

The following Figure 2 shows the imposed strain law (pulsations) and the stress responses. Comparison between elastic response (without taking into account the composite viscosity) and elastoviscous stress response are made.

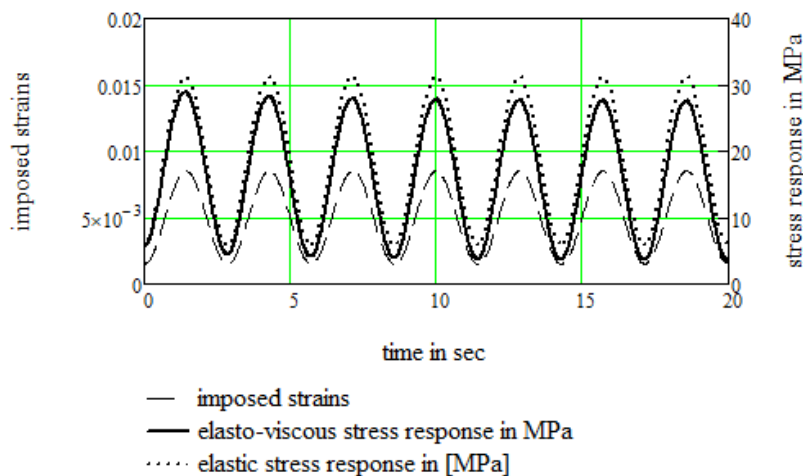


Fig.2. FGC. Imposed strain and stress responses

In the next Figure 3 we have illustrated the final result, namely the temperature rise due to self-heating in three specific cases. The influence of the damage enhancement and heat losses in the surrounding in the case of natural convection can be observed after 1000 seconds (17 minutes). In the case of forced convection the same effect can be observed after 300 seconds (5 minutes).

The imposed strain amplitude was 0.0035 and the imposed angular frequency was 2.2 [rad/s]. For our GFC the density and the thermal capacity were respectively $\rho = 1150$ [kg/m³], $C_p = 998$ [J.kg/deg K] .

The air heat transfer coefficient in the case of natural convection is $h = 6.35$ [W/m² deg K], see the results in the detailed book [15]. In the case of forced convection $hf = 104$ [W/m² deg K] . The latter was calculated using Equation 27, introducing the following parameters: surrounding air velocity $v = 1$ [m/s], cinematic viscosity of the air $\nu = 1.5 \times 10^{-5}$ [m²/s], coefficient of the thermal conductivity $k = 0.3$ [W/m degK], air Prandtl number 0.71 as in [15].

The surface temperature of the samples was measured with laser infrared thermograph [16].

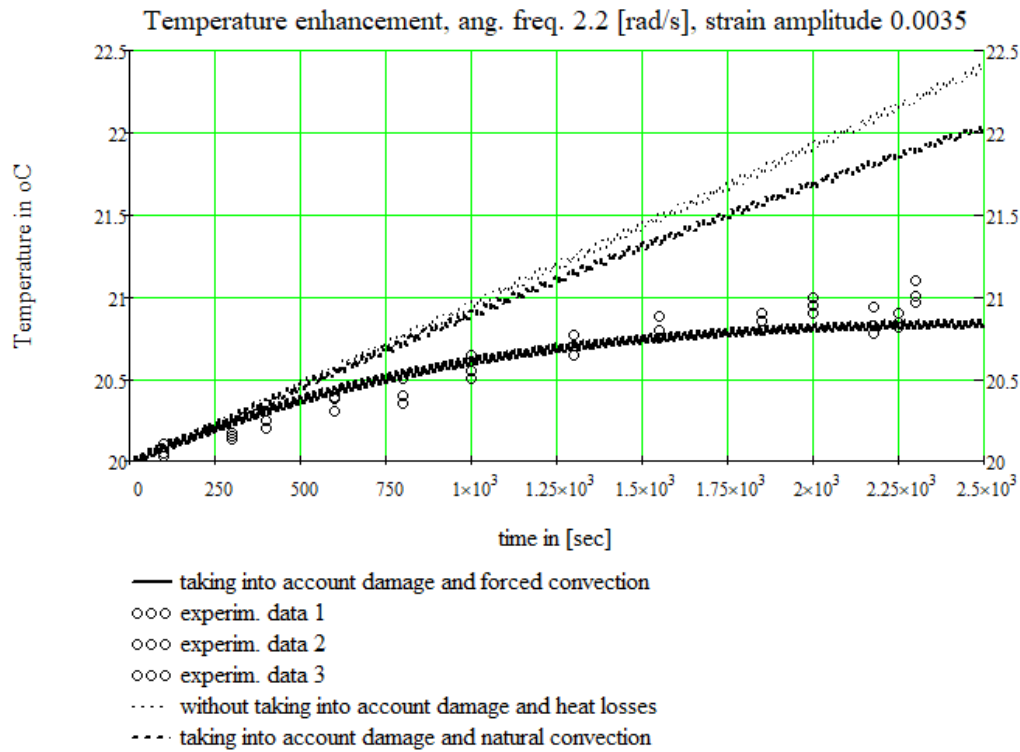


Fig.3 Temperature enhancement with time by cycling. Thin line - without considering damage and heat losses. Intermediate line - considering damage and natural convection. Thick line - considering damage and forced convection

6. CONCLUSIONS

On the basis of the generalized heat equation, after accepting some simplifying assumptions, a differential equation of self-heating during cycling of elastoviscous composites in the nonlinear domain is obtained and resolved. This equation takes into account progressive damage, nonlinear constitutive behavior and heat losses in the surrounding in the cases of natural and forced convection.

Comparison between the above mentioned three cases (no losses in the surrounding, natural convection and forced one) is made. It was established that in the case of forced convection, even with a low blowing air speed around the experimental samples, the self-heating temperature increases significantly more slowly and reaches an equilibrium value much earlier as in the other two cases.

In order to clearly notice the difference between natural convection and no heat transfer to the environment, a significantly longer time is required (comparing with forced convection).

Forced convection can significantly decrease the temperature of self heating of elastoviscous composite materials by cycling loading.

Viscosity has very important impact on the self-heating of composite materials.

Experimental results for glass fiber composite with 30% glass fibers illustrate the applicability of this approach.

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