

Effect of Antenna Diversity and Correlation on Double Scattering MIMO Network

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Abstract- In this paper, a practical scenario of multicast cellular network has been considered to investigate the effect of correlation on scattering network. A source transmits confidential information to its intended multicast users and multiple eavesdroppers try to decode this information. Authors are interested to develop a mathematical model consists of the closed-form analytical expression for the secrecy multicast capacity (SMC) to protect that confidential information from eavesdropping. The results show that the simultaneous effects of correlation significantly reduce the secure performance in multicasting.

Index Terms: Multicasting, Double Scattering, Secrecy Multicast Capacity, Correlation.

I. INTRODUCTION

Advantages of Multiple-antenna systems are impacted by the channel's rank deficit and spatial fading correlation. This study examines, in terms of ESC, how these degradations affect the diversity performance of multiple-input multiple-output (MIMO) systems. Specifically, we take into account a double-scattering channel that includes a range of propagation settings with both spatial correlation effects and rank deficiency. This paper's goal is to examine how double scattering affects MIMO systems' diversity performance in a double scattering scenario.

1.1 Related Work

The channel's rank insufficiency from double scattering or the keyhole effect, as well as the spatial fading correlation brought on by insufficient separation between antenna elements, could restrict the potential benefits of multiple-antenna systems [1]. In [2], the authors theoretically demonstrated that the rank deficiency of the channel correlation matrix deteriorates SE and that the channel hardening condition does not hold for keyhole channels. In [3], the double scattering model was used to study the ergodic rate performance of a multi-user multiple-input single-output (MISO) system. It was discovered that the number of scatterers limits the system performance parameters. Two techniques were suggested by the authors in [4] to address the problem of congestion in the Cognitive Radio Network and examine the impact of the double scattering channel. Considering MIMO multiple access channel (MAC) with doubly-scattering channel model, authors in [5] derived deterministic approximations of the mutual information, the signal-to-noise-plus-interference ratio (SINR) at the output of the minimum-mean square-error (MMSE) detector and the sum-rate with MMSE detection. The spectral efficiency (SE) was derived in [6] considering multi cell Massive MIMO network in order to analyze the key behaviors of the double scatter. In [7] the combined effect of rank deficiency as well as spatial fading correlation on the diversity performance of MIMO system was investigated. To represent spatial correlations at the transmitter and receiver, the Kronecker channel model is widely used when rank deficiency occurs in poor scattering conditions [8].

1.2 Contribution

In all the studies considering the double scattering channel stated in above, the MIMO network has been proposed only with or without antenna correlations for point-to-point communication systems. In spite of great importance of multicasting in MIMO network, no attention has given at all on the security of multi-antenna system incorporating the effect of double scattering channel. Moreover, so far to authors' knowledge, there are no works in the literature yet on this issue. Therefore, motivated by these perspectives, for the first time, we address the problem of secure wireless multicasting in double scattering network.

2. SYSTEM MODEL AND PROBLEM FORMULATION

A double scattering MIMO multicast scenario has been considered, in which base station (BS) communicates with M users in the presence of K eavesdroppers. All the terminals are equipped with multiple antennas. BS is equipped with

n_T antennas and each user has n_D antennas. Each eavesdroppers are equipped with n_E antennas. Let the received signals at the i th user, Y_{m_i} and at the k th eavesdropper, Y_{E_k} are given as

$$Y_{m_i} = \mathbf{H}_i \mathbf{x} + Z_{m_i} \quad (1)$$

$$Y_{E_k} = \mathbf{G}_k \mathbf{x} + Z_{E_k} \quad (2)$$

Where, \mathbf{x} is the transmitted signal vector of order $n_T \times 1$. Z_{m_i} and Z_{E_k} are the Gaussian noises at the i th user and at the k th eavesdropper respectively. In double scattering MIMO channels, the channel matrices \mathbf{H}_i and \mathbf{G}_k can be written as [9],

$$H_i = \frac{1}{\sqrt{s_D}} \mu_D^{\frac{1}{2}} H_{1,i} \mu_S^{\frac{1}{2}} H_{2,i} \mu_T^{\frac{1}{2}} \quad (3)$$

$$G_k = \frac{1}{\sqrt{s_E}} \rho_E^{\frac{1}{2}} G_{1,k} \rho_S^{\frac{1}{2}} G_{2,k} \mu_T^{\frac{1}{2}} \quad (4)$$

Where s_D is the number of scatters on both BS and user side and s_E is the number of scatters on both BS and eavesdropper side. μ_D , μ_S and μ_T are $n_D \times n_D$ receive, $n_T \times n_T$ transmit and $s_D \times s_D$ scatter correlation matrices with all diagonal entries 1, respectively. ρ_E , ρ_S are $n_E \times n_E$ eavesdropper and $s_E \times s_E$ scatter correlation matrices on eavesdropper side with all diagonal entries 1, respectively. The effect of spatial fading correlation on MIMO channels can be analyzed by controlling the number of scatters (s_D) and correlation matrices (μ_D, μ_S and μ_T).

For analyzing the effect of double scattering MIMO channel on the considered model the analytical expressions of the performance parameter (i.e. secrecy multicast capacity) is to be developed. The secrecy multicast capacity in nats per symbol at the i th MU is bound as

$$\langle C_S \rangle \leq E \left[\ln \left\{ \frac{\det \left(\mathbf{I}_{m_1} + \min_{1 \leq i \leq M} \frac{\gamma_i}{n_T} \mathbf{H}_i \mathbf{H}_i^+ \right)}{\det \left(\mathbf{I}_{m_2} + \max_{1 \leq j \leq K} \frac{\gamma_K}{n_T} \mathbf{G}_k \mathbf{G}_k^+ \right)} \right\} \right]$$

Where, γ_i and γ_k are the SNR at the i th user and j th eavesdropper respectively. $m_1 = \min(n_T, n_D, s_M)$ and $m_2 = \min(n_T, n_E, s_E)$. From the definition of determinant [11], it is found that

$$E \left[\det \left(\mathbf{S}_{j_1, \dots, j_k}^{i_1, \dots, i_k} \right) \det \left(\mathbf{S}_{v_1, \dots, v_k}^{u_1, \dots, u_k} \right) \right] = \begin{cases} k! & i_1 = u_1, j_1 = v_1 \\ 0 & \text{otherwise} \end{cases} \quad (5)$$

Using the theorem of [18]

$$E[\det(\mathbf{I}_m + \alpha \mathbf{S})] = \sum_{k=0}^m \left[\zeta^k k! \sum_{1 \leq i_1 \dots \leq i_k \leq m} \det \left(\sum_{l_1 \dots l_k}^{i_1 \dots i_k} \right) \sum_{1 \leq u_1 \dots \leq u_k \leq n} \det(\psi_{u_1 \dots u_k}^{u_1 \dots u_k}) \right] \quad (6)$$

Where α is an arbitrary positive real-valued constant. Applying Jensen's inequality

$$E \left[\det \left(\mathbf{I}_{m_1} + \min_{1 \leq i \leq M} \frac{\gamma_i}{n_T} \mathbf{H}_i \mathbf{H}_i^+ \right) \right] = \sum_{a=0}^{m_1} \min_{1 \leq i \leq M} \left\{ \left(\frac{\gamma_i}{s_D n_T} \right)^a (a!)^2 \sum_{1 \leq i_1 \leq \dots \leq i_a \leq n_T} \det \left(\mu_T^{i_1 \dots i_a} \right) \sum_{1 \leq i_1 \leq \dots \leq i_a \leq n_T} \det \left(\mu_D^{j_1 \dots j_a} \right) \sum_{1 \leq x_1 \leq \dots \leq x_a \leq s_D} \det \left(\mu_{S_{u_1 \dots u_a}}^{u_1 \dots u_a} \right) \right\} \quad (7)$$

In the similar way,

$$E \left[\det \left(\mathbf{I}_{m_2} + \max_{1 \leq j \leq K} \frac{\gamma_K}{n_T} \mathbf{G}_k \mathbf{G}_k^+ \right) \right] = \sum_{b=0}^{m_2} \max_{1 \leq j \leq K} \left\{ \left(\frac{\gamma_K}{s_E n_T} \right)^a (b!)^2 \sum_{1 \leq x_1 \leq \dots \leq x_b \leq n_T} \det \left(\mu_T^{x_1 \dots x_b} \right) \sum_{1 \leq y_1 \leq \dots \leq y_b \leq n_E} \det \left(\rho_E^{y_1 \dots y_b} \right) \sum_{1 \leq z_1 \leq \dots \leq z_b \leq s_E} \det \left(\rho_{S_{z_1 \dots z_b}}^{z_1 \dots z_b} \right) \right\} \quad (8)$$

Hence, the SMC at the users is expressed as

$$\langle C_S \rangle \leq \ln \left[\frac{\sum_{a=0}^{m_1} \min_{1 \leq i \leq M} \left\{ \left(\frac{\gamma_i}{s_D n_T} \right)^a (a!)^2 \sum_{1 \leq i_1 \leq \dots \leq i_a \leq n_T} \det \left(\mu_T^{i_1 \dots i_a} \right) \sum_{1 \leq i_1 \leq \dots \leq i_a \leq n_T} \det \left(\mu_D^{j_1 \dots j_a} \right) \sum_{1 \leq x_1 \leq \dots \leq x_a \leq s_D} \det \left(\mu_{S_{u_1 \dots u_a}}^{u_1 \dots u_a} \right) \right\}}{\sum_{b=0}^{m_2} \max_{1 \leq j \leq K} \left\{ \left(\frac{\gamma_K}{s_E n_T} \right)^a (b!)^2 \sum_{1 \leq x_1 \leq \dots \leq x_b \leq n_T} \det \left(\mu_T^{x_1 \dots x_b} \right) \sum_{1 \leq y_1 \leq \dots \leq y_b \leq n_E} \det \left(\rho_E^{y_1 \dots y_b} \right) \sum_{1 \leq z_1 \leq \dots \leq z_b \leq s_E} \det \left(\rho_{S_{z_1 \dots z_b}}^{z_1 \dots z_b} \right) \right\}} \right] \quad (9)$$

For constant correlation determinant is given by

$$\det \{ \sigma_n(\partial)_{e_1, \dots, e_l}^{e_1, \dots, e_l} \} = (1 - \partial)^{l-1} (1 - \partial + l\partial) \quad (10)$$

Now, considering $\mu_T = \beta_{n_T}^{cons}(\Omega_T)$, $\mu_D = \beta_{n_D}^{cons}(\Omega_D)$, $\mu_S = \beta_{s_D}^{cons}(\Omega_S)$, $\rho_S = \beta_{s_E}^{cons}(\theta_S)$ and $\rho_E = \beta_{n_E}^{cons}(\theta_E)$ equation (9) can be expressed as

$$\langle C_S \rangle \leq \ln \left[\frac{\sum_{a=0}^{m_1} \min_{1 \leq i \leq M} \left\{ \left(\frac{\gamma_i}{s_D n_T} \right)^a (a!)^2 \binom{n_T}{a} \binom{n_D}{a} \binom{s_D}{a} (1 - \Omega_T)^{a-1} (1 - \Omega_D)^{a-1} (1 - \Omega_S)^{a-1} (1 - \Omega_T + a\Omega_T) (1 - \Omega_D + a\Omega_D) (1 - \Omega_S + a\Omega_S) \right\}}{\sum_{b=0}^{m_2} \max_{1 \leq j \leq K} \left\{ \left(\frac{\gamma_K}{s_E n_T} \right)^a (b!)^2 \binom{n_T}{b} \binom{n_E}{b} \binom{s_E}{b} (1 - \Omega_T)^{b-1} (1 - \theta_E)^{b-1} (1 - \theta_S)^{b-1} (1 - \Omega_T + b\Omega_T) (1 - \theta_E + b\theta_E) (1 - \theta_S + b\theta_S) \right\}} \right] \quad (11)$$

3. NUMERICAL RESULTS

The numerical results from the closed-form analytical expressions given in Sections 2 are shown in this section. We have stated before that our proposed model is a newer one in the literature, and our derived expressions are novel adopting double scattering scenario in the study of security performance MIMO multicast network. For the simulation purpose, a correlated Rayleigh fading channel is generated using MATLAB code. The channel is then used to find the security parameters such as SMC.

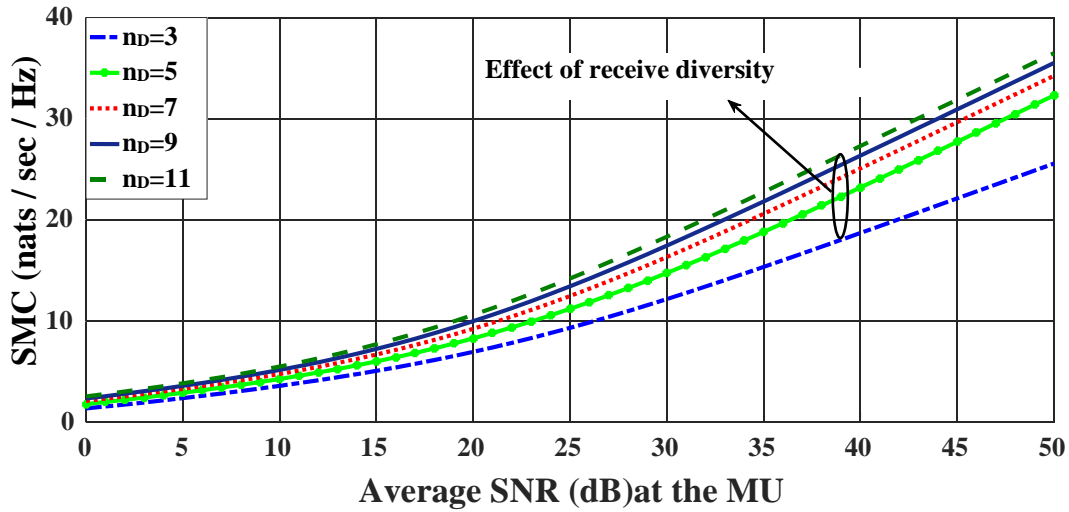


Figure 1: Effect of changing number of user antenna

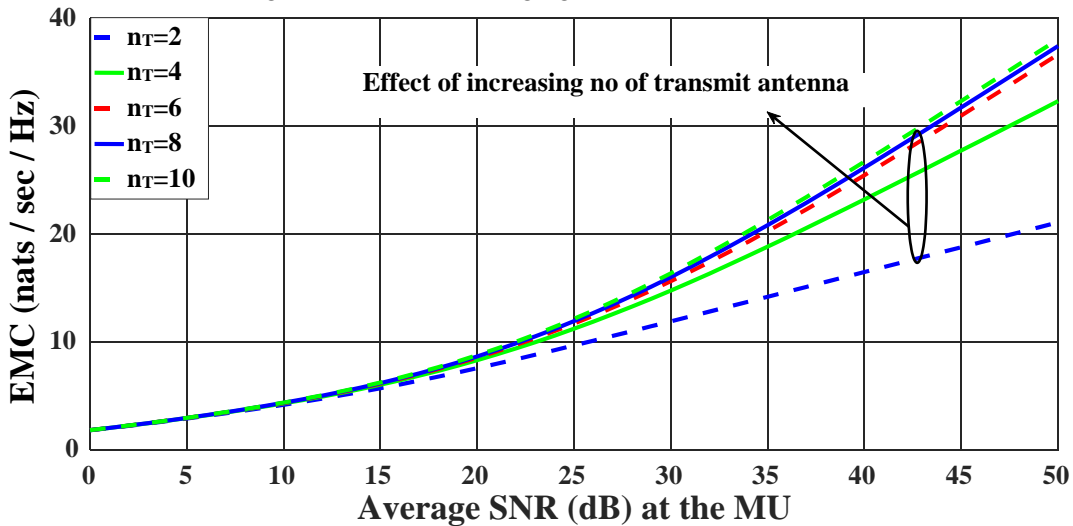


Figure 2: Effect of changing number of transmit antenna

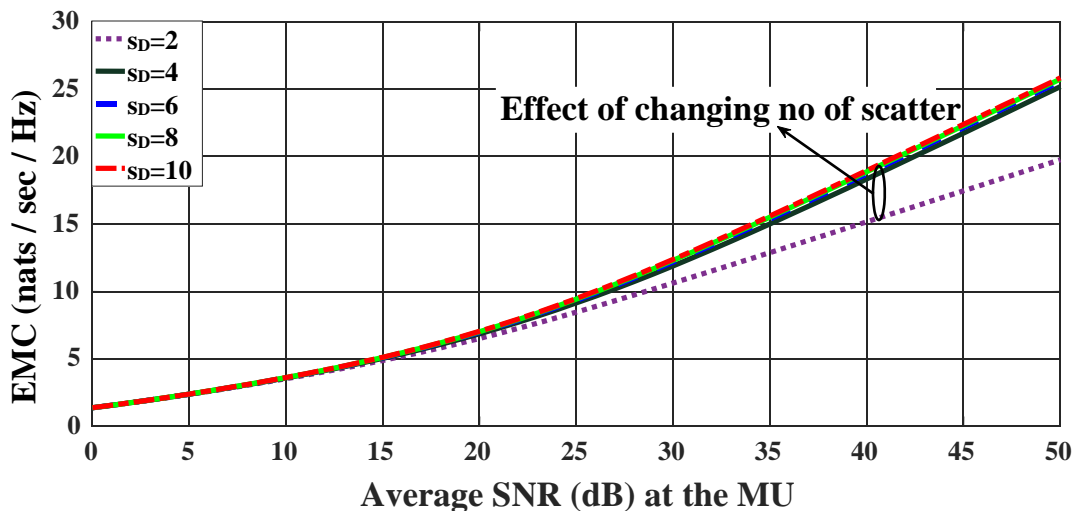


Figure 3: Effect of changing number of scatterer between base station and users

SMC vs average SNR at users is shown in Figure 1, where all multicast channels are taken into account as doubly scattered and consistently correlated. The effects of varying the number of users' antennas on the SMC are observed as a function of the users' average SNR (in dB). This figure illustrates how the rank insufficiency of the channel affects the behavior of MIMO capacity. Because of correlation and the double scattering effect, it is seen that the asymptotic slopes of the SMC curves for $n_D = 5$ are lower than those for $n_D = 3$. However, because of the diversity impact of utilizing, the asymptotic slopes of the SMC curves increase as soon as the number of receive antenna equals or exceeds $\min(n_T, n_D) = 6$. A similar situation can be observed in the case of increasing number of transmitting antennas (Figure 2) and number of scatter (Figure 3).

4. CONCLUSIONS

In this study, we primarily concentrate on creating a mathematical model to assess multicast network security while taking diversity and correlation into account. Therefore, we developed a mathematical model of a MIMO multicast scheme taking into consideration of a base station, multiple receivers and multiple eavesdroppers. The case of constant correlation has been taken into consideration when deriving the generalized closed-form expression for the MSC. Based on the analysis of the results section, it can be inferred that the effect of double scattering channel and antenna correlation parameter for correlated double scattering correlated channel causes channel capacity to decrease as the number of transmit and receive antennas increases until the number of transmit and receive antennas does not exceed the number of scatterers. However, the diversity effect alone is responsible for the increase in channel capacity after the number of transmit and receive antennas is equal to or larger than the number of scatterers.

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